# Introductory Course: Using LS-OPT® on the TRACC Cluster

## 2.5a - Probabilistic Analysis

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# What is Different?

Variable

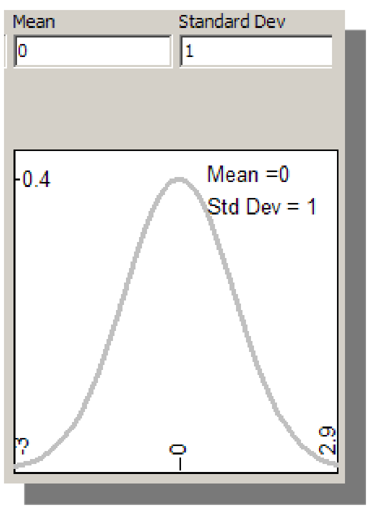
Deterministic Analysis

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=\{F(t)\}$$

***One result***

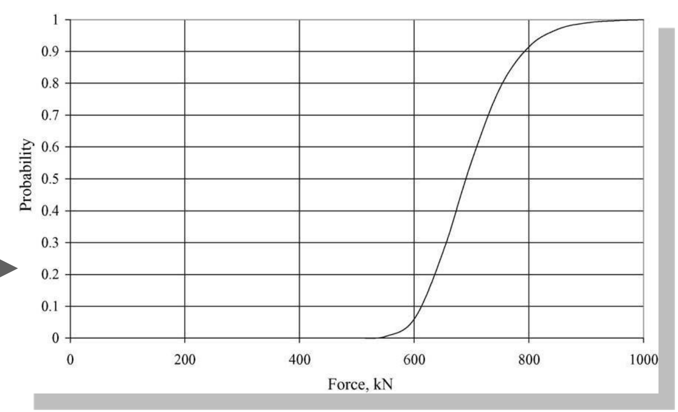
Parameters

Probabilistic Analysis




Random Variables
Input Parameters
Design Variables

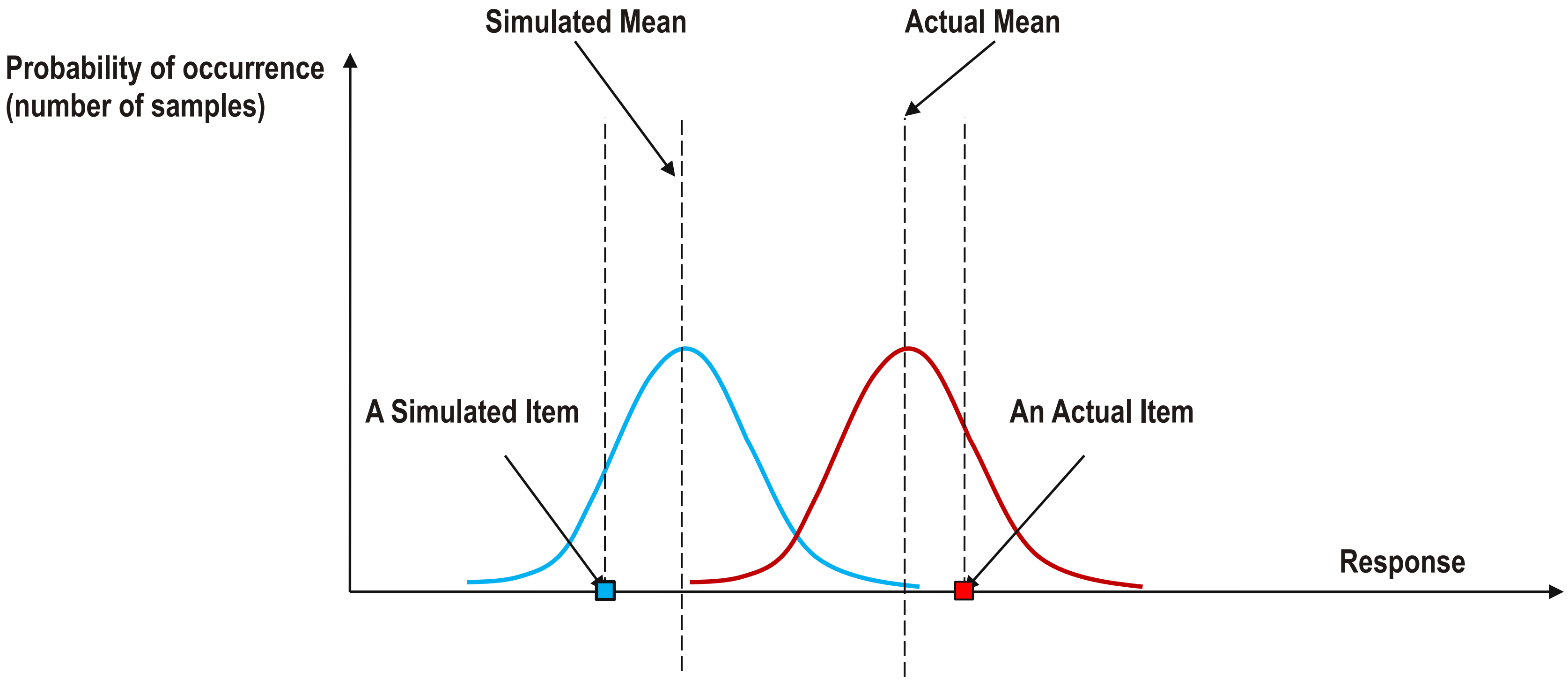
$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=\{F(t)\}$$



# Introduction

- Deterministic Analysis:  one set of input parameters>>>>>one result
- Probabilistic Analysis:  multiple inputs based on parameter ranges>>>>>range of response
- No two structural events will be exactly similar, nor will structural event occur exactly as designed or analyzed.
- Adverse combinations of design and loading variation may lead to undesirable behavior or failure.
- Therefore if significant variation is expected – a probabilistic evaluation is required.
- Oftentimes, it is wise to do a probabilistic analysis to get a warm feeling about deterministic results.
- Differences in structural performances can be attributed to deterministic and random effects. Understanding their sources is crucial for successful analysis, yet it is very challenging.

# Simulation and Experimental Results



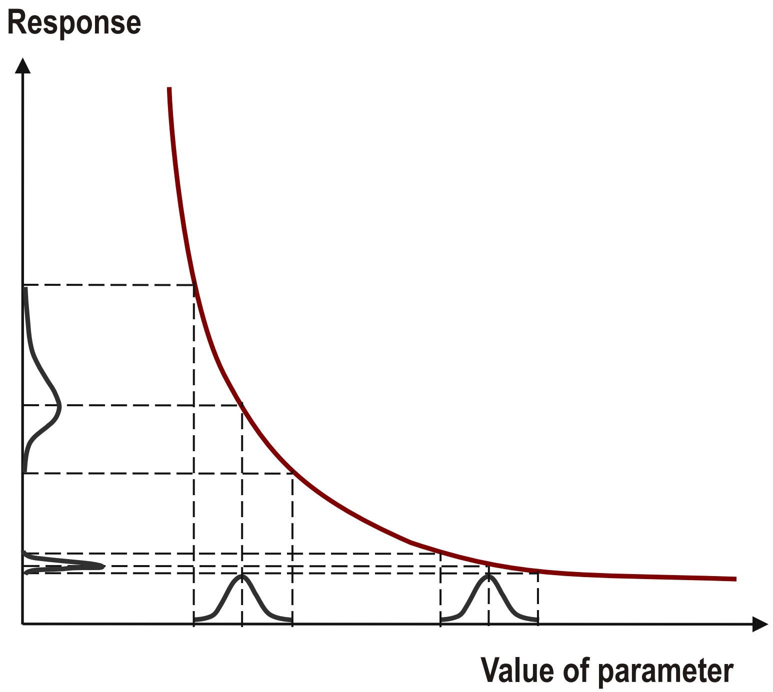

- Many times a single simulation is compared to a single experiment, and, if the results are different, the analysts isn't sure if his simulation is reasonable.
- Insight can be gained if a probabilistic analysis is performed.
- Increasing the number of experiments is also very useful.

# Response Variation

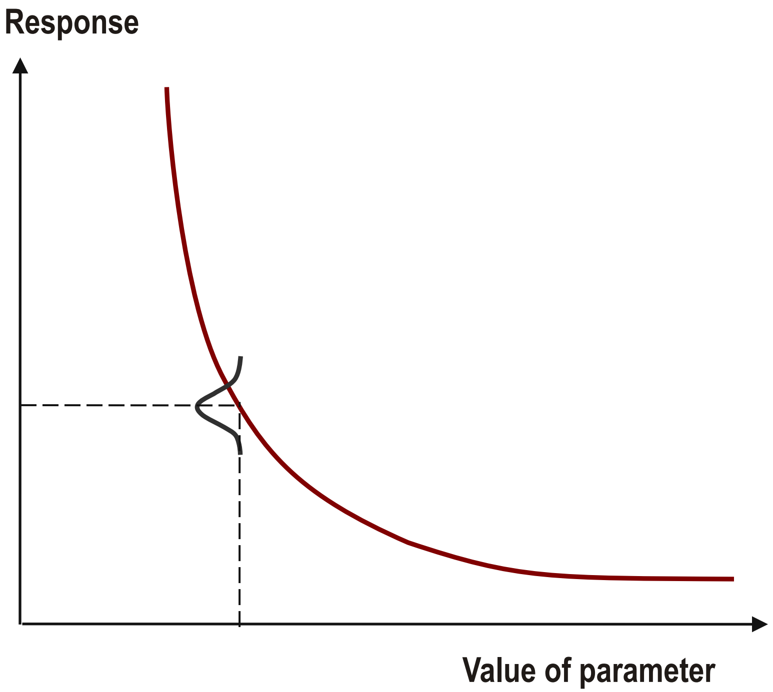
- Deterministic variation
  - Expected, predictable and repeatable variation in a response associated with a variation in a parameter.
  - Can be controllable or uncontrollable
- Random variation
  - Variation that cannot be associated with a change in system parameters
    - Regular random variation – not associated with the physics of the system
    - Chaotic random variation – noise caused by bifurcation behavior in the structure

# Response Variation

- Deterministic variation



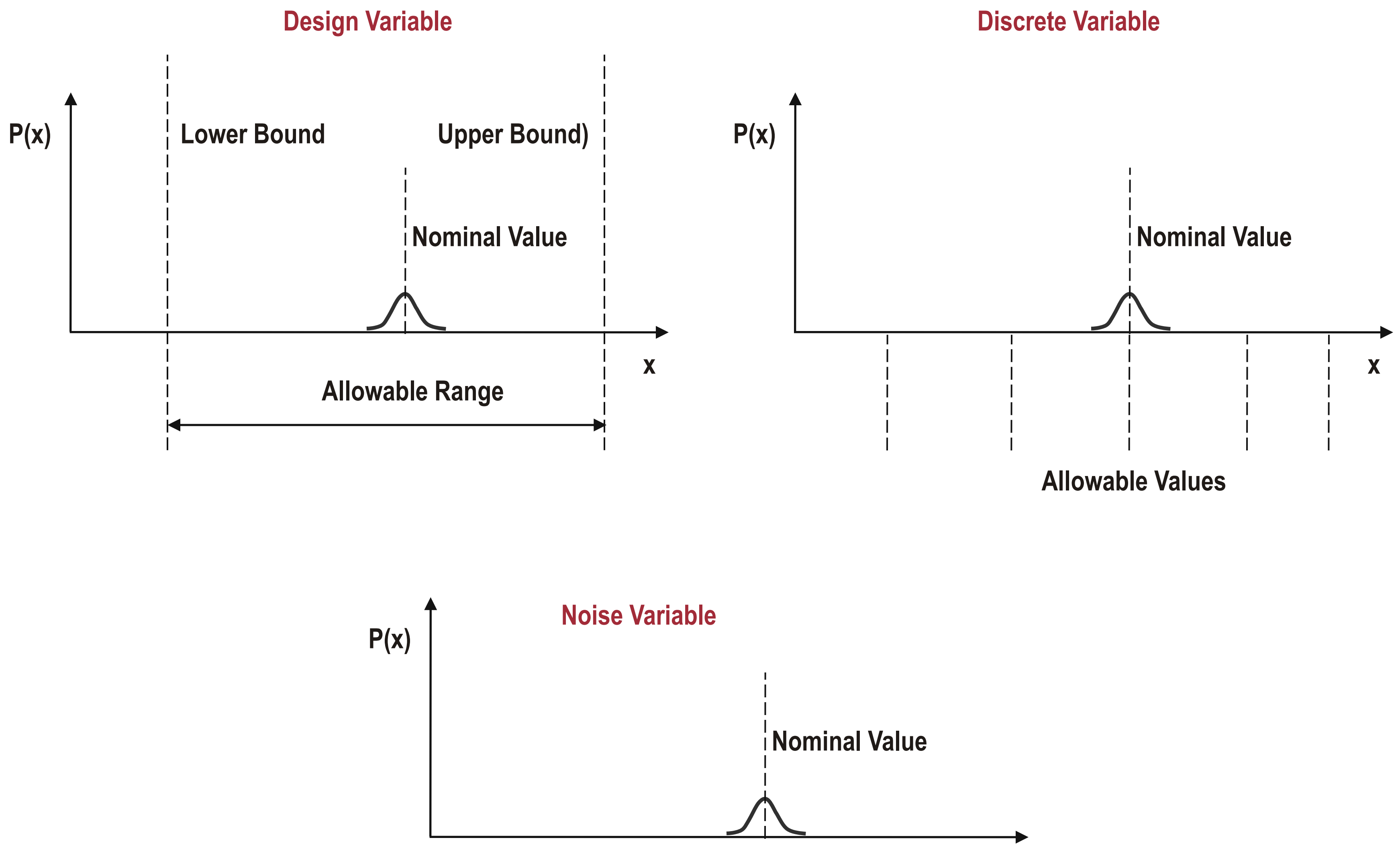

- Random variation (buckling, contact etc.)



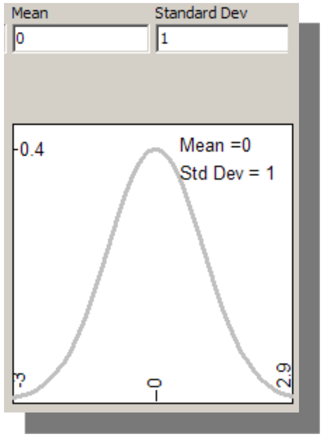
# Sources of Variation

- Design Parameter Variation
  - Control variables
    - can be controlled in the design, analysis and production level.
    - it can be assigned a nominal value and will have variation around it.
    - example: shell thickness
  - Noise variables
    - are difficult or impossible to control at the design and/or production level.
    - but can be controlled on the analysis level.
    - will  have a nominal value and will follow exactly a distribution.
    - example: variations in loads and material properties (Young’s modulus, yield stress, failure, strain, etc.)

# Sources of Variation



# Distributions

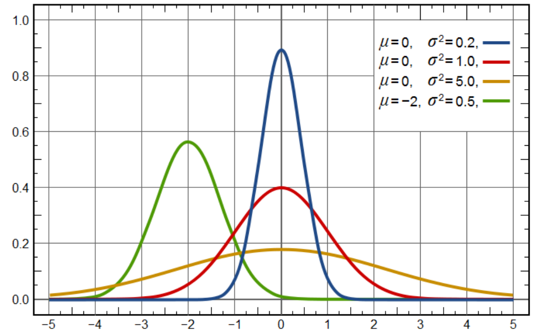


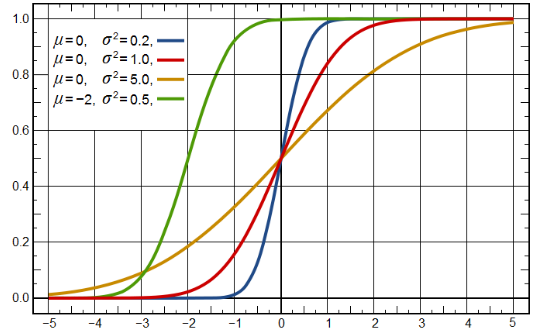

- Property of a Probability density function (PDF):

$$\int_{-\infty}^{+\infty} f(x)dx = 1 \qquad f(x) \geq 0$$

- Cumulative Distribution function (CDF) of the random variable:

$$F(x) = \int_{-\infty}^{x} f(t)dt$$




- Mean of a probability density function:

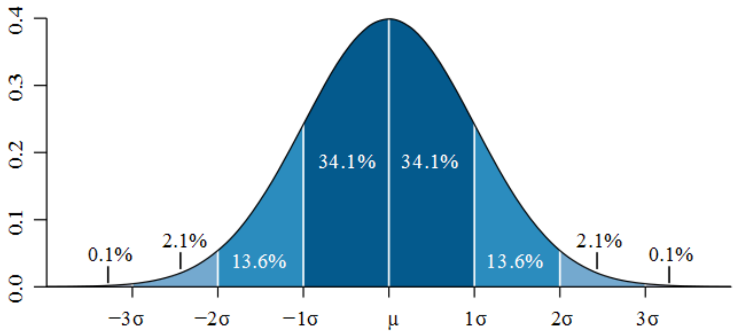
$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mu = \int_{-\infty}^{+\infty} x \cdot f(x)dx$$

# Distributions

- Variance - the second moment about the mean:

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu)^2 \qquad \sigma^2 = \int_{-\infty}^{+\infty}(x-\mu)^2 \cdot f(x)dx$$

Area under PDF




- Standard Deviation

$$\sigma = \sqrt{\sigma^2}$$

$$\sigma \Rightarrow 68\%$$
$$2\sigma \Rightarrow 96\%$$
$$3\sigma \Rightarrow 99.7\%$$

