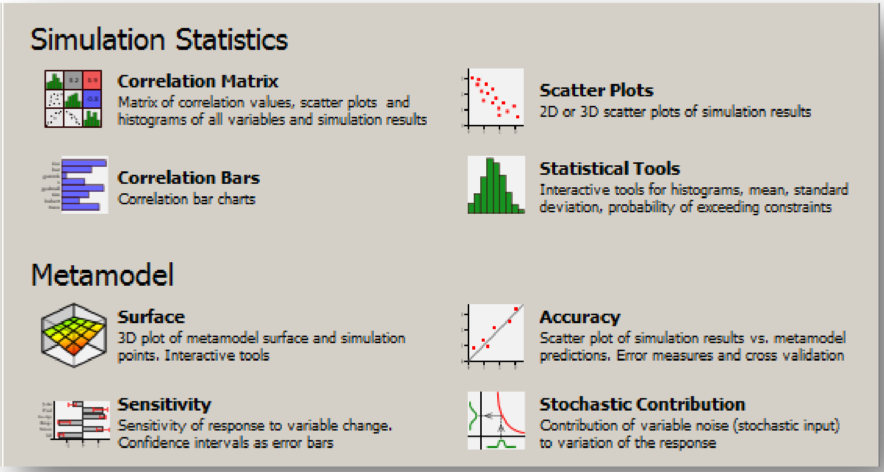
Introductory Course: Using LS-OPT® on the TRACC Cluster

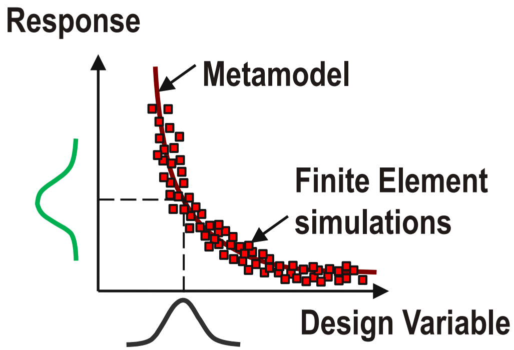
2.6 – Introduction to Reliability Based Design Optimization (RBDO)

By: Cezary Bojanowski, PhD

Goals of Stochastic Investigations

- The stochastic investigations are performed to obtain information on the:
  - Variation of the responses due to variation of input (variables, parameters).
  - Significance / Contribution of the parameters with respect to specific responses.
  - Robust Parameter Design (Objective → min standard deviation of the response)
  - Assessment of reliability of structure
  - Design optimization subject to reliability based constrains





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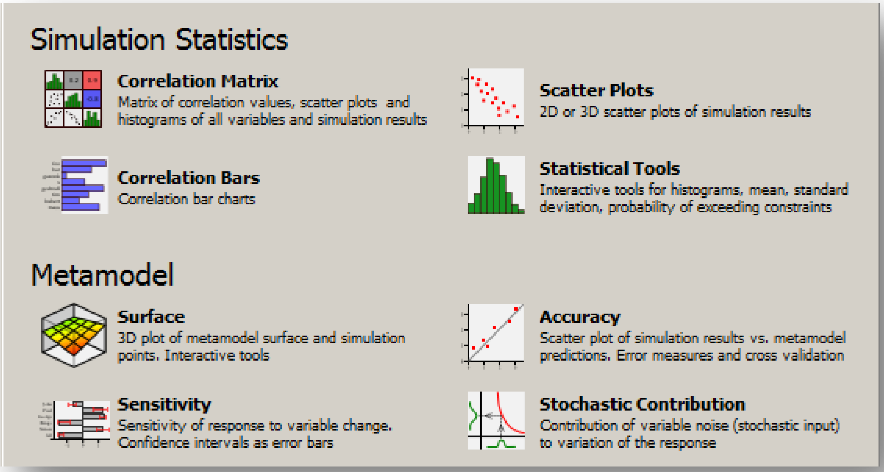
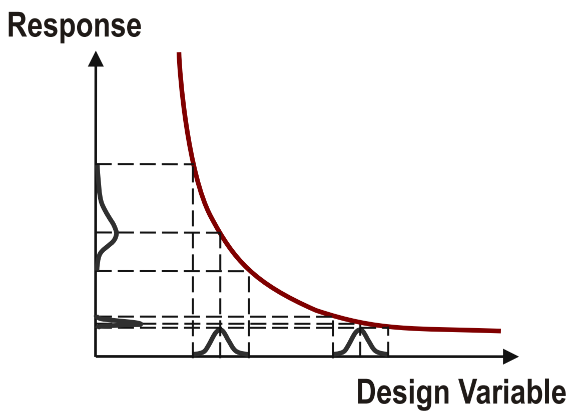




Reliability Assessment

- The reliability of a given design is defined as:

$$Reliability = 1 - probability\ of\ failure$$

- It may be assessed by comparing a numerically determined failure probability with a given target probability of an event. Reliability of a specific design is achieved if condition below is satisfied:

$$P_f < P_t$$

- The selection of the target probability is problem dependent and often oriented to the desired product quality vs. product cost.
- Sometimes safety distance is defined based on these definitions as:

$$d_s = P_t - P_f$$

- Positive values of d_s indicate a permissible design, and higher positive values stand for a more reliable design.