- Coefficient of Variation, COV, indicates amount of uncertainty

$$COV = \frac{\sigma}{\mu}$$

In engineering problems, a COV less than 0.3 is common for a random variable

# Correlation of Responses

- The covariance of two variables indicates whether a change in the one is associated with a change in the other:

$$Cov(y_1, y_2) = E[(y_1 - \mu_1)(y_2 - \mu_2)]$$

- Coefficient of correlation is used instead as scaled quantity (always between -1 and 1)

$$\rho = \frac{Cov(y_1, y_2)}{\sigma_1 \sigma_2}$$

# Random Variables, Distributions, Means, Standard Deviations, Coefficient of Variations

| Random Variable | Distribution | Mean | Standard Deviation | COV |
|---|---|---|---|---|
| Young's modulus: Golden Gate Bridge | Normal<br>Lognormal | 29576 ksi | 1,507 | 0.051 |
| Young's modulus:  C4 Posterior | Lognormal | 3500 | 735 | 0.21 |
| Young's modulus:  C45 annulus | Lognormal | 4.7 | 0.705 | 0.15 |
| Thickness:  Strap in bonded joint | Normal | 0.125 | 0.005 | 0.04 |
| Radius: Containment vessel | Normal | 37.0 in | 0.0521 | 0.00141 |
| Yield Stress: Containment vessel | Normal | 106e3 psi | 4e3 | 0.03774 |
| Mass:  Missile | Lognormal | 182 kg | 9.1 | 0.05 |
| Density:  Concrete | Lognormal | 2000 $kg/m^3$ | 200 | 0.10 |
| Depth: Target | Normal | 1.5 m | 0.075 | 0.05 |

# Choosing a Distribution

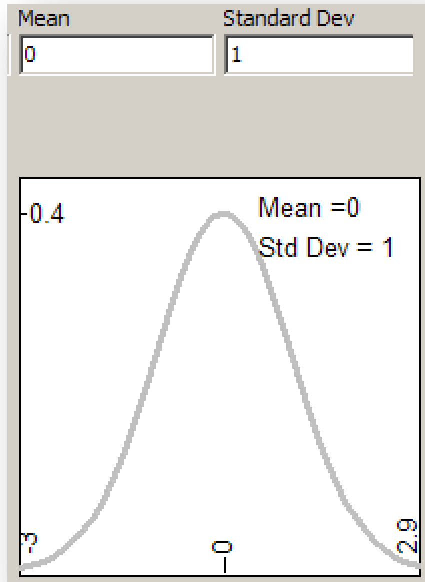
- The analysts must specify the PDFs for all the random variables

- PDFs can be obtained from:
  - Experiments
  - Manufacturing data
  - Literature

- If a PDF is not available, then engineering judgment must be used:
  - Normal, Lognormal, Uniform, etc.
  - Mean value
  - Coefficient of variation (less than 0.3)

# Distributions in LS-OPT

- **Normal**
- Uniform
- Lognormal
- Weibull
- Beta
- Binomial
- Truncated normal
- User defined




$$f(x) = a e^{-\frac{(x-b)^2}{2c^2}}$$

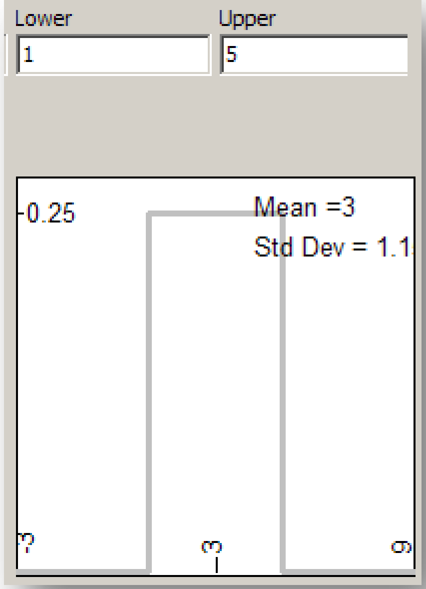
$a$ – Height of the peak

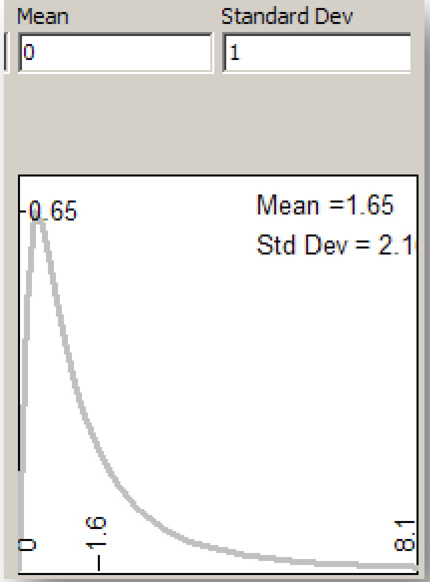
$b$ – Position of center (mean)

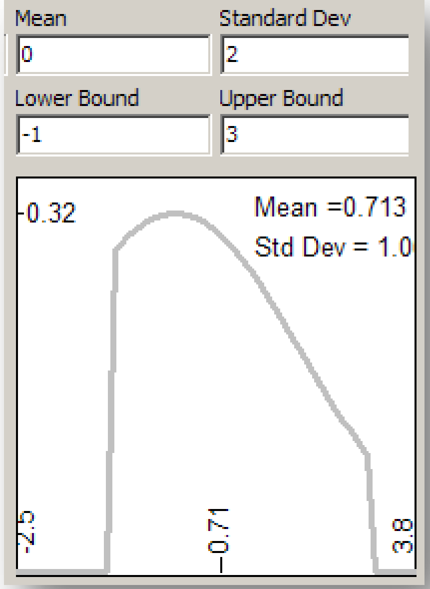
$c$ – Controls the width of the “bell” (standard deviation)

# Distributions in LS-OPT

- Uniform distribution
- Lognormal distribution
- Truncated normal distribution

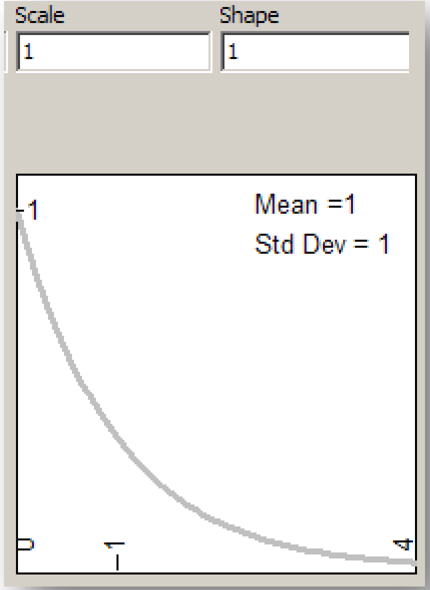


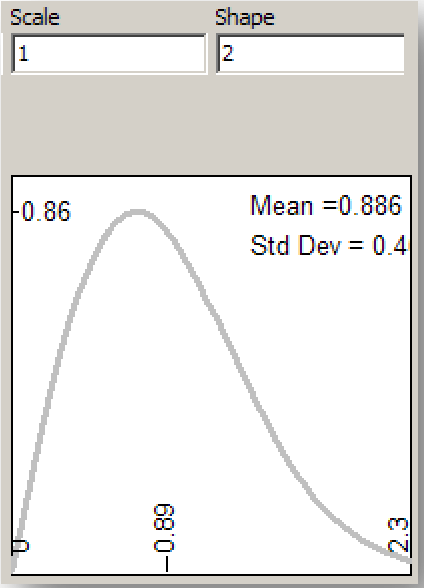



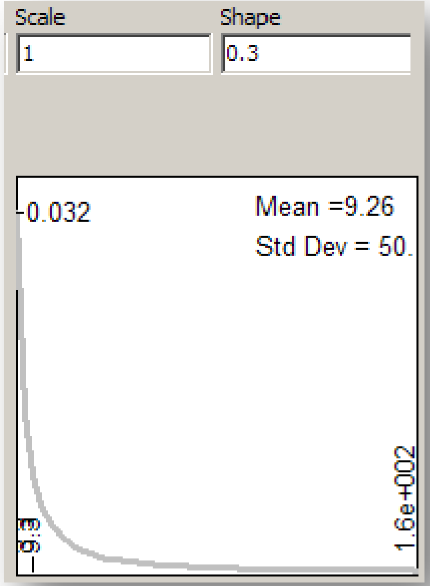



# Distributions in LS-OPT

- Weibull distribution



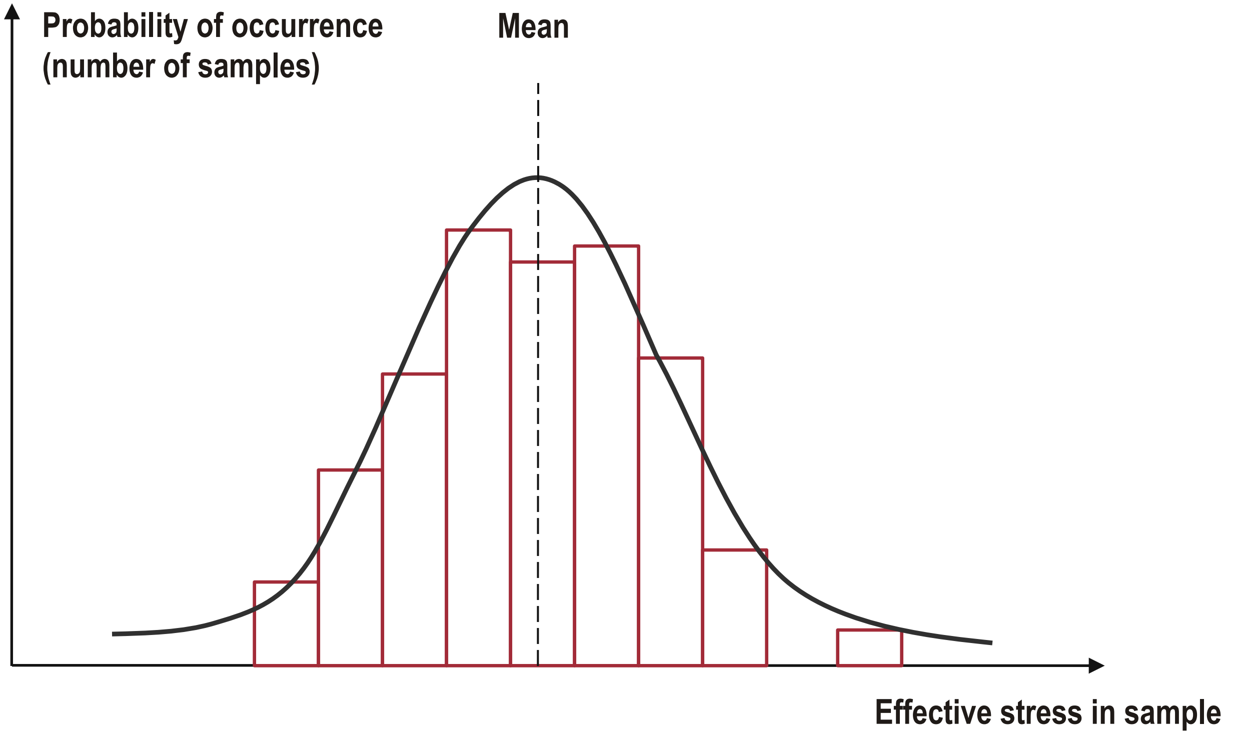
# Sources of Variation

- Modeling variation
  - Differences in modeling will give different results as well as introduce noise to the results.
    - mesh density
    - solution algorithm: FEA, SPH, MMALE
    - resolution of output data
- Analysis variation
  - Variations in initial conditions can lead to noticeable differences in responses.
    - Physical: bifurcation events, sequence of impact (contact) due to changes in the design variables
    - Algorithmic: due to discretization a node can come into contact with an element or adjacent to it

# Sources of Variation

- Pure Variation
  - change in the results nonrelated to the input
    - different computer
    - different math library
    - machine precision
    - round-off error
    - different versions of solver

# Response Variation



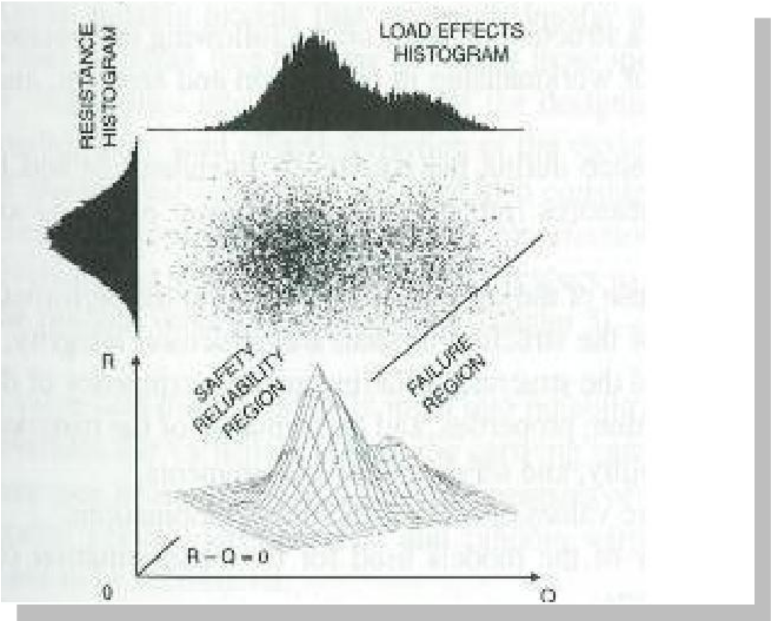

- What is the distribution of the response, given the distribution of parameters?
- What is the probability of structure failure?
- What are efficient redesign strategies?

# Monte Carlo Analysis

- Monte Carlo simulation aims to compute results with the same scatter as what will occur in practice.
- Multiple analyses are conducted using values of the input variables selected considering their probability density function.
- The results from these analyses should have the scatter expected in practice.
- Requires large number of runs and <u>random</u> sampling.

# Direct Monte Carlo Analysis

- The direct Monte Carlo method consist of the following steps:
  - Based on their probability distribution functions, generate the values of the input random variables
  - Perform a deterministic analysis and check for failure
  - Repeat for N times with different values of the input random variables and count the number of failures, $N_f$
  - The estimate of the mean probability of failure is: $p_f = N_f/N$
  - Note, each "ant" would represents a complete FE analysis ☹



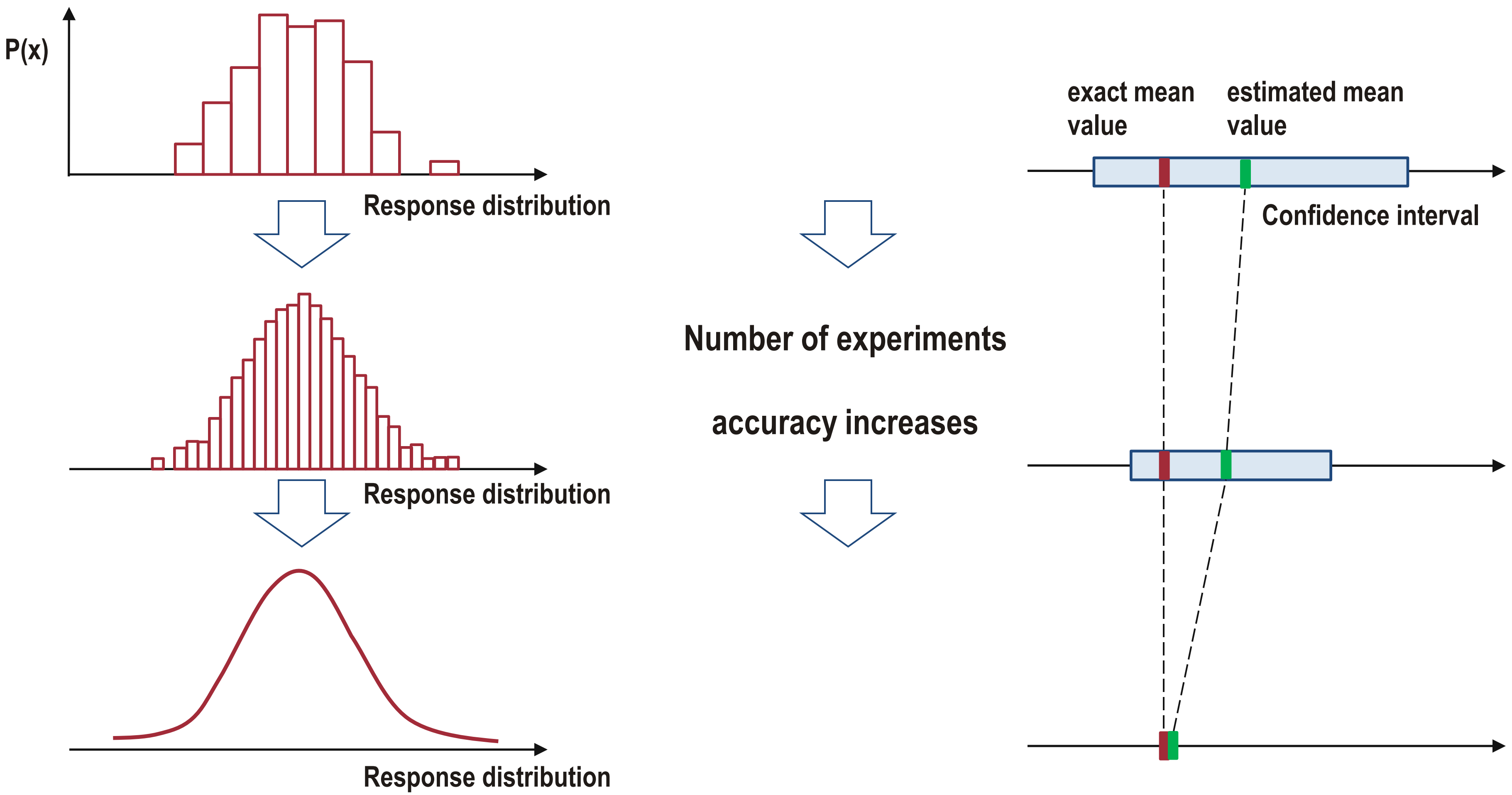
# Direct Monte Carlo

- For a specified error in the true failure probability, the number of required MC simulations is given by

$$N = \frac{1 - p_f^T}{(\varepsilon\%)^2 \, p_f^T} 200^2$$

| Probability of Failure | Percent Error | Number of Simulations |
|---|---|---|
| 0.01 | 20 | 10,000 |
| 0.01 | 10 | 39,600 |

# Monte Carlo Analysis

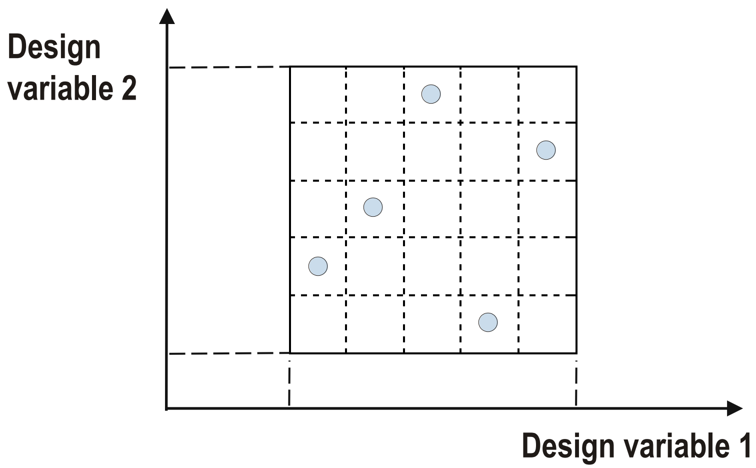


# Latin Hypercube sampling

- The Latin Hypercube method is a random experimental design process. It was developed to generate a reasonable collection of parameter values from a multidimensional distribution.
- A square grid containing sample points is a Latin square iff there is only one sample in each row and each column.
- A Latin hypercube sampling is the generalization of this concept to an arbitrary number of dimensions.
- LH designs are independent of the mathematical model of the approximation and allow estimation of the main effects of all factors in the design in an unbiased manner

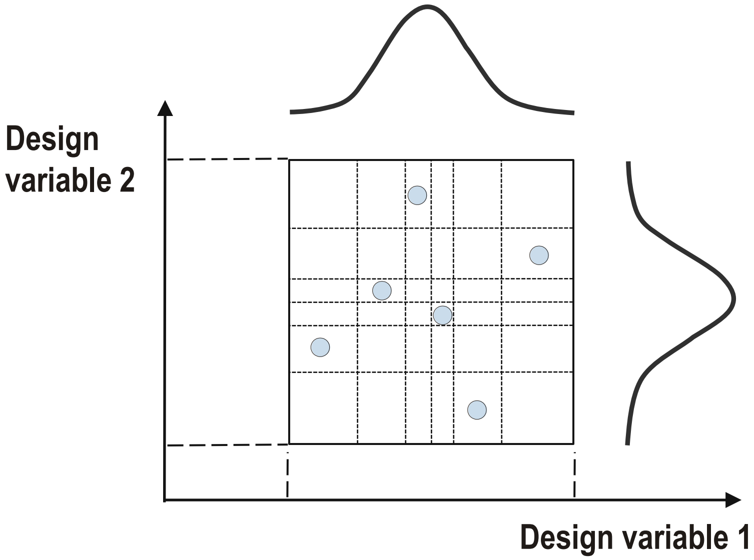
# Latin Hypercube sampling

- When sampling a function of $n$ variables, the range of each of them is divided into $P$ equally probable intervals. $P$ sample points are then placed to satisfy the LH requirements.
- This forces the number of divisions to be equal for each variable.
- LH sampling does not require more samples for more dimensions (variables)



# Latin Hypercube sampling

- If the variable has assigned distribution, the partitions for LH sampling will be subdivided as to have equal probability

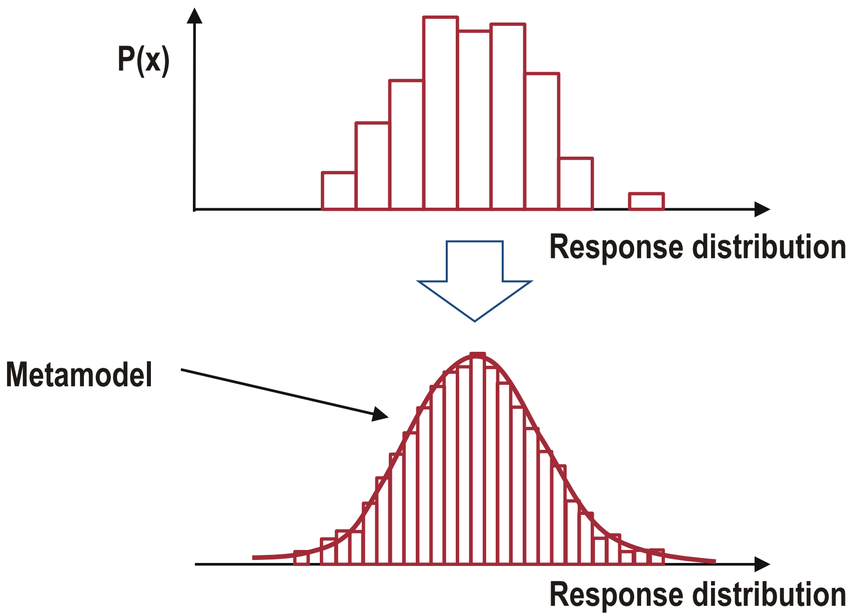



- LH sampling may be used further for space filling designs

# Metamodel-based Monte Carlo

- Problem: Monte Carlo analysis requires large number of runs. MC is usually prohibitive for large models (100s to 1000s+ runs).
- Solution: Use Monte Carlo analysis with reduced number of runs and construct metamodel (10-30+ runs). Very large number of function evaluations are possible when using metamodels ($10^6$).
- The results are exact for linear or quadratic responses approximated with linear or quadratic response surfaces.
- For random shapes of response use Neural Networks (NN) or Radial Basis Function (RBF) Networks

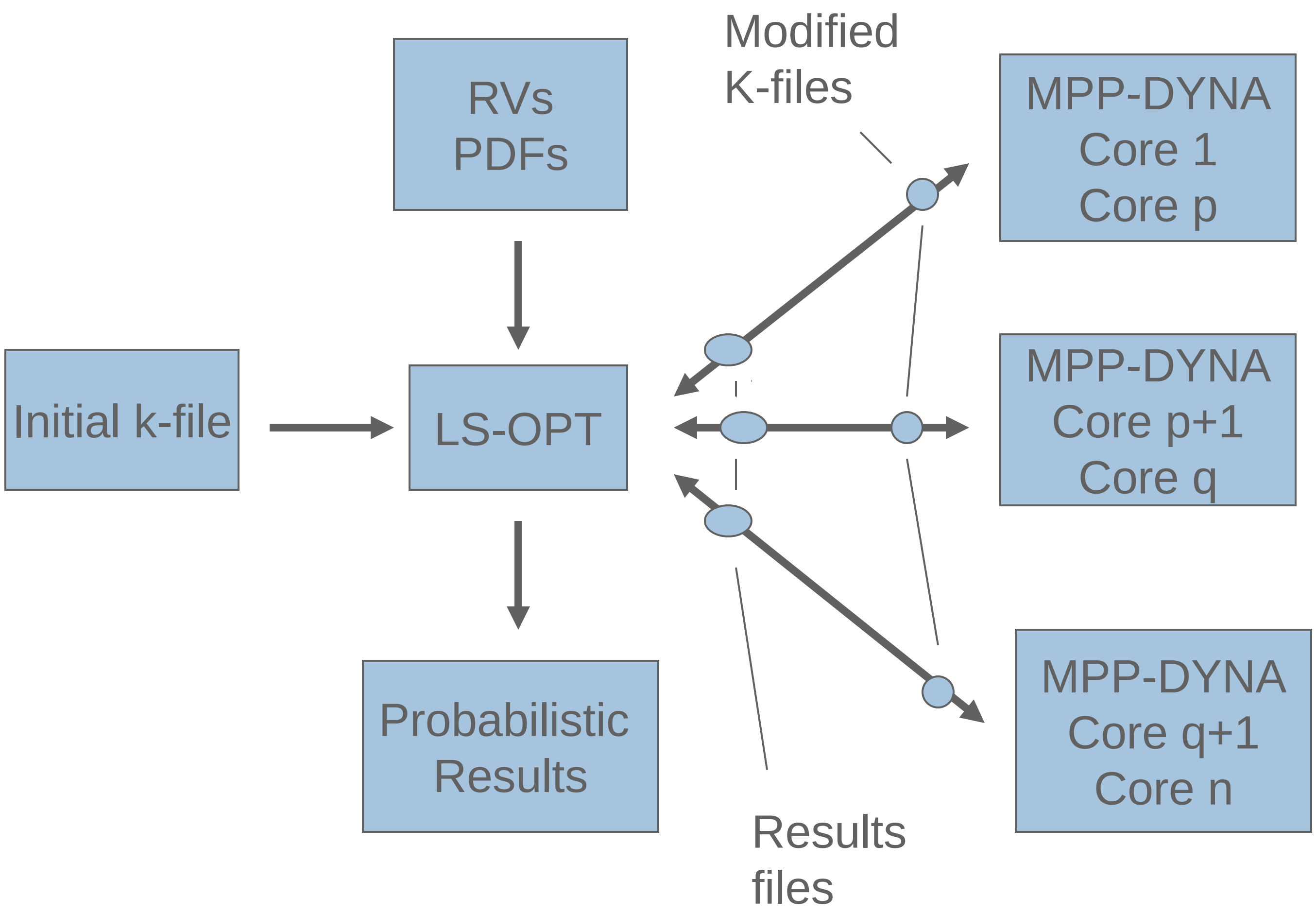
# Metamodel Based Monte Carlo Analysis




**Build a metamodel on medium number of simulation points**

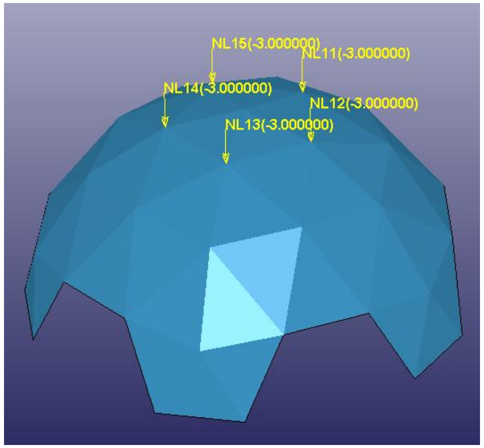
**Perform hundreds of function evaluations on cheap metamodel**

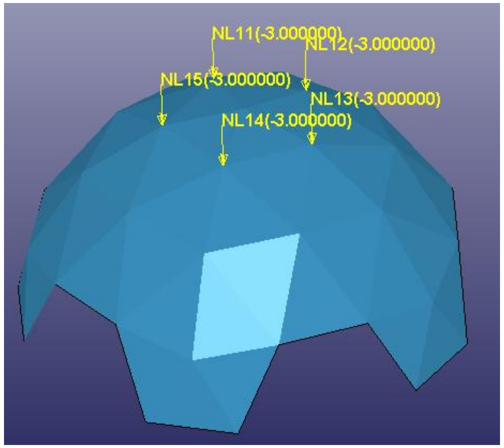
# LS-OPT and MPP-DYNA on the TRACC Cluster

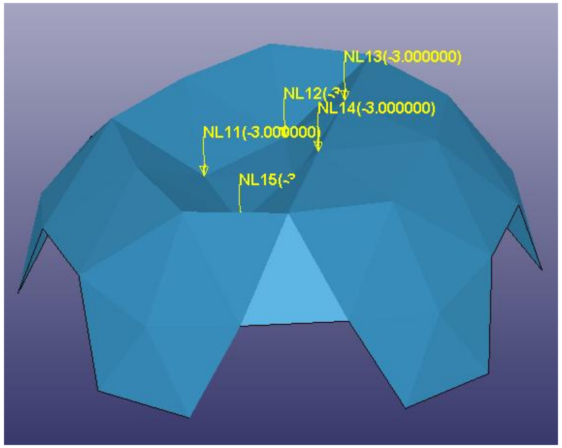


# Monte Carlo Analysis with LS-OPT Direct Simulations

Example: Deflection of the dome structure.



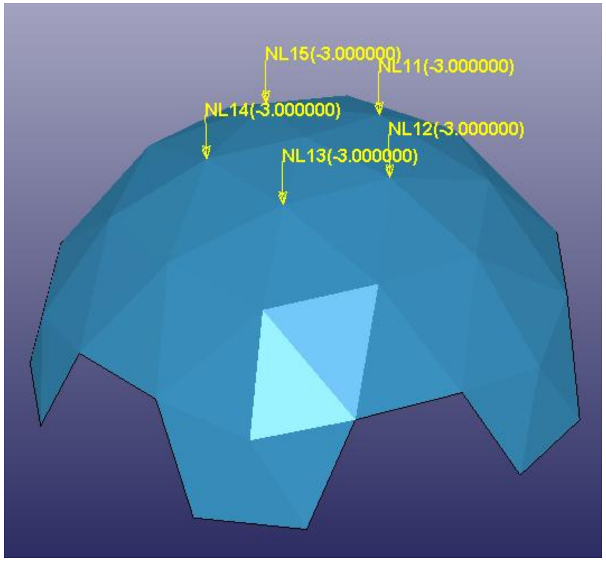



# Problem description

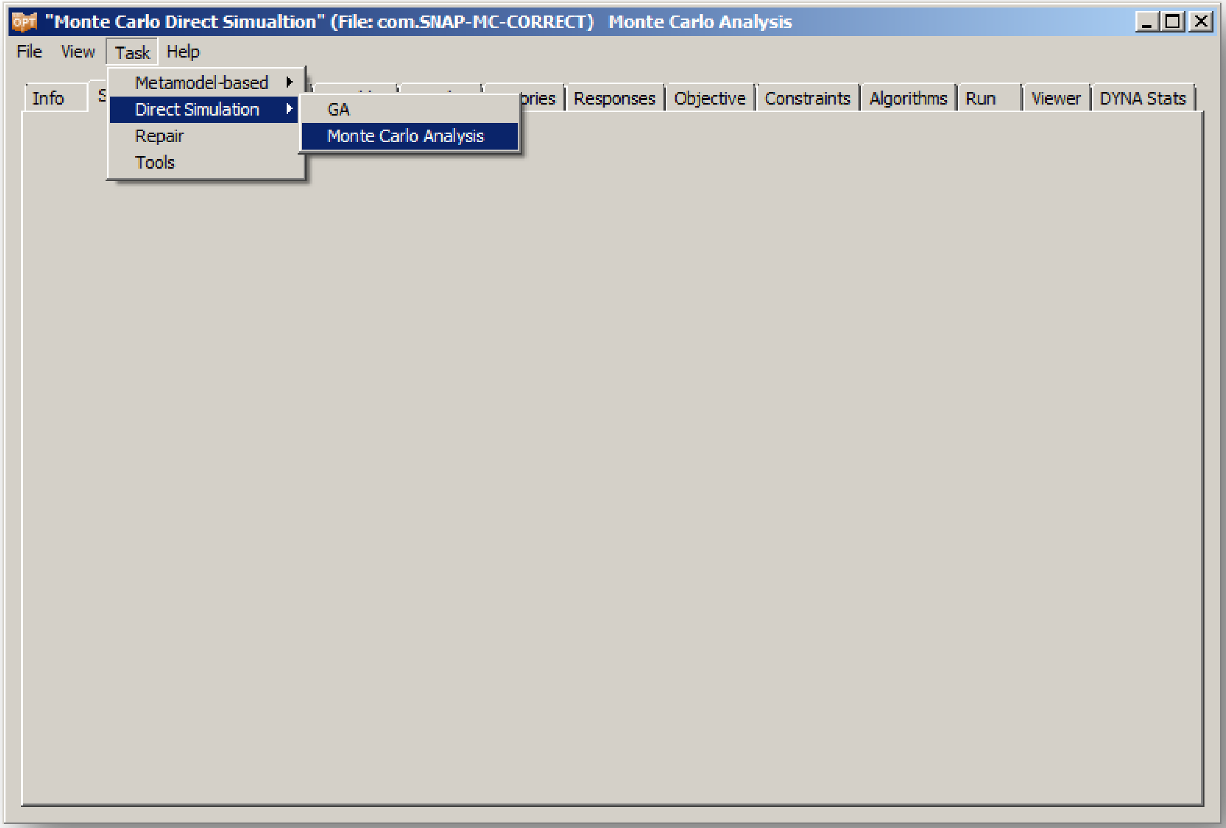
- This example is a Monte Carlo analysis of a dome structure being loaded from the top. The effect of variations in:
  - material properties,
  - thickness,

  are investigated.
- The geometry is shown in Figure below.
- The x, y and z – displacements are restrained at the bottom of the structure.
- It is loaded from the top in five nodes.
- Maximum z – displacement of the central node is used as the response variable