Reliability Based Design Optimization

- The objective of the reliability based design optimization (RBDO) may be formulated regarding two different aspects:

  - In order to achieve a maximum reliability of an investigated subject with respect to a set of problem dependent constraints the objective is given:

    $$\max\, (d_s) \mid \mathbf{c}(x_i) > 0$$

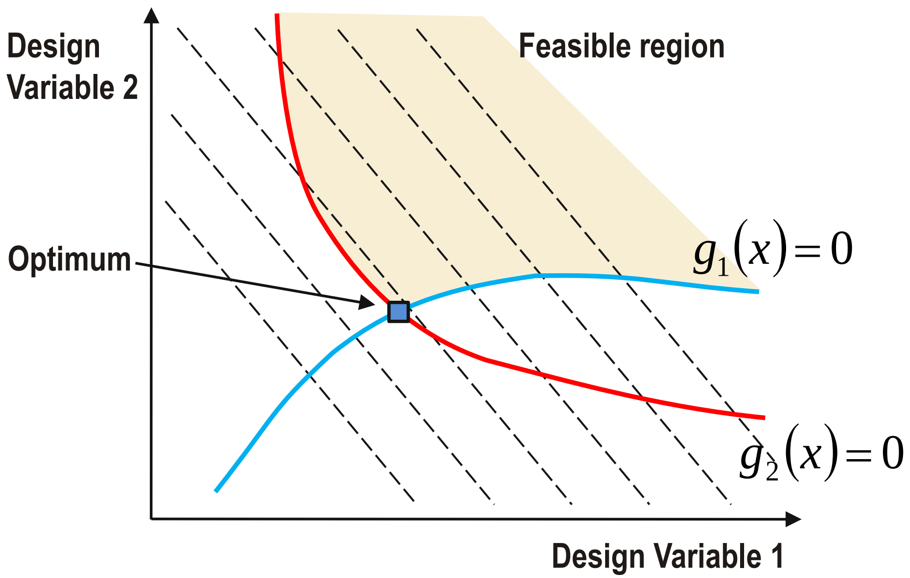
    where: $c(x) > 0$ is set of constraints, and safety level is maximized under the condition that the constraints are met.
  - Conventional objectives q concern with e.g. the reduction of cost due to minimization of the mass. In order to combine these optimization goals with the idea of a reliable design, the objective of RBDO may also be formulated as:

    $$\min(q) \mid d_s\,,\, \mathbf{c}(x_i) > 0$$

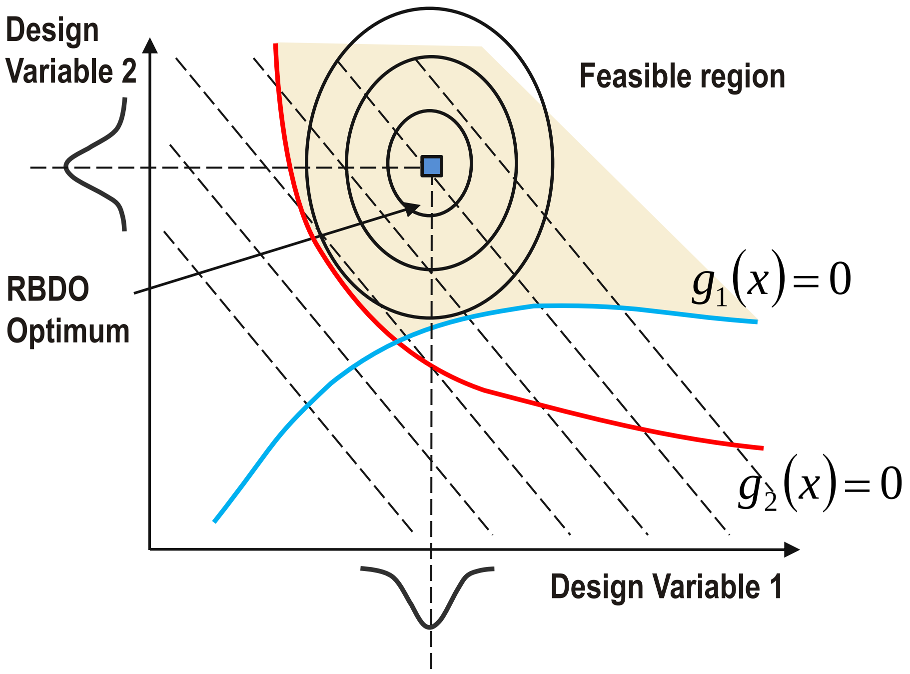
    The safety distance is additionally considered as constraint of an actual optimization problem.

Reliability Based and Deterministic Optimization

- Deterministic optimum



- RBDO optimum



- What is the probability of failure?
- Which point is likely to fail first?