# Approach 1 – Monte Carlo Direct Analysis

- Go to Task tab
- Select Monte Carlo Analysis from Direct Simulations group

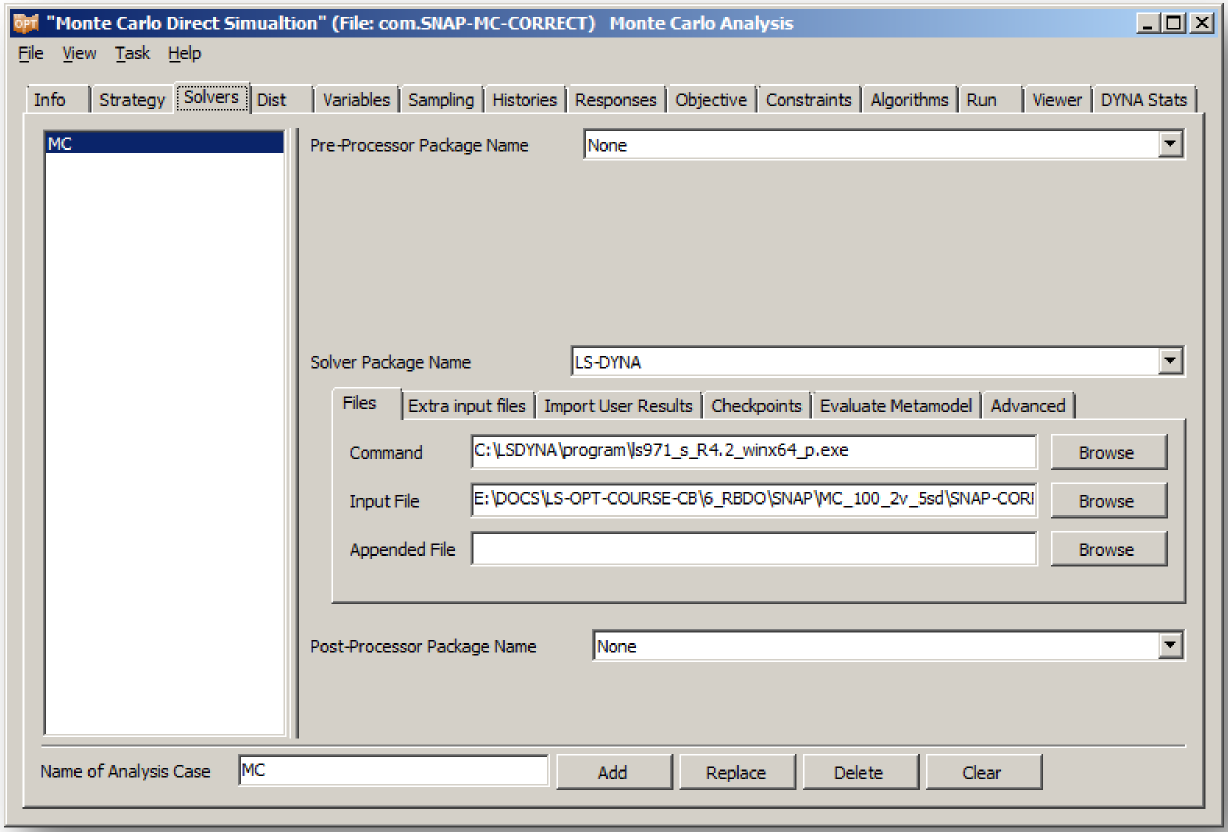


# Info Panel

- Describe the problem
- Give author's name

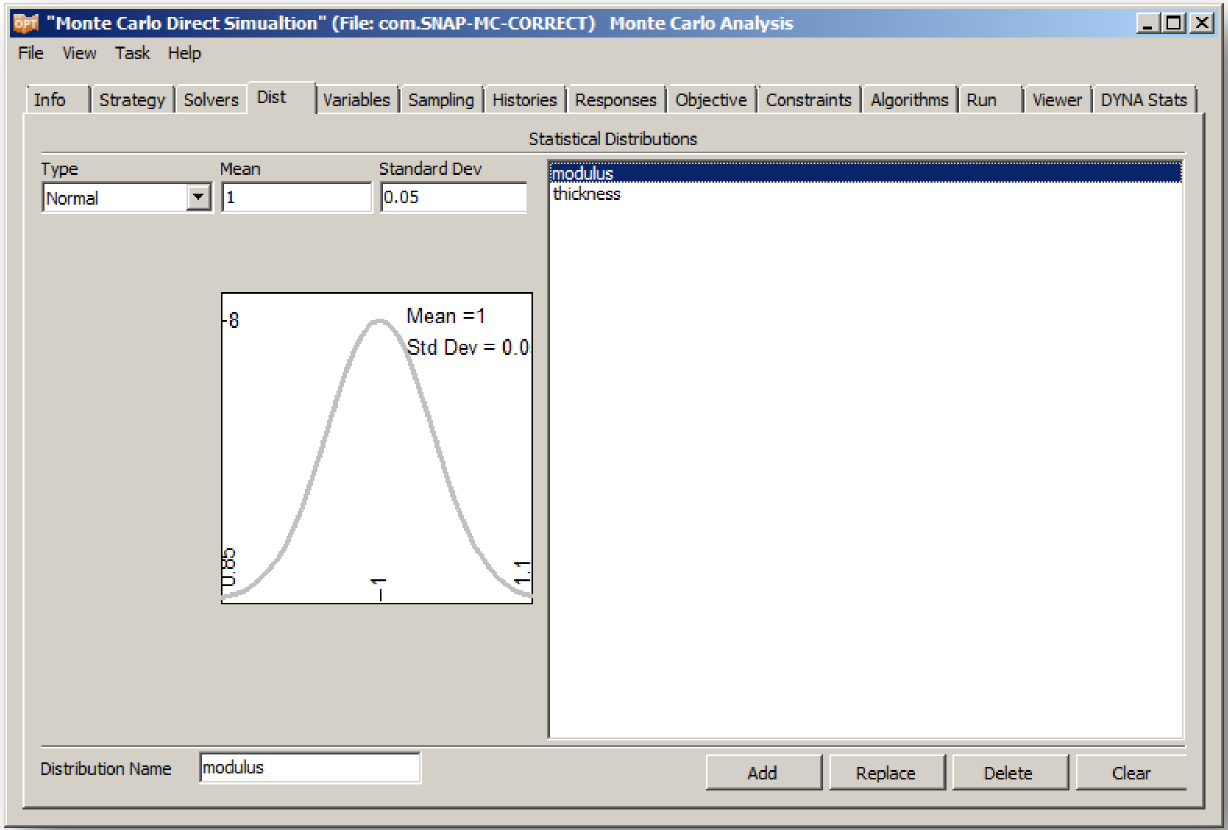
# Solvers Panel

- On TRACC cluster specify the command as full path to `lsoptscript`
- Navigate to the input file
- Name the analysis `MC` and hit Add



# Distributions Panel

- In distributions panel select Type Normal, type 1 for Mean and 0.05 for Std. dev.
- Name the distribution – modulus and hit Add button
- Repeat the same procedure for thickness distribution



# LS-DYNA k-file

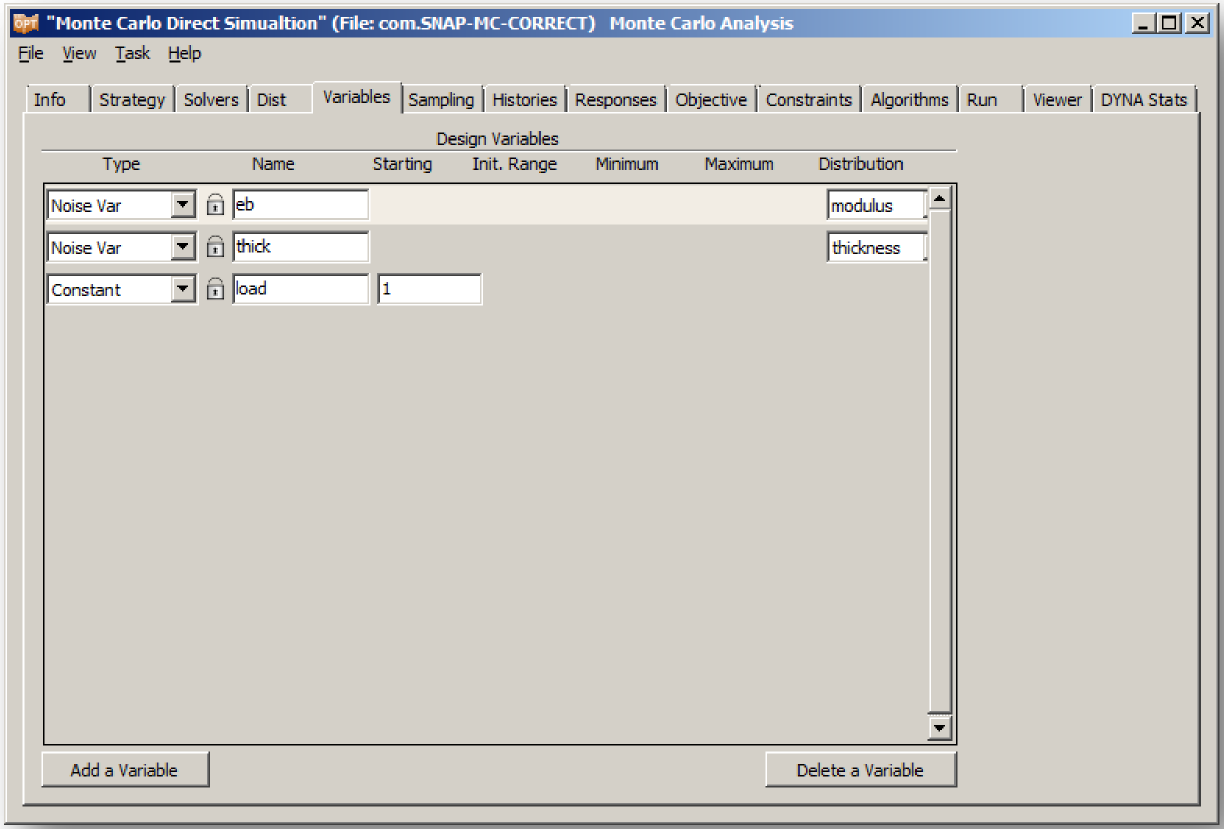
- Defined noise variables will be the multipliers of the design parameters – thickness and elastic modulus

```
*SECTION_SHELL
$#   secid    elform      shrf       nip     propt   qr/irid     icomp     setyp
         2         4     0.000         3         0         0         0         0
$#      t1        t2        t3        t4      nloc     marea      idof    edgset
<<65*thick>>,<<65*thick>>,<<65*thick>>,<<65*thick>>
```

```
*MAT_ORTHOTROPIC_ELASTIC
$#     mid        ro        ea        eb        ec      prba      prca      prcb
         2 2.1500E-6 10.000000 <<0.3*eb>> 1.0000E-5  0.050000 1.0000E-5 1.0000E-5
$#     gab       gbc       gca      aopt         g      sigf
  0.500000  0.001000  0.001000     0.000     0.000     0.000
$#      xp        yp        zp        a1        a2        a3      macf
     0.000     0.000     0.000     0.000     0.000     0.000         1
$#      v1        v2        v3        d1        d2        d3      beta       ref
     0.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
```

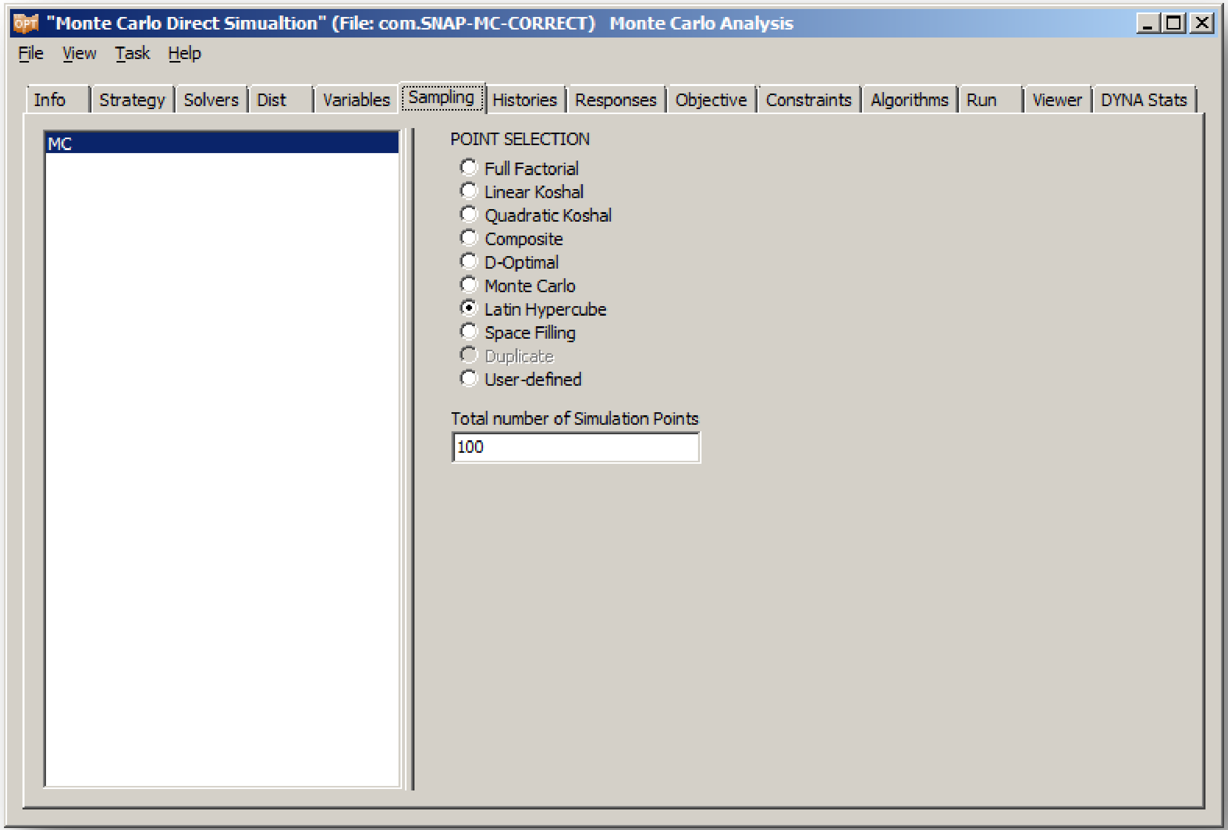
# Variables Panel

- After reading the k-file, variables should be defined already.
- Change type for variable `eb` to Noise variable
- Select distribution type `modulus`
- For `thick` variable change type to Noise variable and select `thickness` distribution
- For Constant load type `1` as starting value



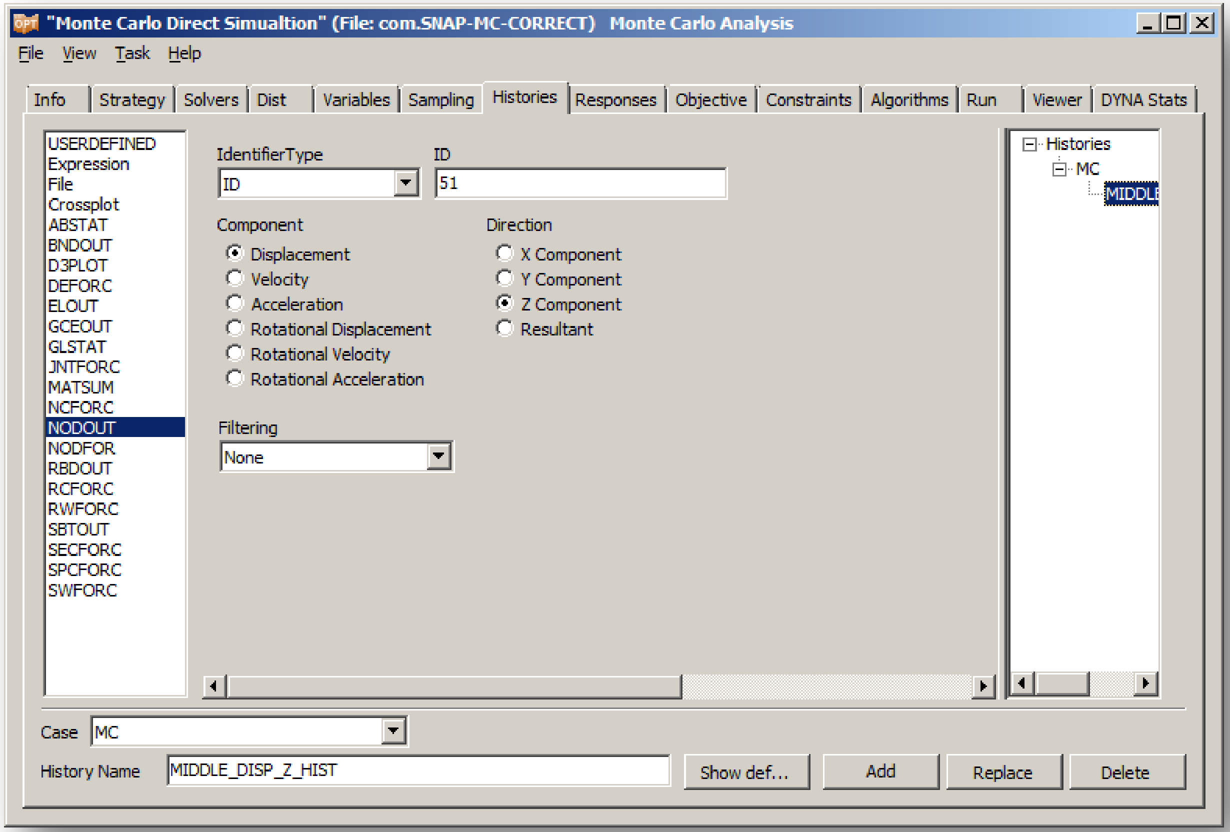
# Sampling Panel

- In Sampling Panel pick Latin Hypercube as Point selection method
- Type `100` for Total number of Simulation Points



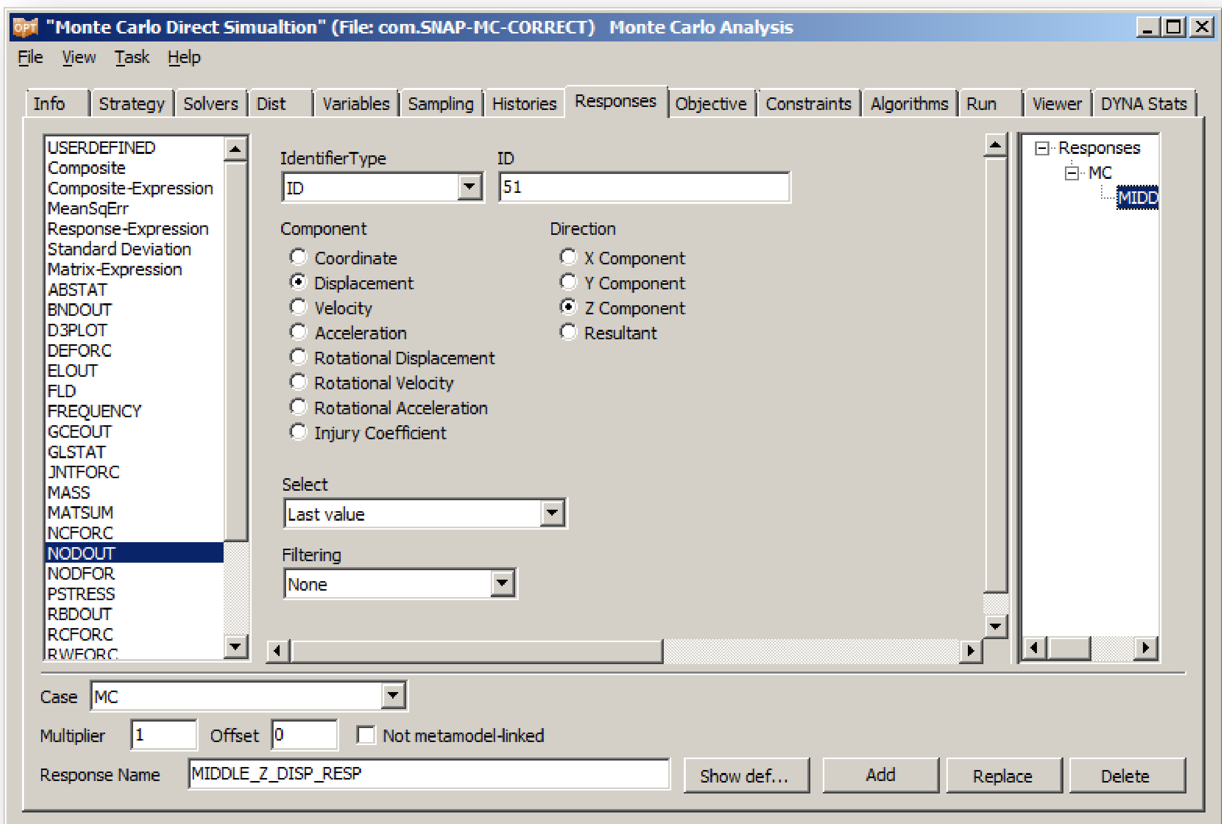
# Histories Panel

- In Histories Panel pick NODOUT from left window
- Choose ID for Identifier Type
- Type 51 for ID and select Z Component as Direction of Displacement
- Give name to the history MIDDLE_DISP_Z_HIST and press Add



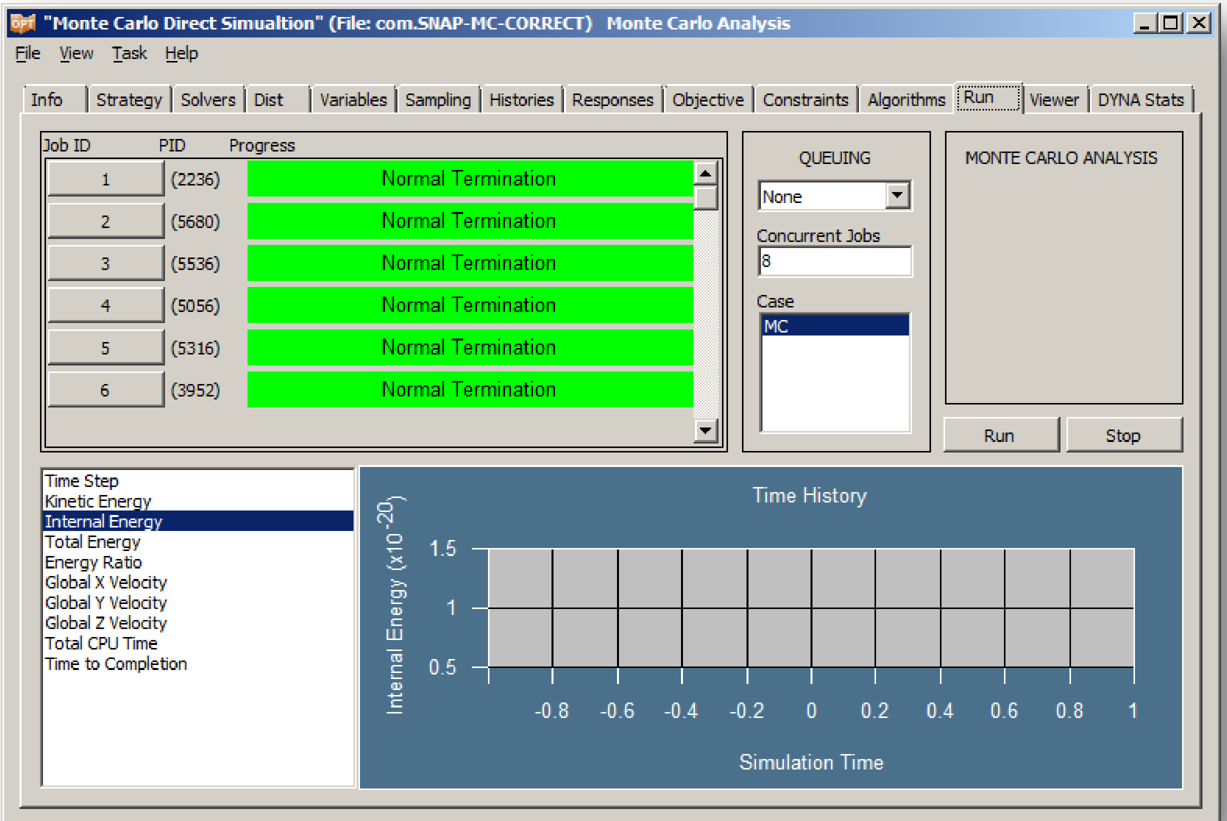
# Responses Panel

- In Histories Panel pick NODOUT from left window
- Choose ID for Identifier Type
- Type 51 for ID and select Z Component as Direction of Displacement
- Give name to the history MIDDLE_DISP_Z_RESP and press Add



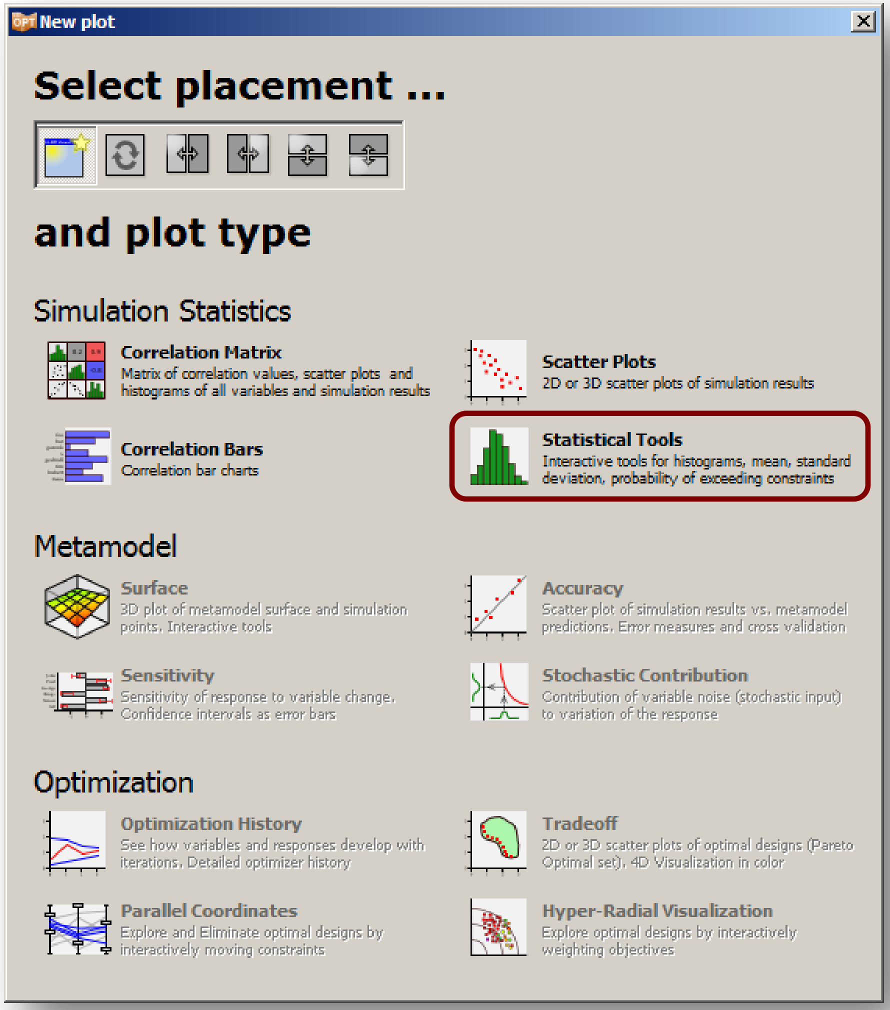
# Run Panel

- In Run Panel select PBS for Queuing system if run on TRACC cluster
- Leave none if run locally on Windows machine
- Type 8 for concurrent jobs and press Run button (one core is enough for this job)



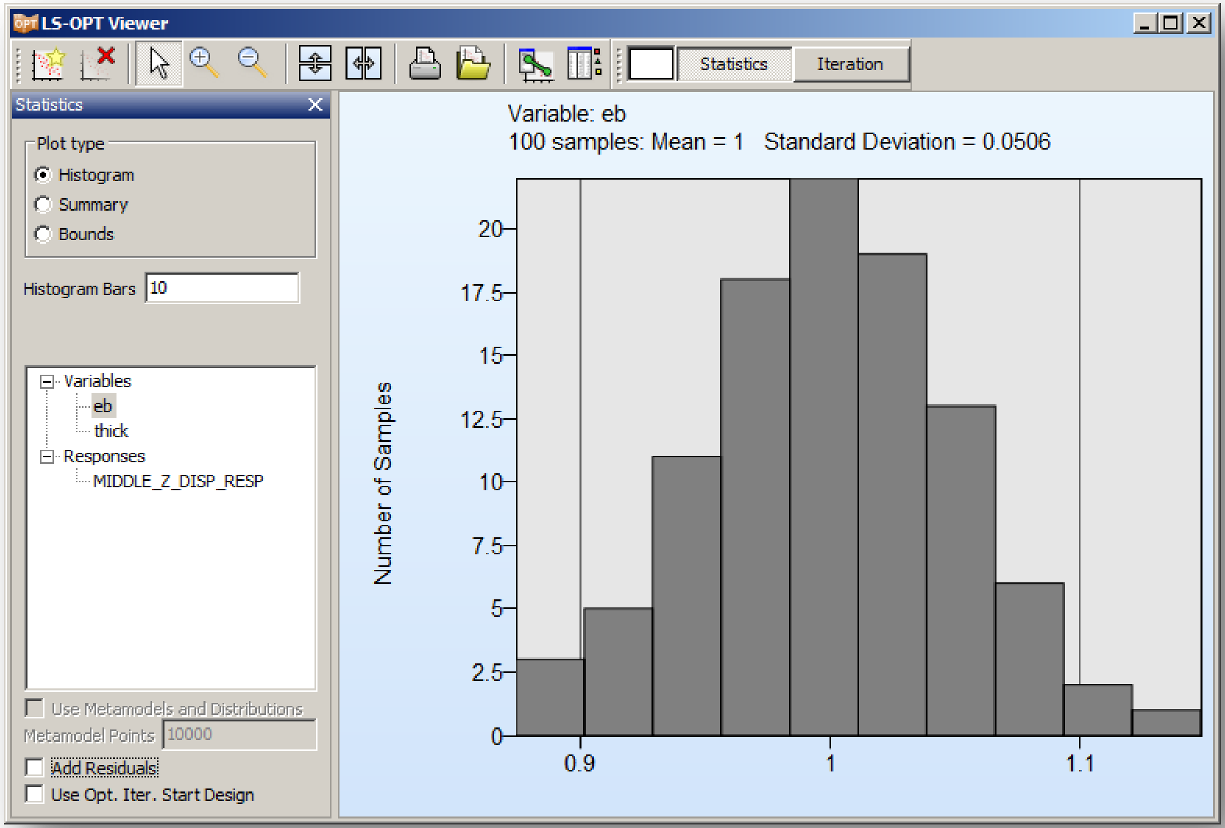
# Viewer

- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select "Statistical Tools"



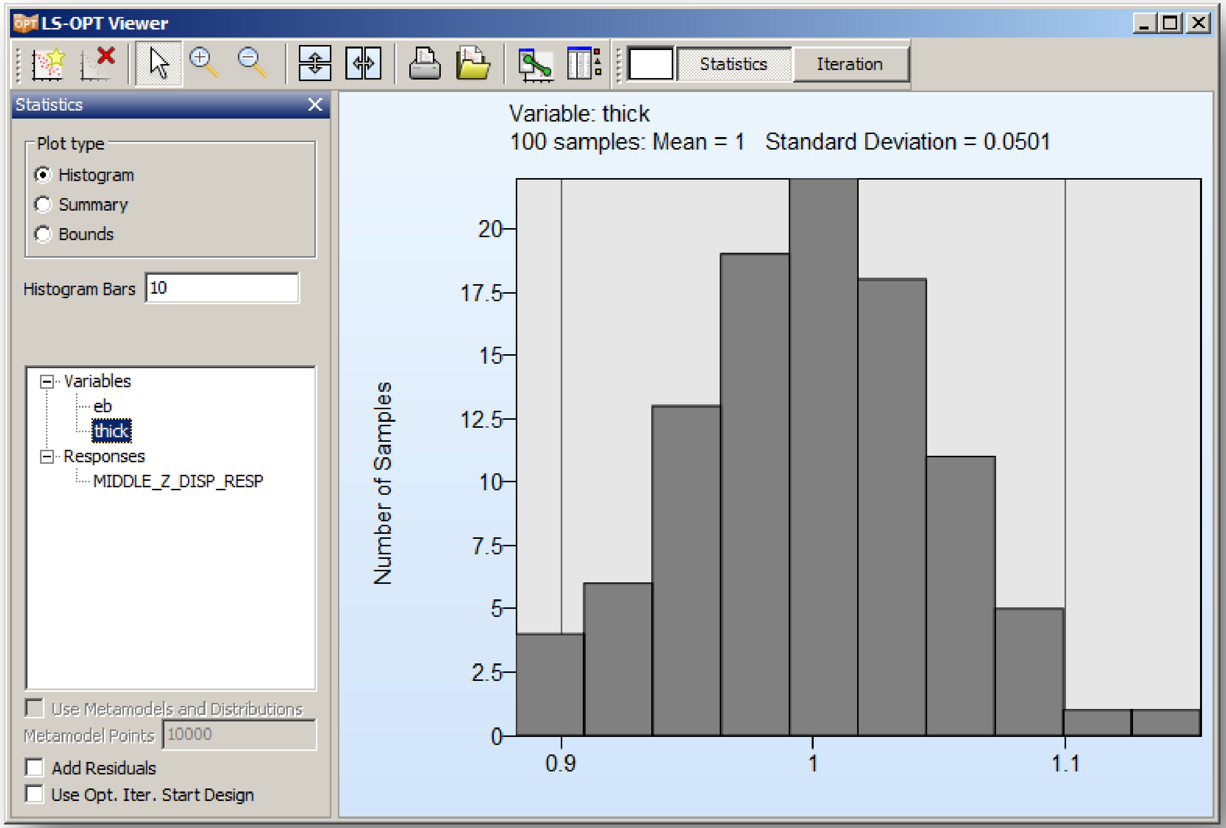
# Statistical Tools

- From Histogram Plot type select eb Variable statistics to display
- As requested mean is 1 and the std. deviation is ~0.05



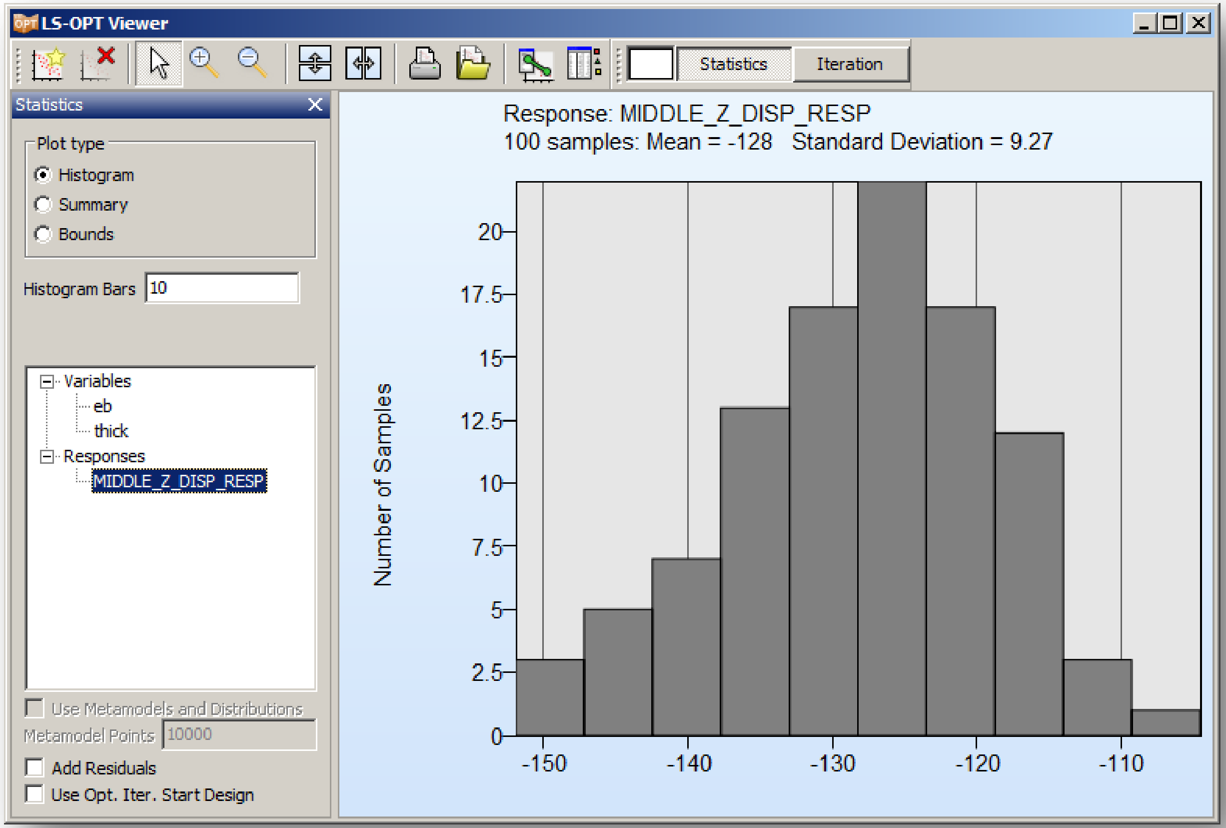
# Statistical Tools

- From Histogram Plot type select `thick` Variable statistics to display
- As requested mean is `1` and the std. deviation is `~0.05`