RBDO and Deterministic Optimization

- Deterministic optimization problem:

$\min f(x)$	Objective function
$g_j(x) \geq 0; \quad j = 1,2,\ldots,m$	Inequality constraints
$h_k(x) = 0; \quad k = 1,2,\ldots,l$	Equality constraints
$x_{i,L} \leq x_i \leq x_{i,U}$	Side constraints - Bounds on variables

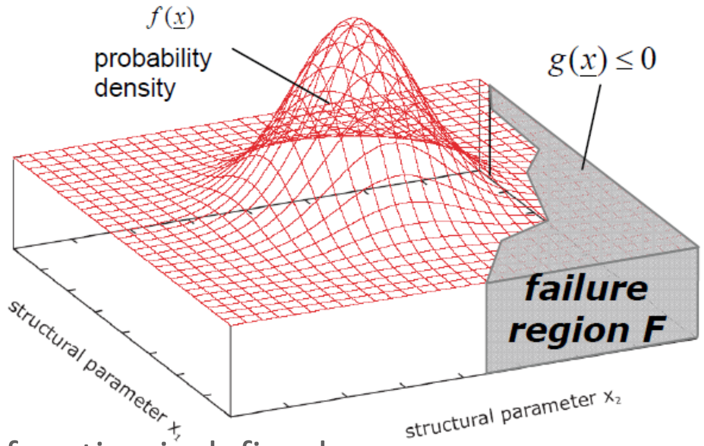
- RBDO optimization problem:

$\min f(x)$	Objective function
$P(g_j(x) \leq 0) \leq P_j; \quad j = 1,2,\ldots,m$	Reliability constraint
$h_k(x) = 0; \quad k = 1,2,\ldots,l$	Equality constraint
$x_{i,L} \leq x_i \leq x_{i,U}$	Side constraints - Bounds on variables

Reliability Assessment

- In order to determine the safety distance d_s, in the general case the failure probability has to be computed by numerical evaluation of the integral:

$$P_f = P[g(x) \le 0] = \int_{g(x)\le 0} f(x)dx$$



- Where *f(x)* denotes joint probability density function of the random variables *x* and *g(x)* represents limit state function.
- The limit state function is usually highly non-linear and is only given in non closed form. Usually the indicator function is defined as:

$$I_f(x) = \begin{cases} 1 & \text{if } x \in F \\ 0 & \text{if } x \notin F \end{cases} \text{ with } F = \{x \mid g(x) \le 0\}$$

- Then, probability of failure in simulation based problem is re-defined as:

$$P_f = \int_x I_f(x) \cdot f(x)dx$$

Reliability Assessment

- This enables the point estimation of the failure probability based on the sampling results of a Monte Carlo simulation according to:

$$\hat{P}_f = \frac{1}{N}\sum_{k=1}^{N} I_f(x_k)$$

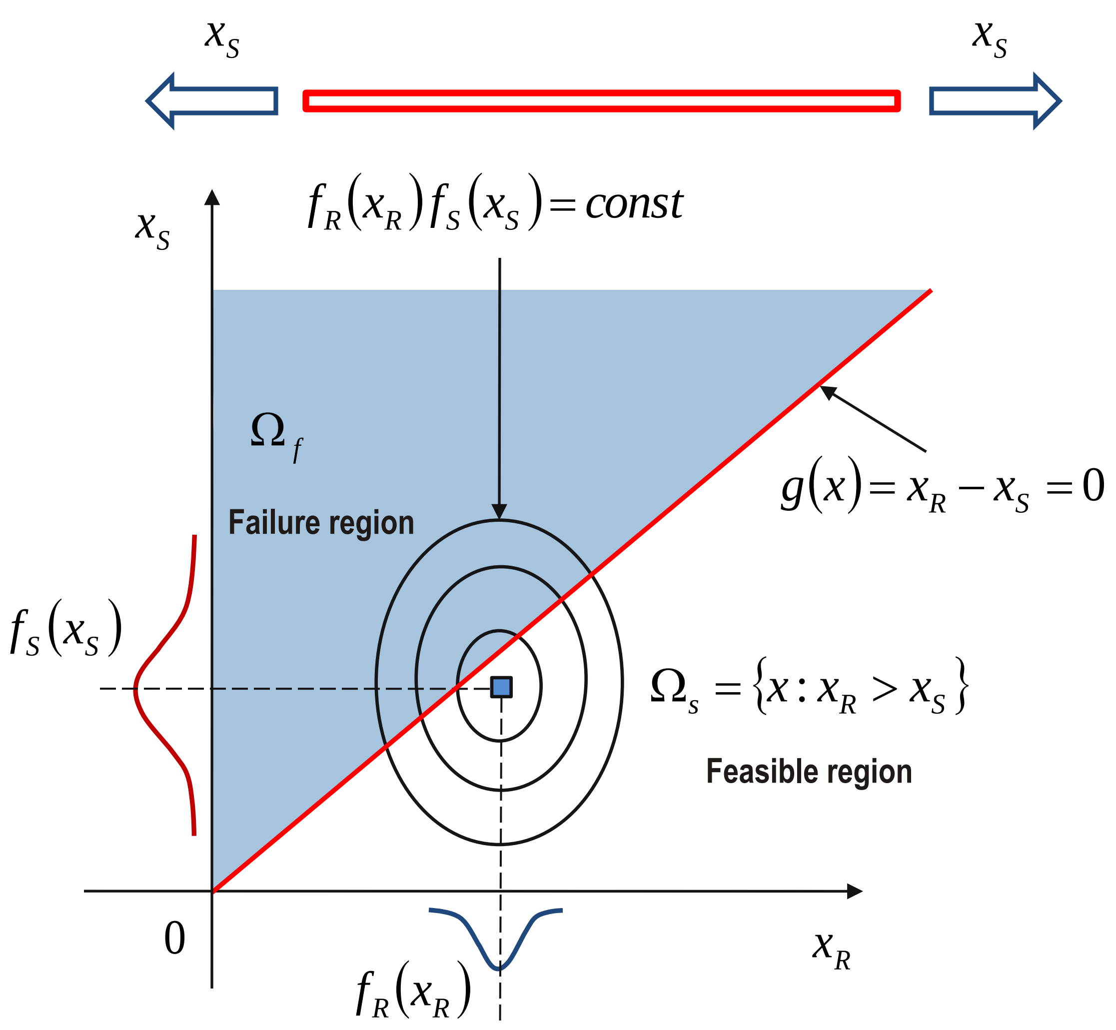
With N – sample size.