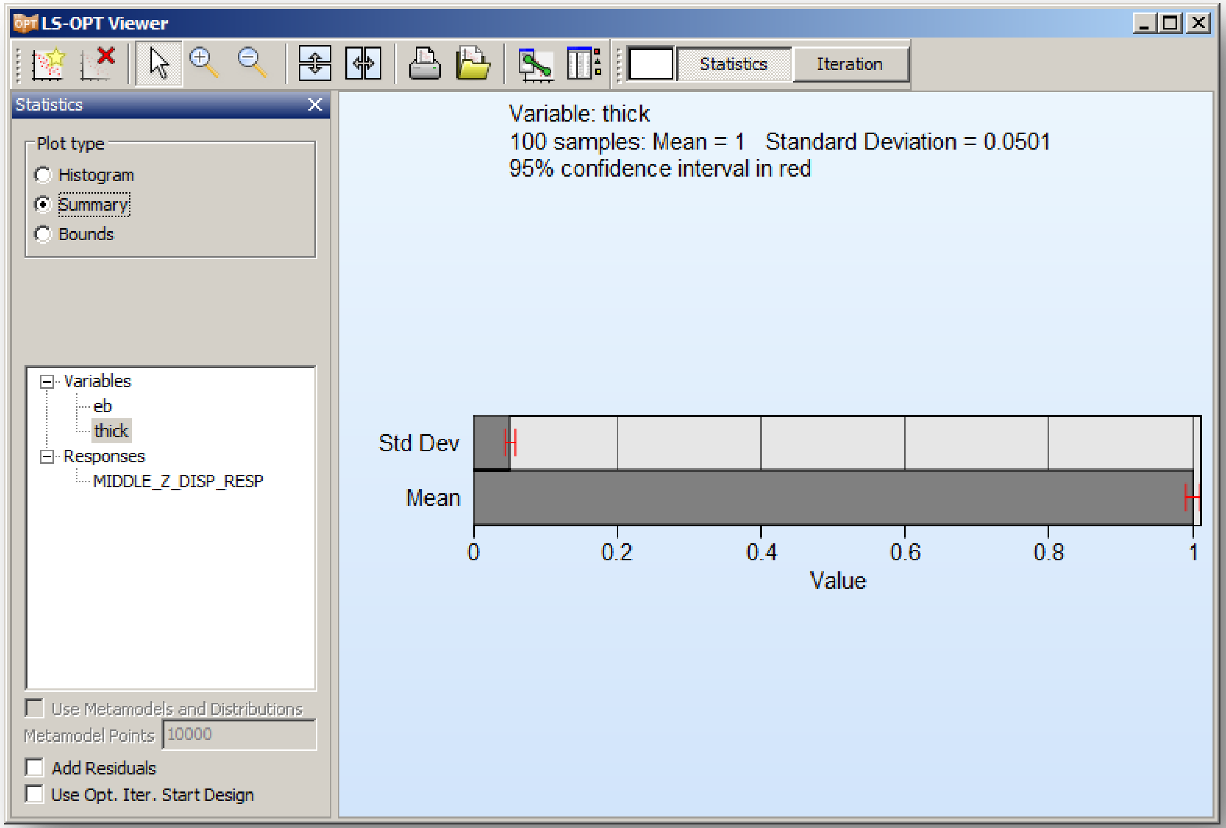
# Statistical Tools

- From Histogram Plot type select MIDDLE_Z_DISP_RESP Response statistics to display
- The mean of the responses is -128 with standard deviation of 9.27



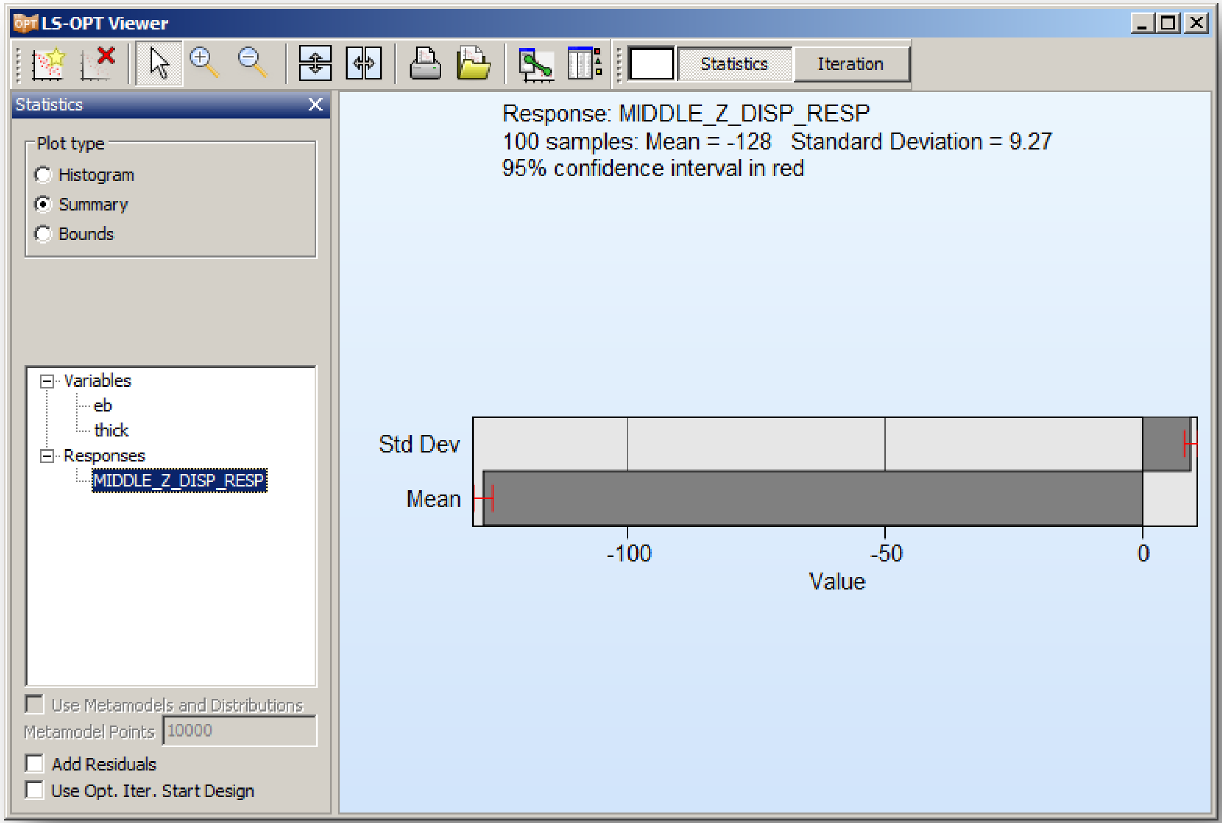
# Statistical Tools

- From Summary Plot type select thick Variable



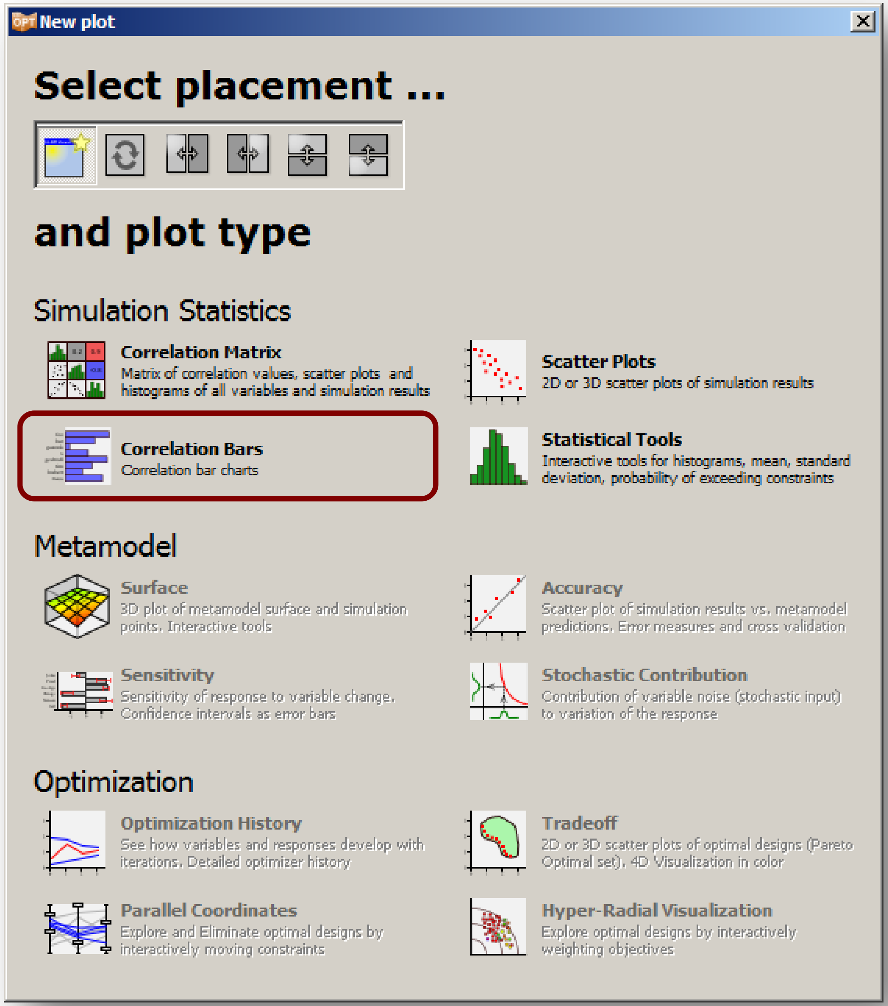
# Statistical Tools

- From Summary Plot type select MIDDLE_Z_DISP_RESP Response to see as bars mean and standard deviation of the item



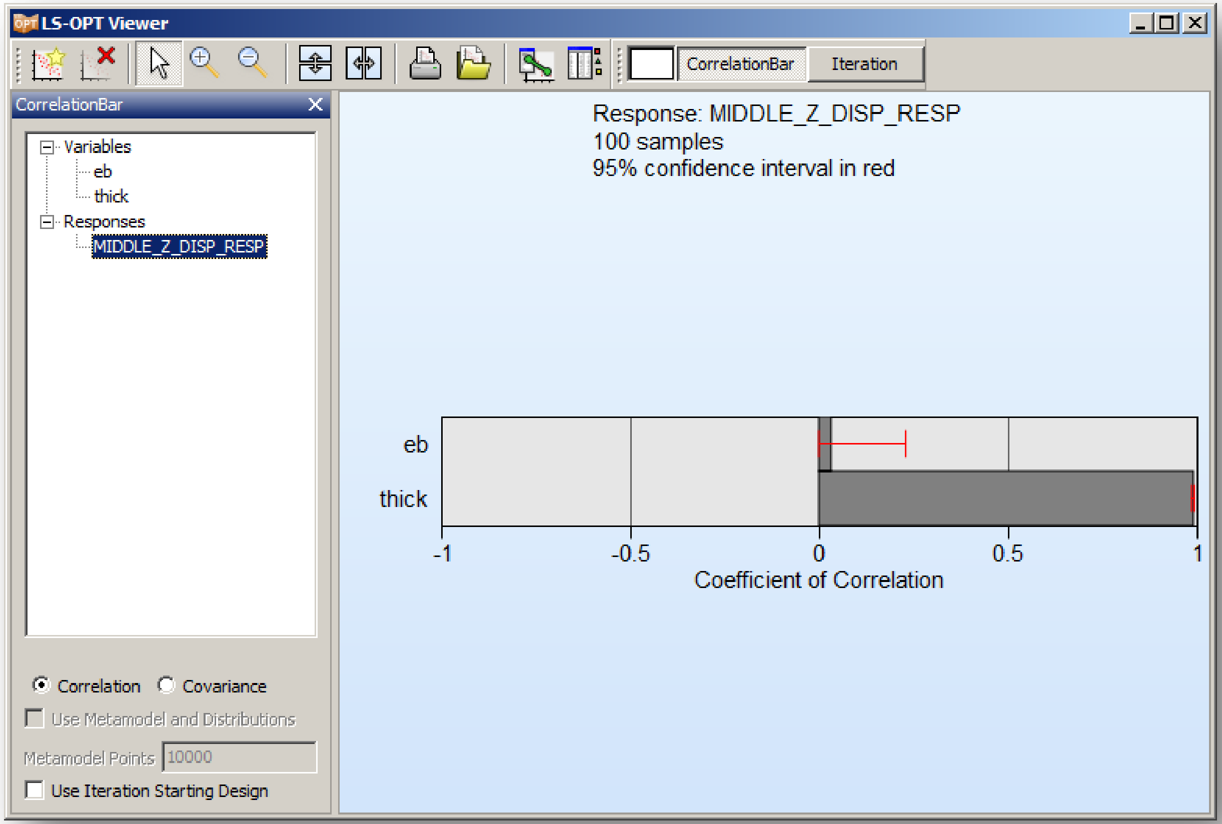
# Viewer

- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select "Correlation Bars"



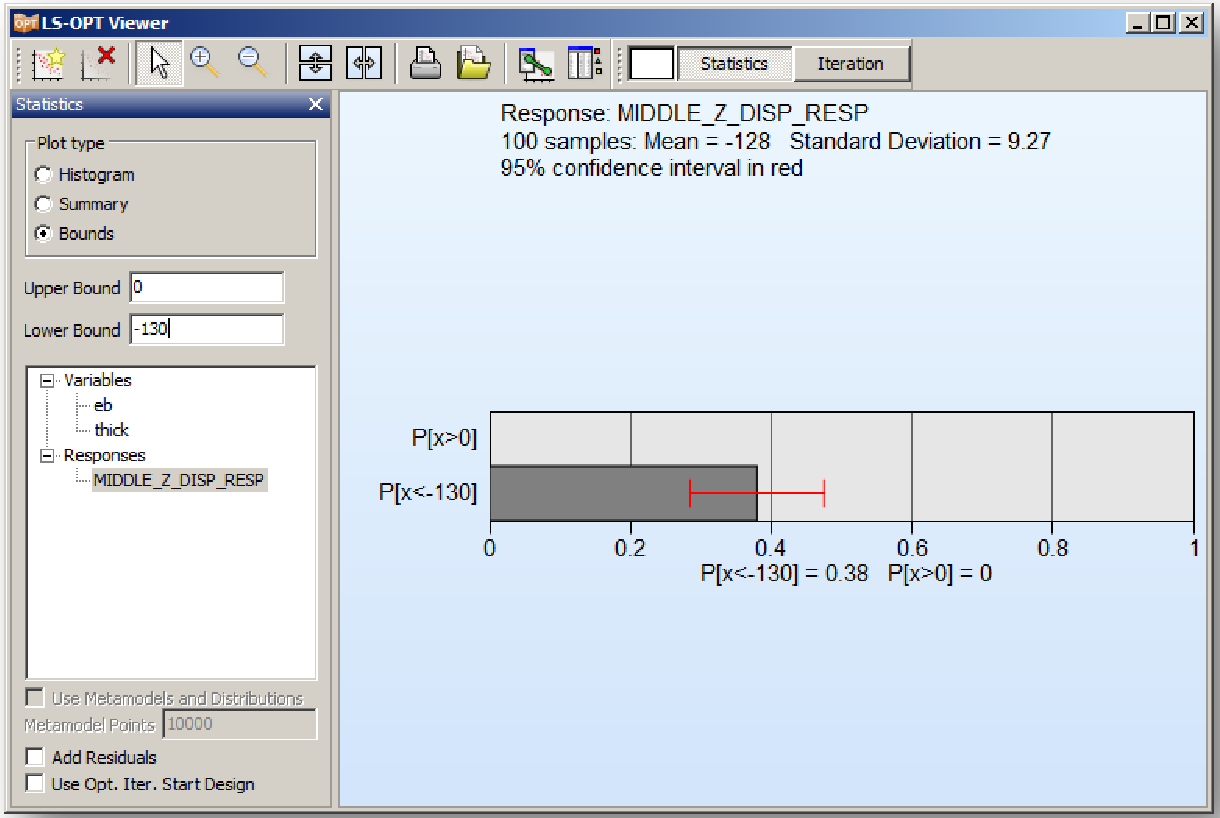
# Correlation Bars

- From Correlation Bar menu select MIDDLE_Z_DISP_RESP Response to see correlation bars between the response and design variables



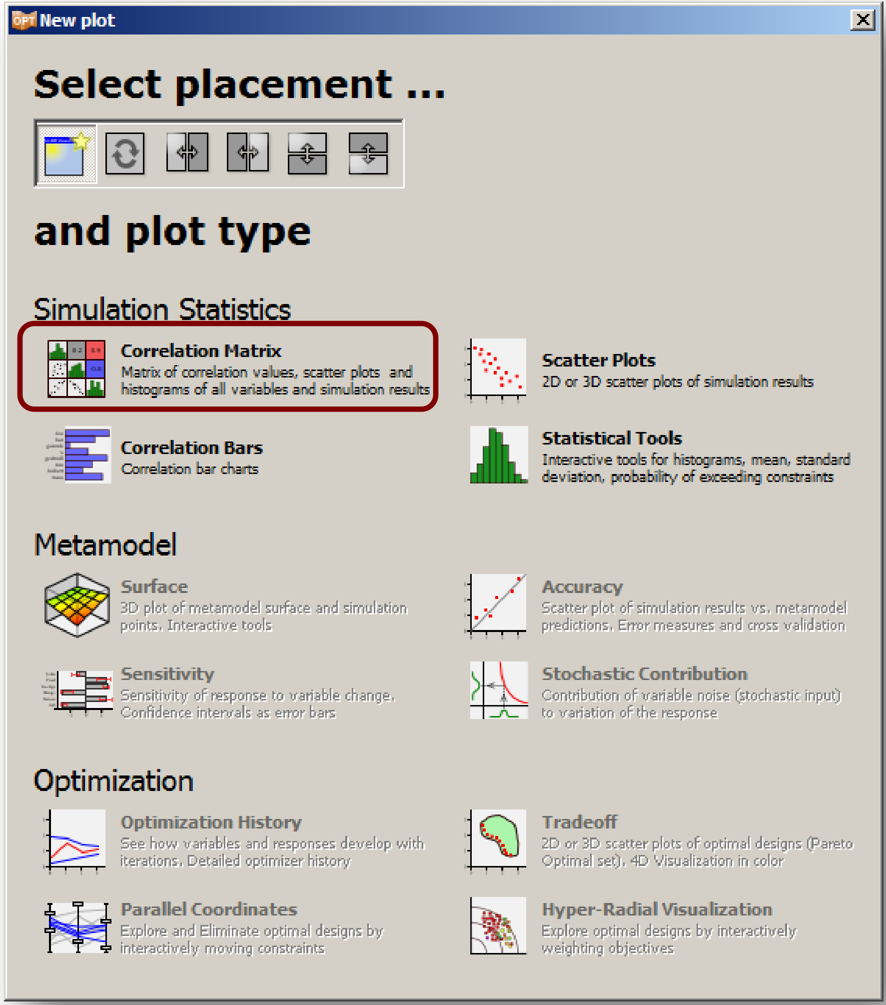
# Statistical Tools

- Go back to Statistical Tools
- Pick Bounds and type `-130` as Lower bound for `MIDDLE_Z_DISP_RESP` Response
- Probability of z-displacement exceeding `-130` is 38%