- A minimum of sample size is estimated by:

$$N \geq \frac{1-\hat{P}_f}{\hat{P}_f \cdot \delta^2_{\hat{P}_f}}$$

- Where $\delta_{\hat{P}_f} = \frac{\sigma}{\mu}$ is a coefficient of variation. N becomes very large for small values of the failure probability . Thus, it is advisable to apply metamodel based stochastic simulation techniques.

Basic Structural Reliability Problem



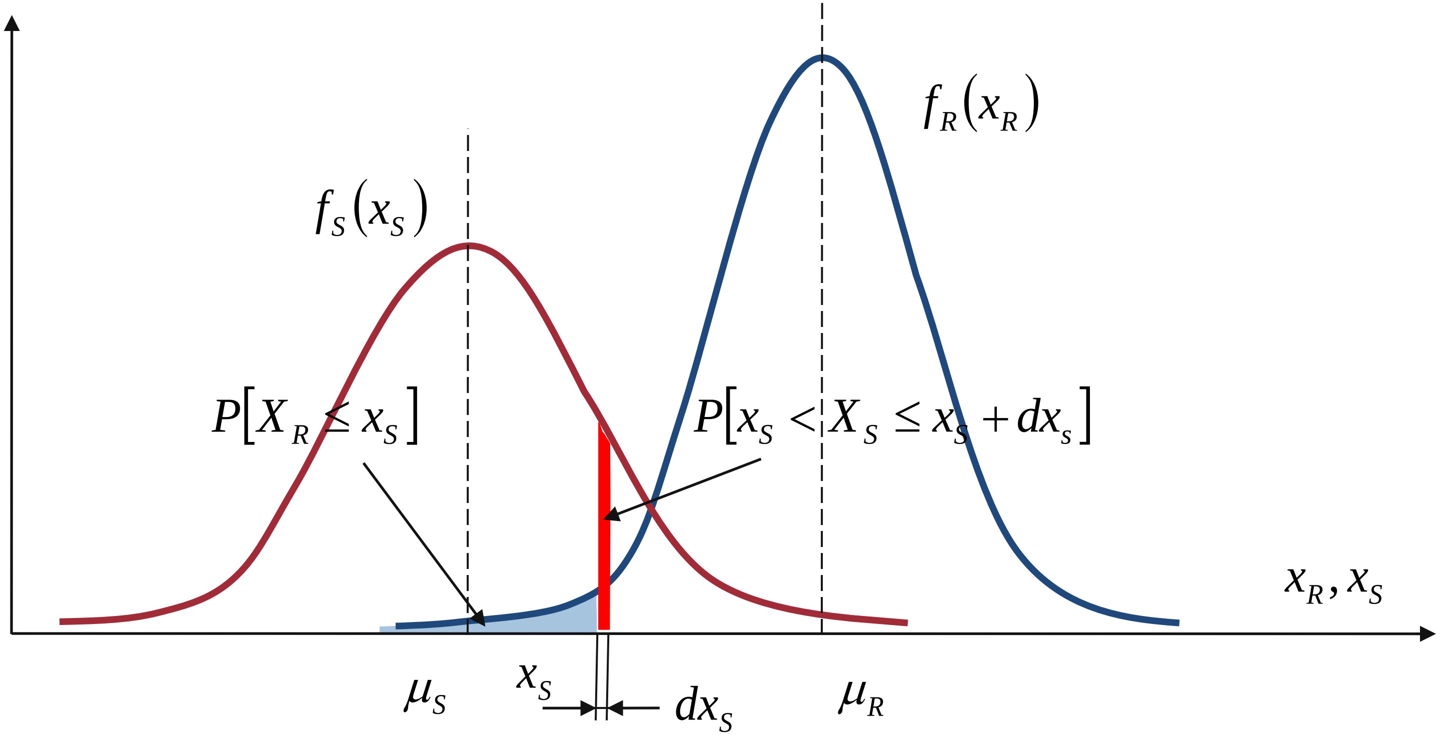
x_S – Tension load

x_R – Tensile strength

x_R, x_S – Non-negative, independent random variables with probability density functions: $f_R(x_R), f_S(x_S)$

$x_R \leq x_S$ – failure

Probability of Failure



- Probability of failure: $P_f = P[X_R \le X_S] = \int_{x_R \le x_S} f_R(x_R) f_S(x_S) dx_R dx_S = \int_0^\infty F_R(x_S) f_S(x_S) dx_S$
- The integral is hard (if not impossible) to compute for most of the real cases.