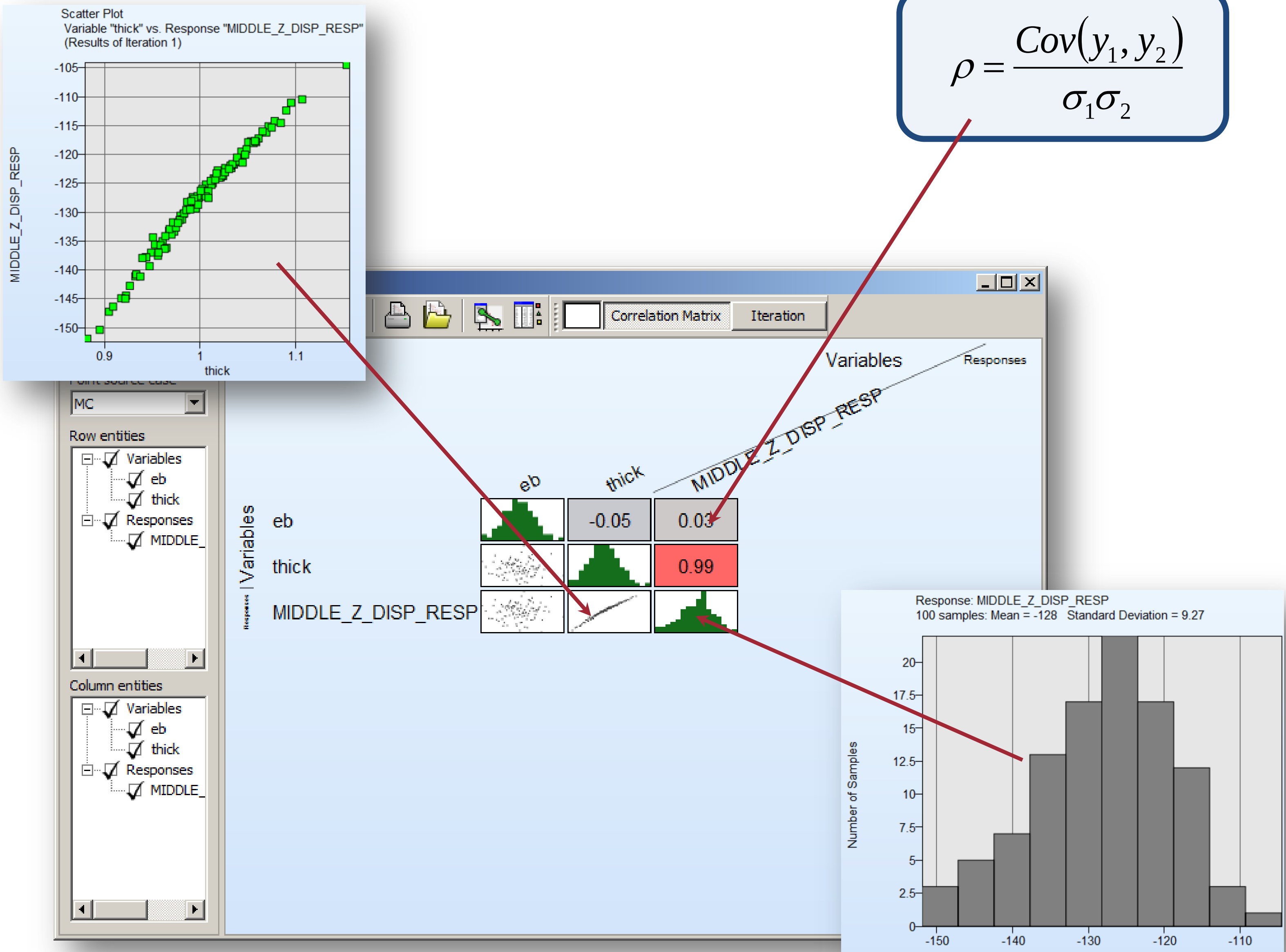
# Viewer

- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select “Correlation Matrix”



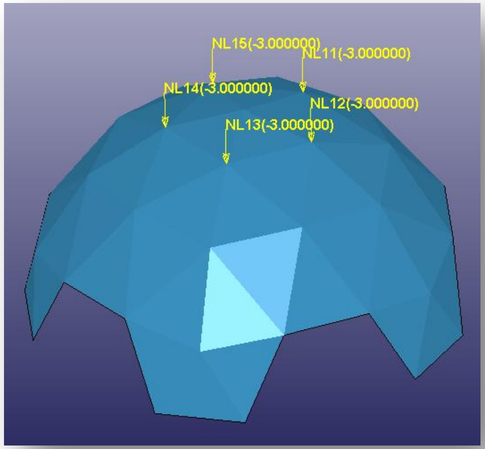
# Correlation Matrix

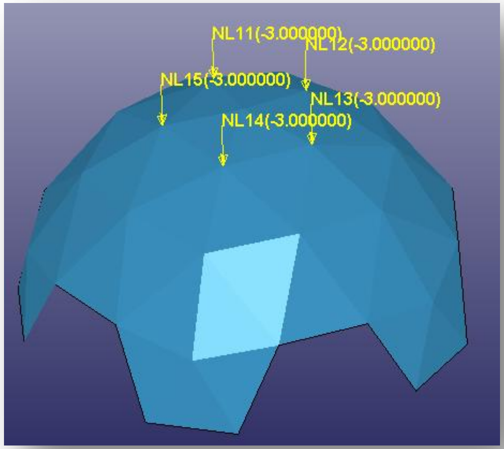
$$\rho = \frac{Cov(y_1, y_2)}{\sigma_1 \sigma_2}$$

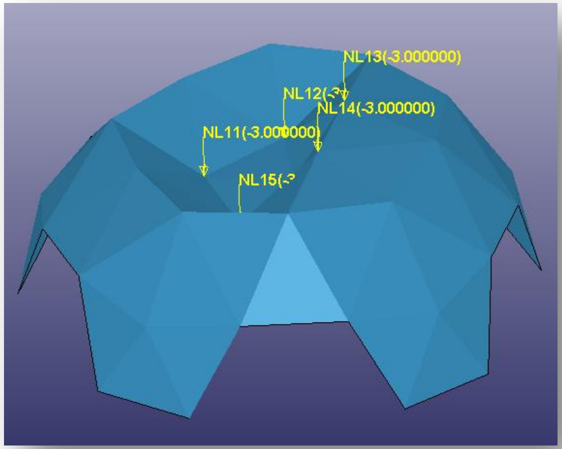


# Monte Carlo Analysis with LS-OPT Metamodel Based Simulation

Example: Deflection of the dome structure.

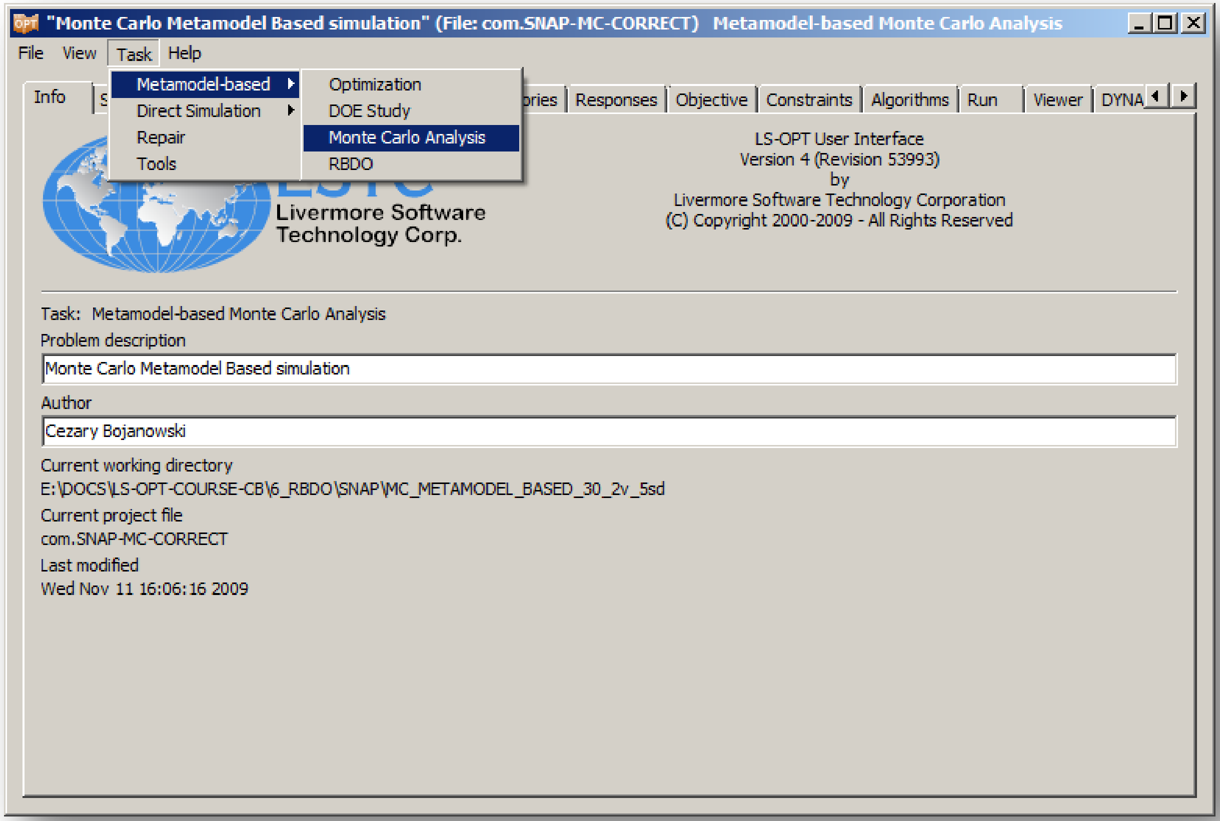


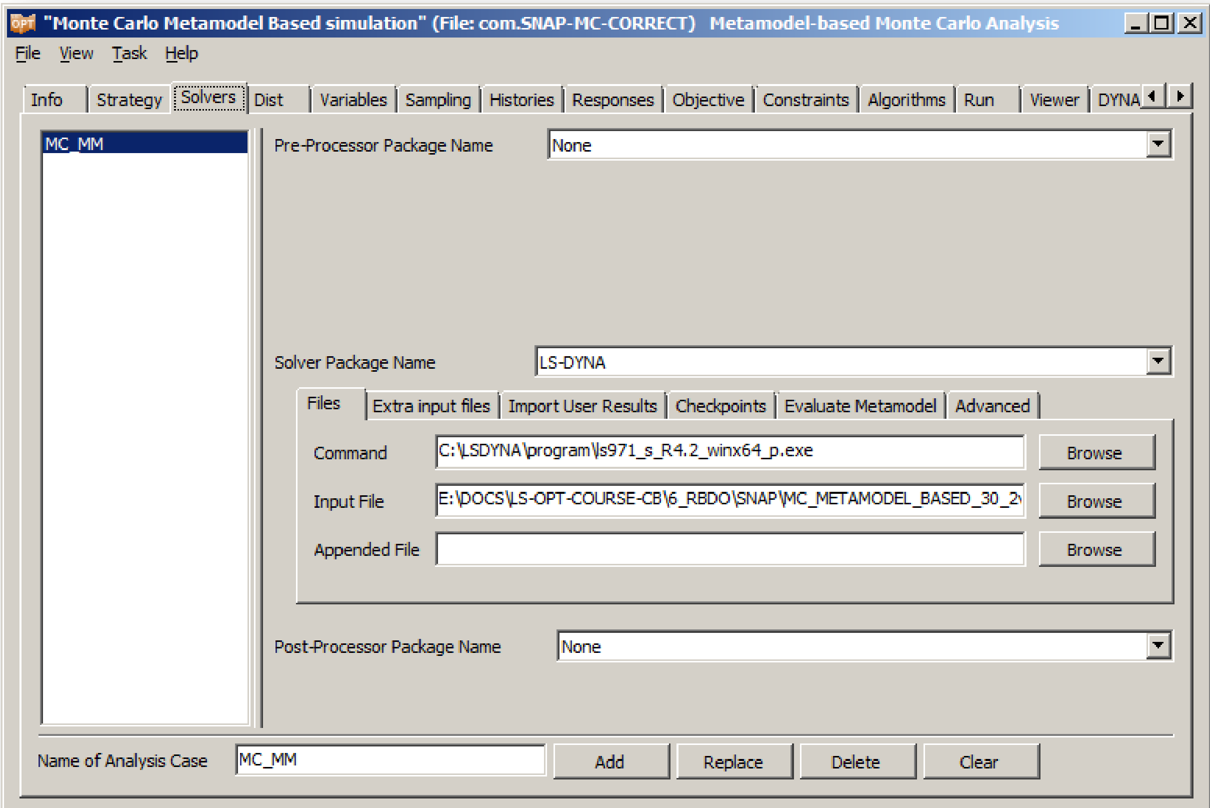
# Approach 2 – Metamodel based Monte Carlo Analysis

- Go to Task tab
- Select Monte Carlo Analysis from Metamodel based group



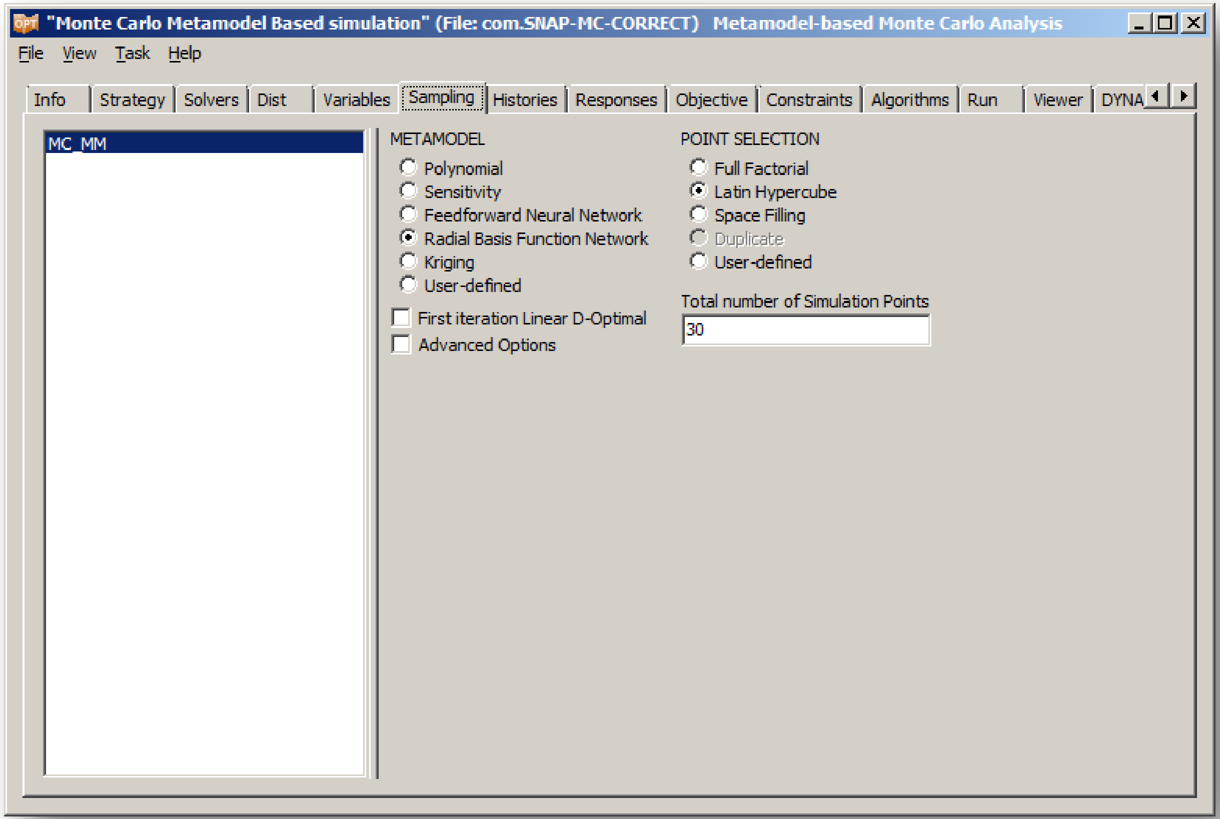
# Solvers Tab

- Go to Solvers tab
- Navigate to the `lsoptscript` (or ls-dyna executable) in Command field
- Find the k-file in Input file tab
- For Name of Analysis Case type `MC_MM` and press Add



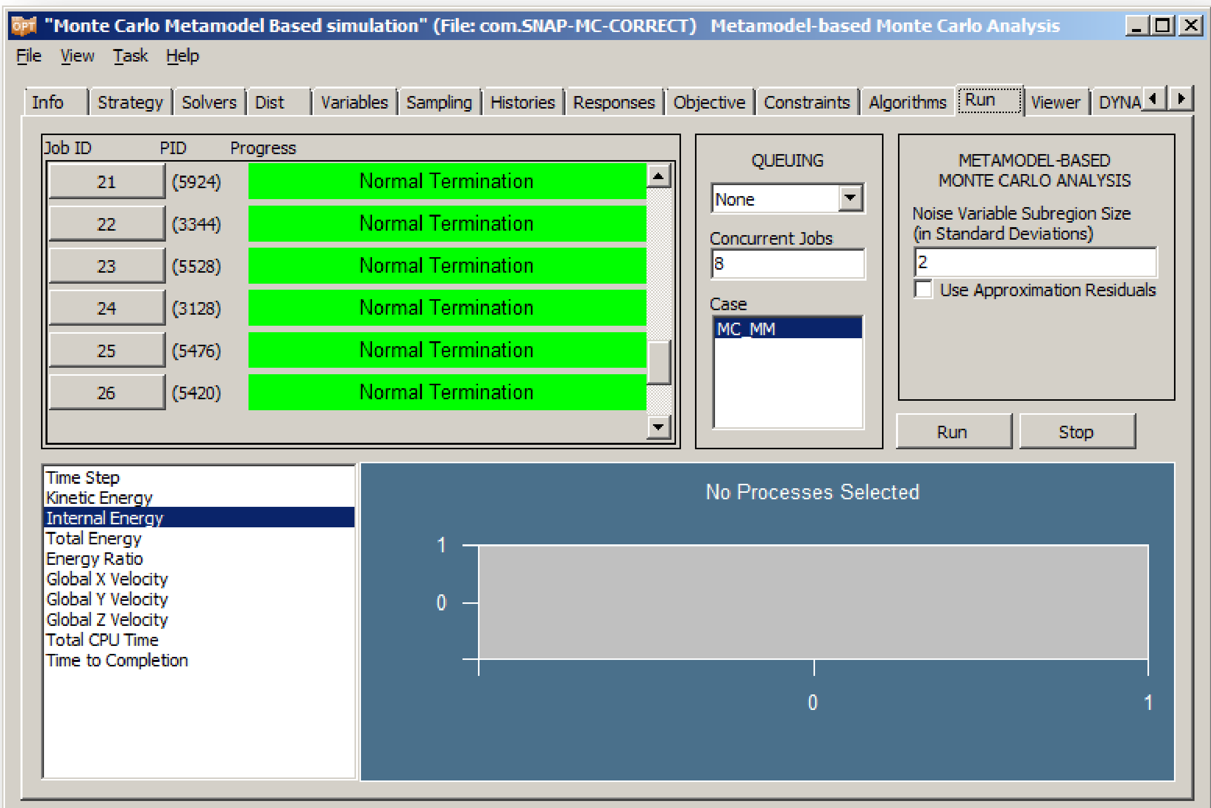
# Sampling Tab

- Go to Sampling tab
- Select Radial basis Function Network for Metamodel type and
- Choose 30 for Total number of Simulation Points