Probability of Failure

- Alternative formulation in terms of limit state function $g(X_R, X_S) = X_R - X_S$
- Since $g \leq 0$ defines the failure region, probability of failure can be defined as:

$$P_f = P[g(X_R, X_S) < 0]$$

- The mean of the limit state function (mean margin of safety):

$$\mu_g = \mu_R - \mu_S$$

- When resistance and load are not correlated, the standard deviation of the limit state function is:

$$\sigma_g = \sqrt{\sigma_R^2 - \sigma_S^2}$$

Reliability Index

- The probability of failure can be computed as follows:

$$P_f = \int_{-\infty}^{0} f_g(g)dg = \Phi\left(-\frac{\mu_g}{\sigma_g}\right) = \Phi(-\beta)$$

Where: $\Phi(\cdot)$ is cumulative distribution function

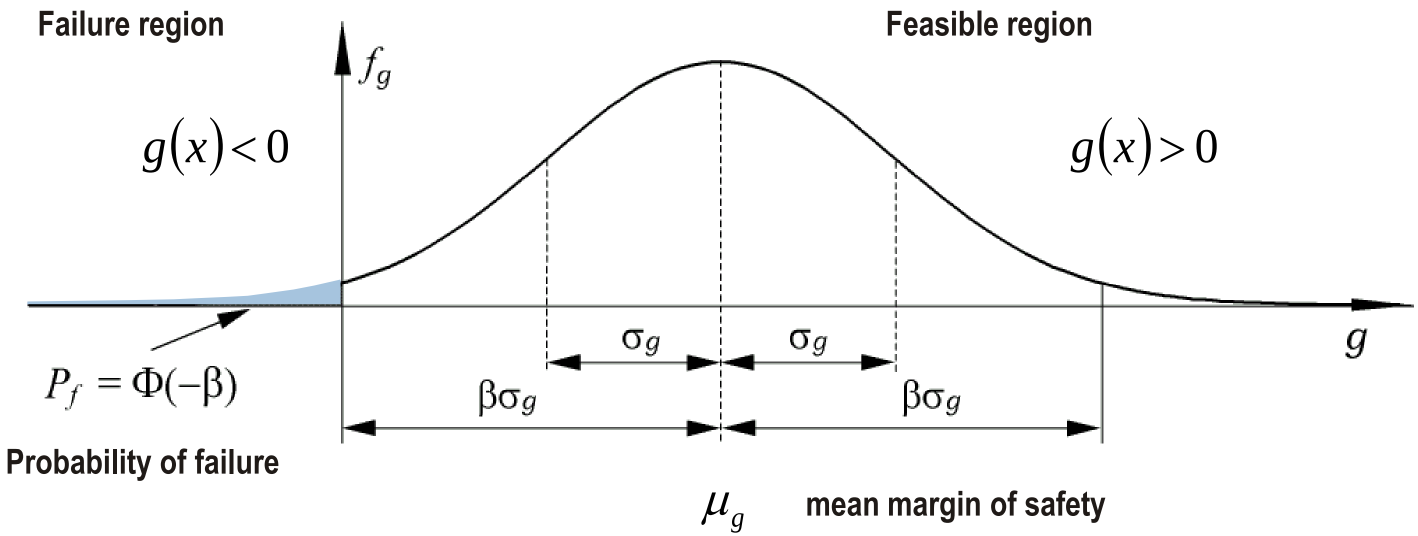
- The Safety Index or Reliability Index is defined as:

$$\beta = \frac{\mu_g}{\sigma_g}$$

- The Reliability index indicates the distance of the mean margin of safety from the failure region

Reliability Index - Graphical Interpretation

Reliability Index $$\beta = \frac{\mu_g}{\sigma_g}$$

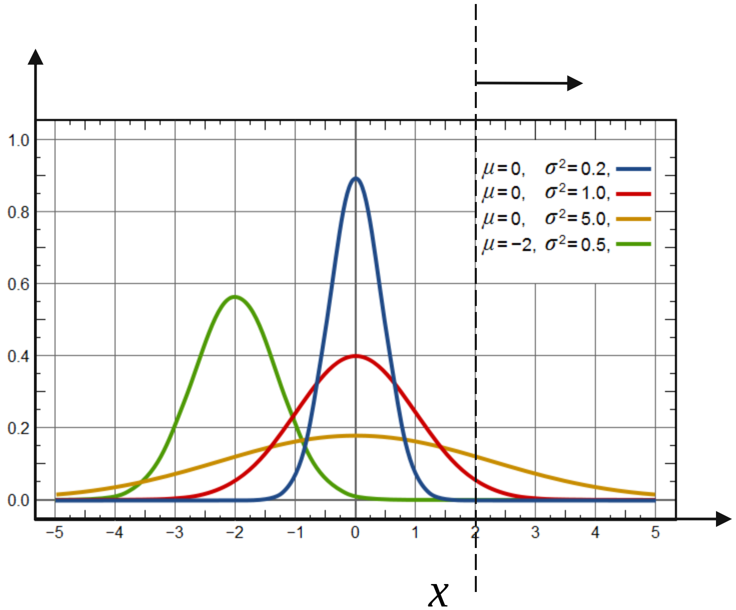


$$Reliability = 1 - probability\ of\ failure$$

Probability and Cumulative Distribution Function

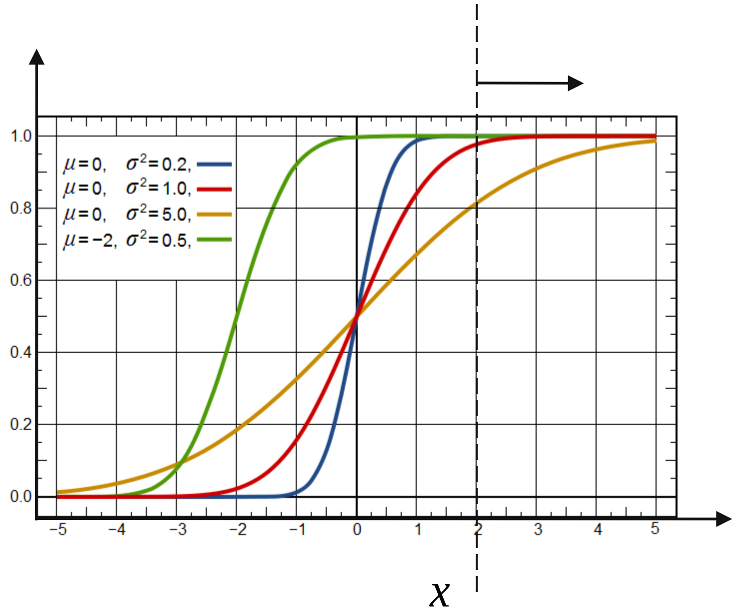
Probability distribution function (PF)

$$f(t), \phi(t)$$



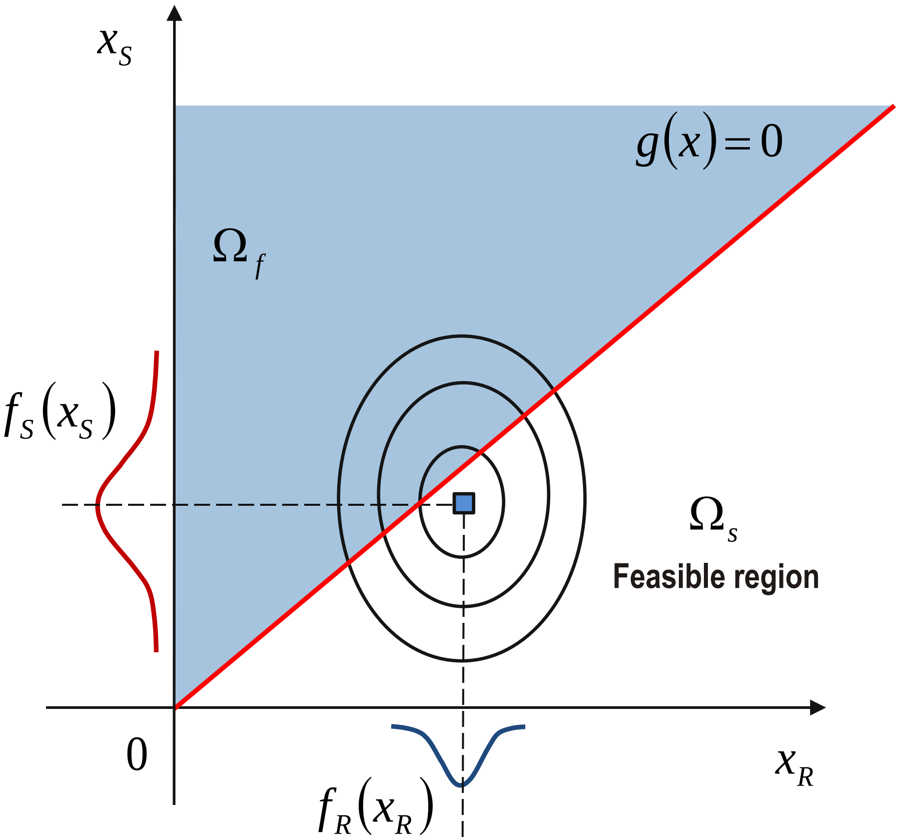
Cumulative distribution function (CDF)

$$F(x), \Phi(x) = \int_{-\infty}^{x} \phi(t)dt$$



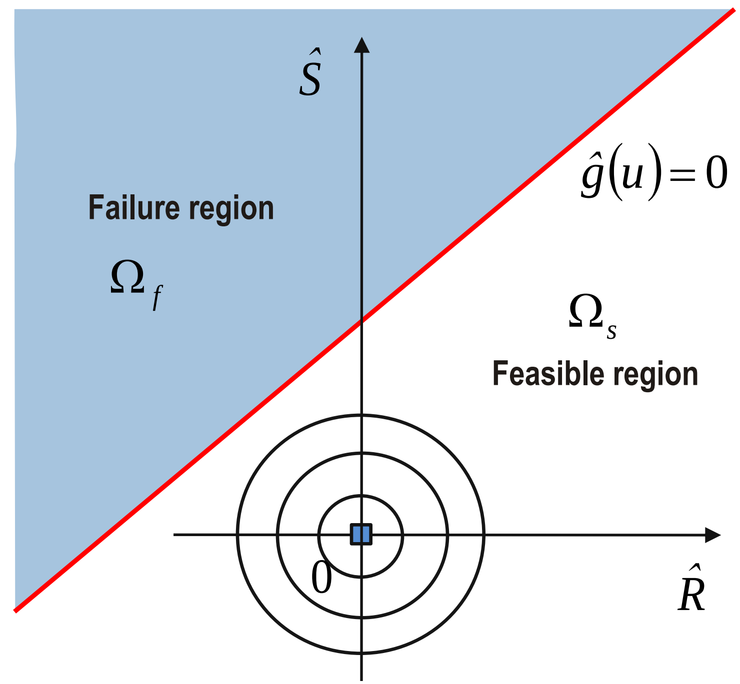
Hasofer and Lind (HL) Transformation

- **X - space**



- **U - space**

$$\hat{R} = \frac{x_R - \mu_R}{\sigma_R} \qquad \hat{S} = \frac{x_S - \mu_S}{\sigma_S}$$