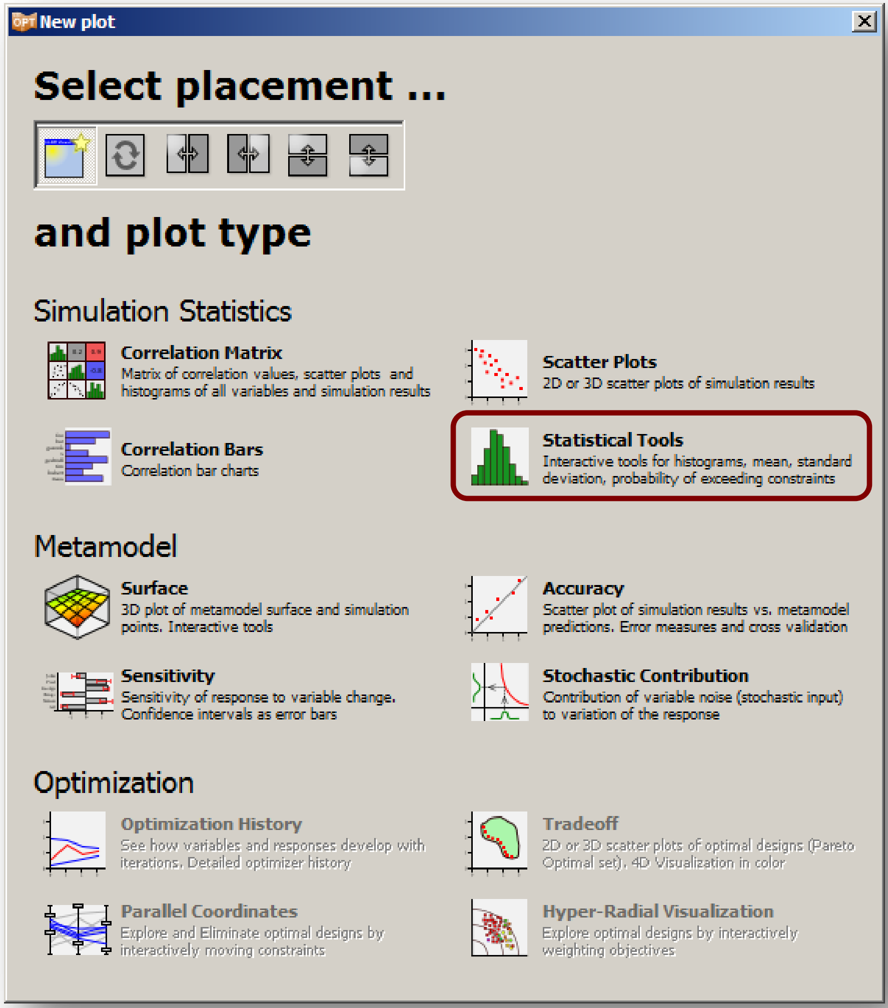
# Run Tab

- Go to Run tab
- Select PBS for your Queuing system (if on TRACC cluster) or leave none
- Press Run button



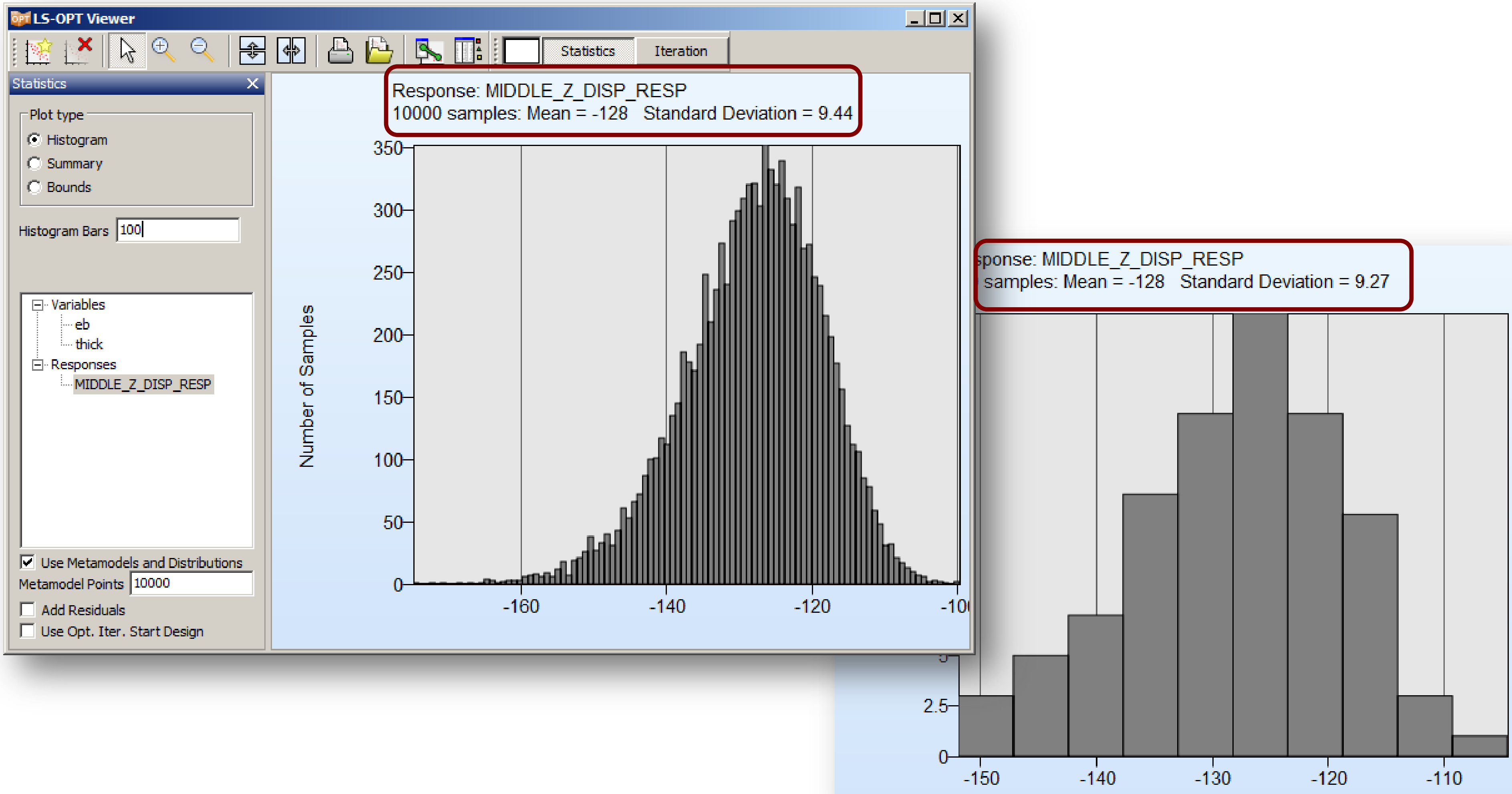
# Viewer

- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select “Statistical Tools”



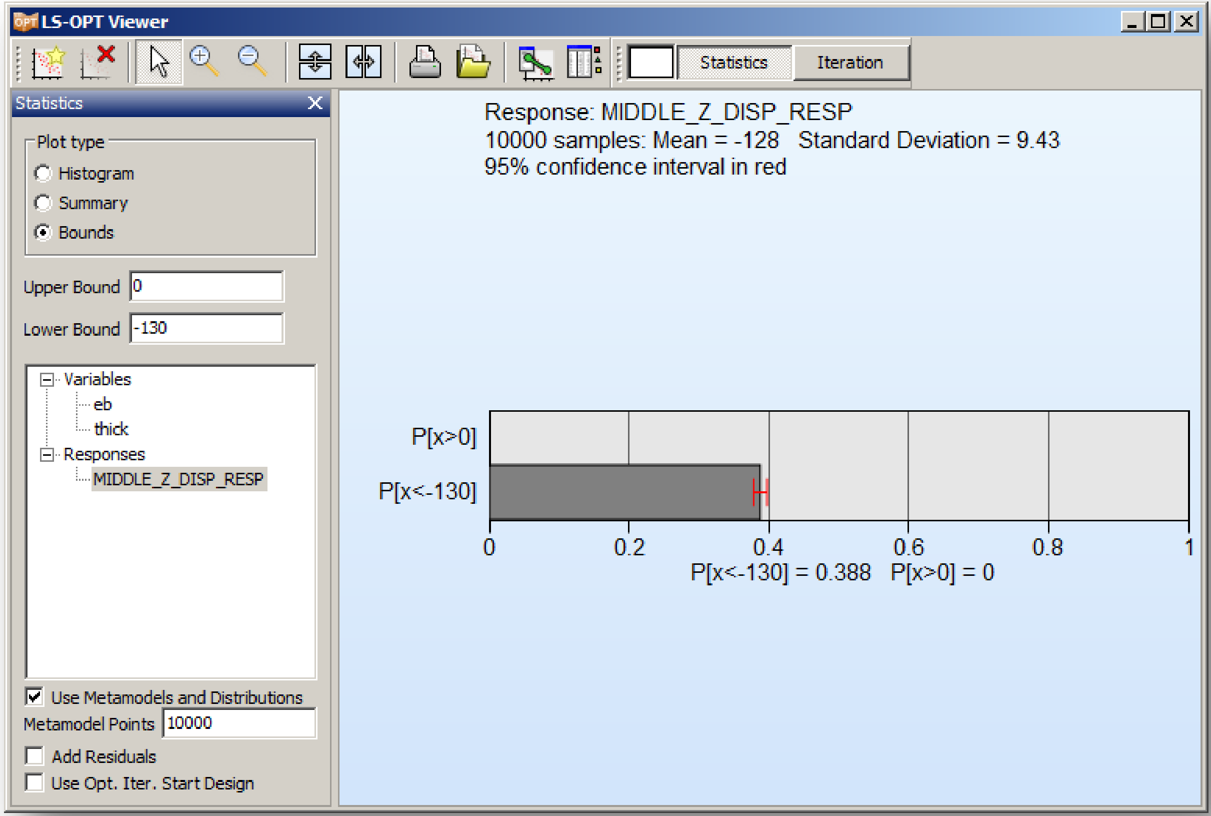
# Statistical Tools

- Plot histograms for Variables and for Response `MIDDLE_Z_DISP_RESP`
- In Histogram Bars field type `100`



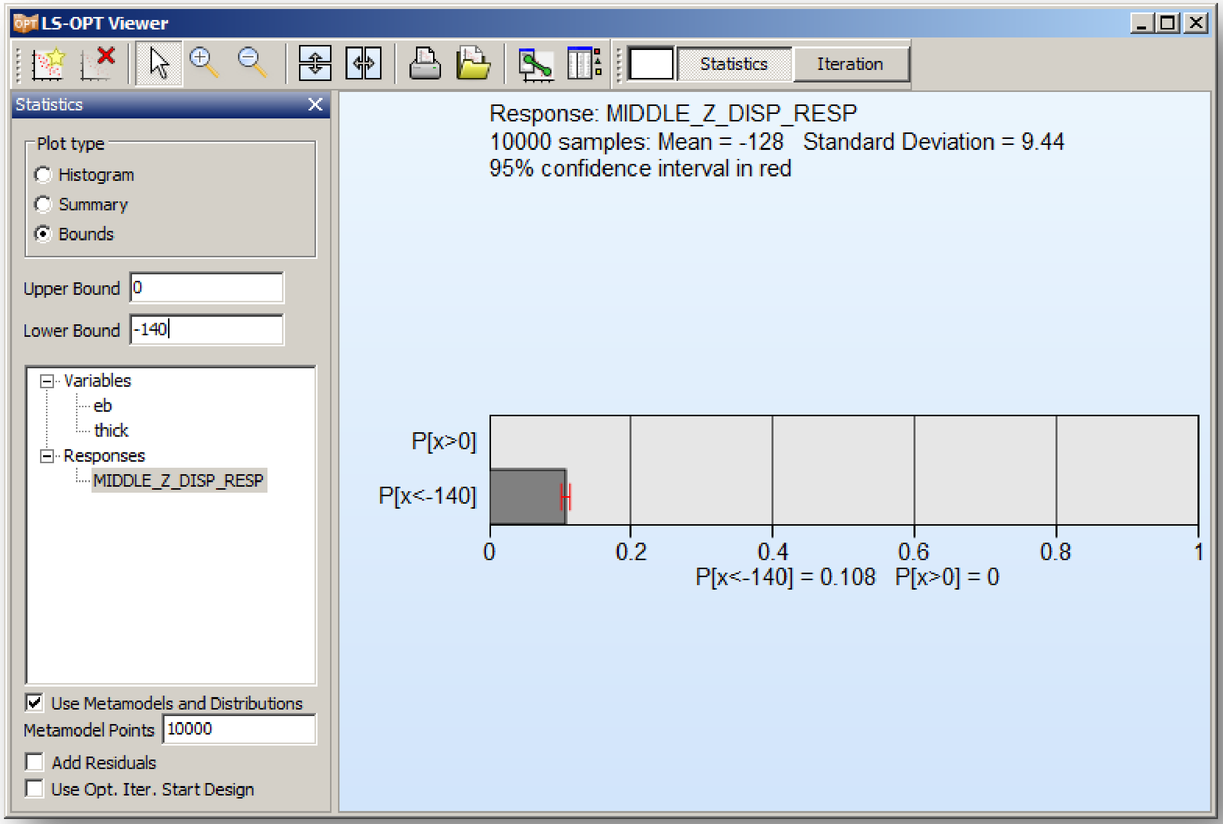
# Statistical Tools

- Go back to Statistical Tools
- Pick Bounds and type `-130` as Lower bound for `MIDDLE_Z_DISP_RESP` Response
- Probability of z-displacement exceeding `-130` is `38.8%` (previously `38.0%`)



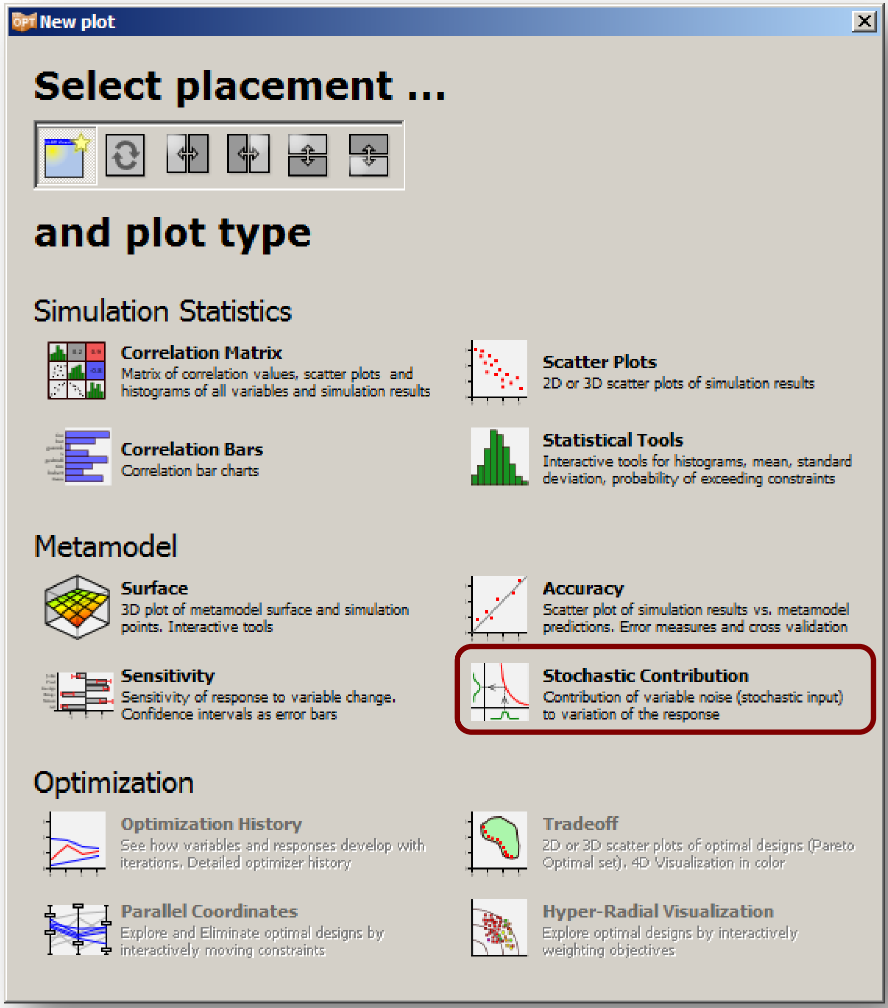
# Statistical Tools

- Go back to Statistical Tools
- Pick Bounds and type `-140` as Lower bound for `MIDDLE_Z_DISP_RESP` Response
- Probability of z-displacement exceeding `-140` is `10.8%`



# Viewer

- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select "Stochastic Contribution"



# Stochastic Contribution

- Standard deviation in response due to distributions of variables should be shown