Hasofer and Lind (HL) Transformation

- The random variables are mapped into set of normalized and independent variables:

$$\hat{R} = \frac{x_R - \mu_R}{\sigma_R} \qquad \hat{S} = \frac{x_S - \mu_S}{\sigma_S}$$

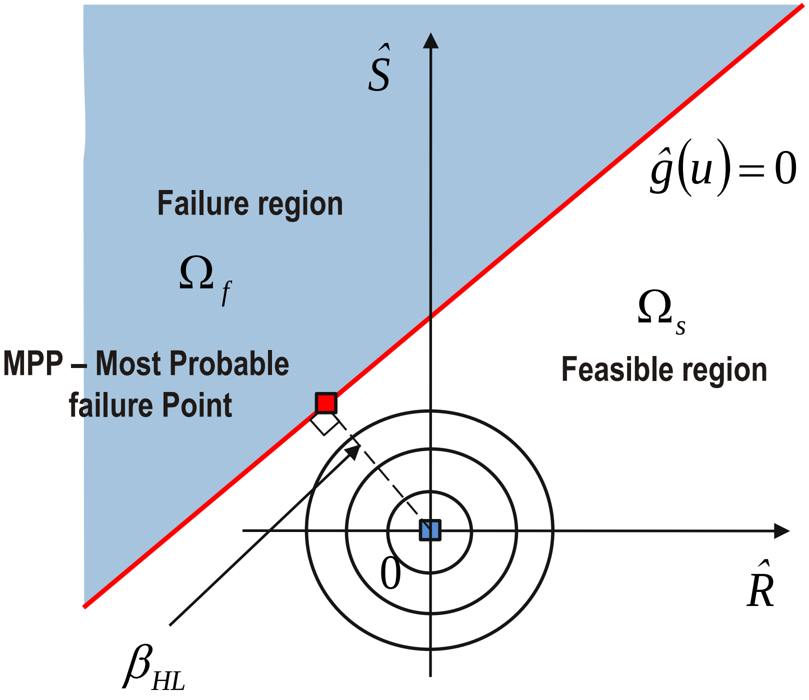
- The limit state function takes form:

$$\hat{g}(u) = \hat{R}\sigma_R + \mu_R - \hat{S}\sigma_S - \mu_S$$

- The shortest distance from the origin to the failure surface is equal to the safety index:

$$\beta_{HL} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

- The closes point on this surface is called Most Probable Point (MPP) of failure



Hasofer and Lind (HL) Transformation

- In general case the limit state function is nonlinear and can be defined as:

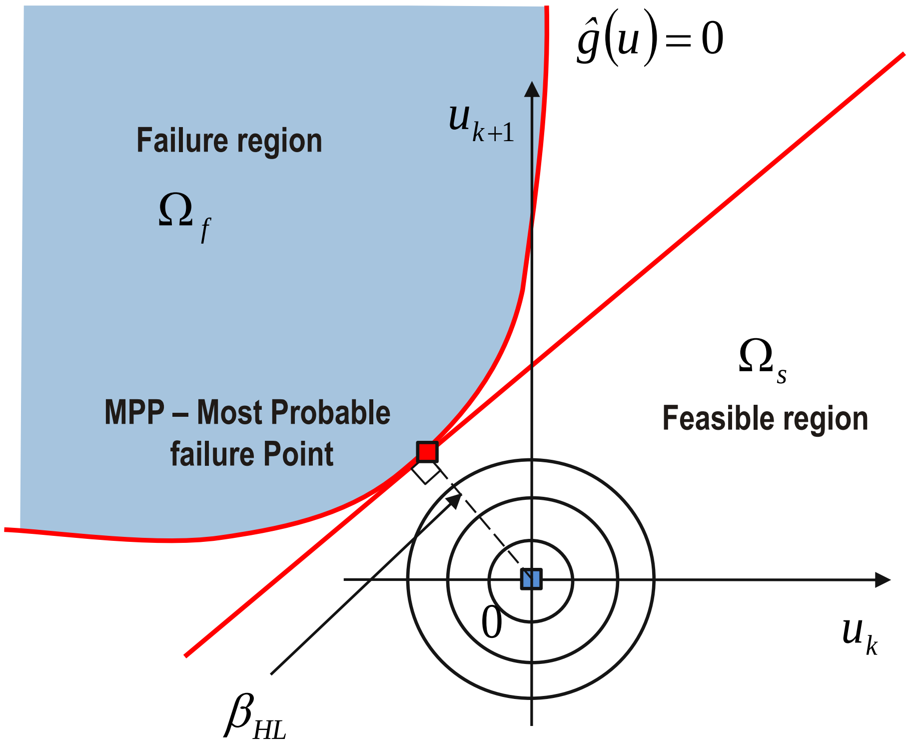
$$\hat{g}(u) = g\left(\left\{u_1\sigma_{x_1} + \mu_{x_1}, u_2\sigma_{x_2} + \mu_{x_2}, \ldots, u_n\sigma_{x_n} + \mu_{x_n}\right\}^T\right) = 0$$

- Where:

$$u_i = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}}$$

- First order Taylor series of expansions of $\hat{g}(u)$ at the MPP is considered

- The method is called First Order Second Moment (FOSM) since only mean and standard deviation (second moment about the mean) are used in description of inputs and outputs.



Reliability Based Design Optimization

- RBDO optimization problem can be reformulated into:

$\min\ f(x)$	Objective function
$P\left(g_j(x) \le 0\right) - \phi\left(-\beta_{t_j}\right) \le 0; \quad j = 1,2,\ldots,m$	Reliability constraint
$h_k(x) = 0; \quad k = 1,2,\ldots,l$	Equality constraint
$x_{i,L} \le x_i \le x_{i,U}$	Side constraints - Bounds on variables

- Safety index is the solution of a constrained optimization problem in the standard normal space:

$$min\ \beta(u) = \left(u^T u\right)^{\frac{1}{2}}$$

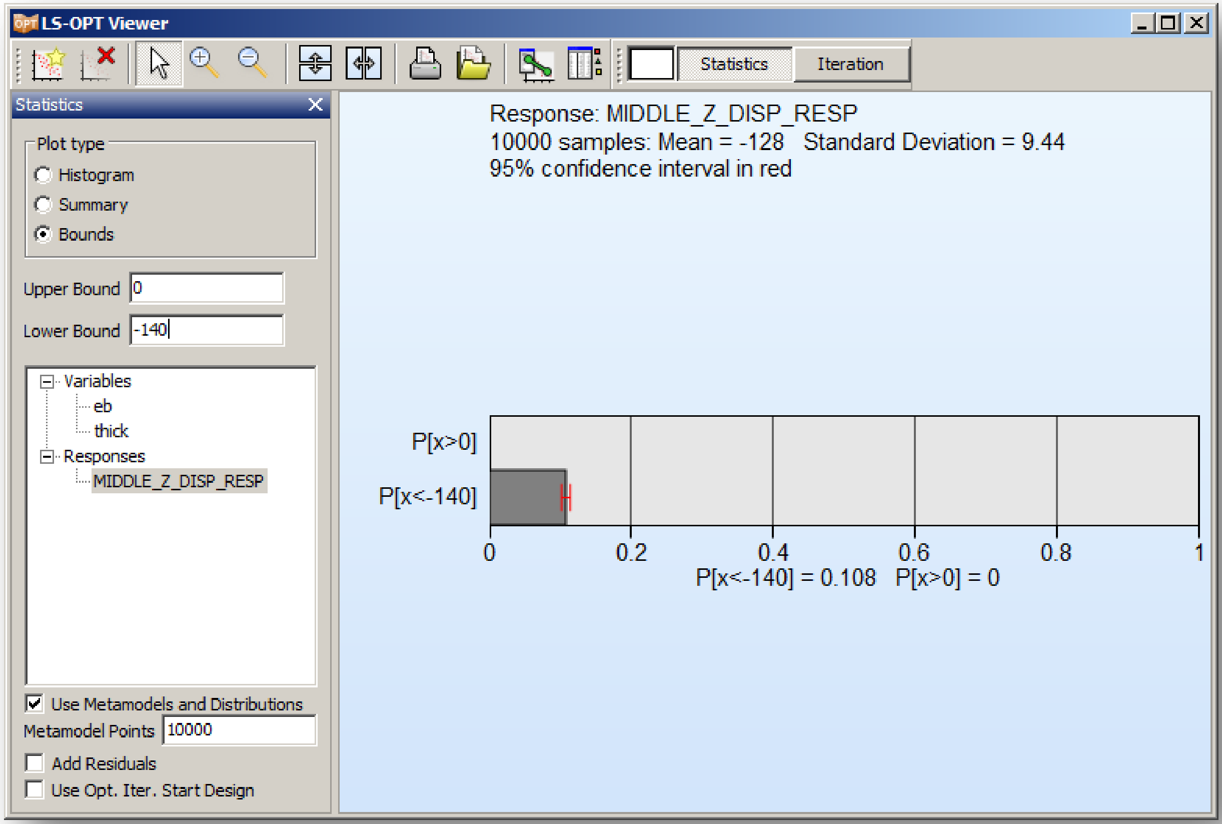
$$g(u) = 0$$

$u^* -$ MPP

- Checking reliability constraints in design optimization becomes inner level optimization.
- There are several methods of solving RBDO problems: Double Loop, Sequential Optimization and Reliability Assessment (SORA), Single Loop.

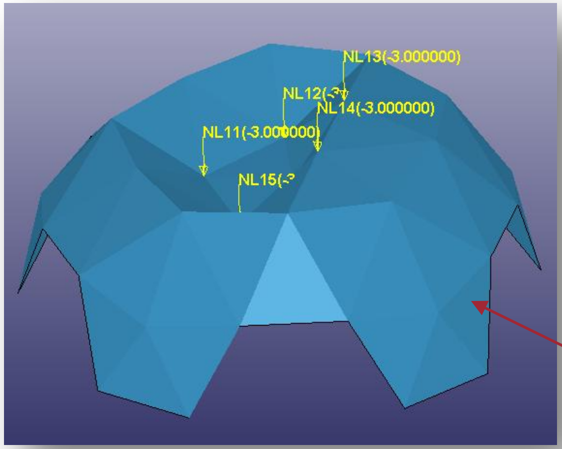
Problem

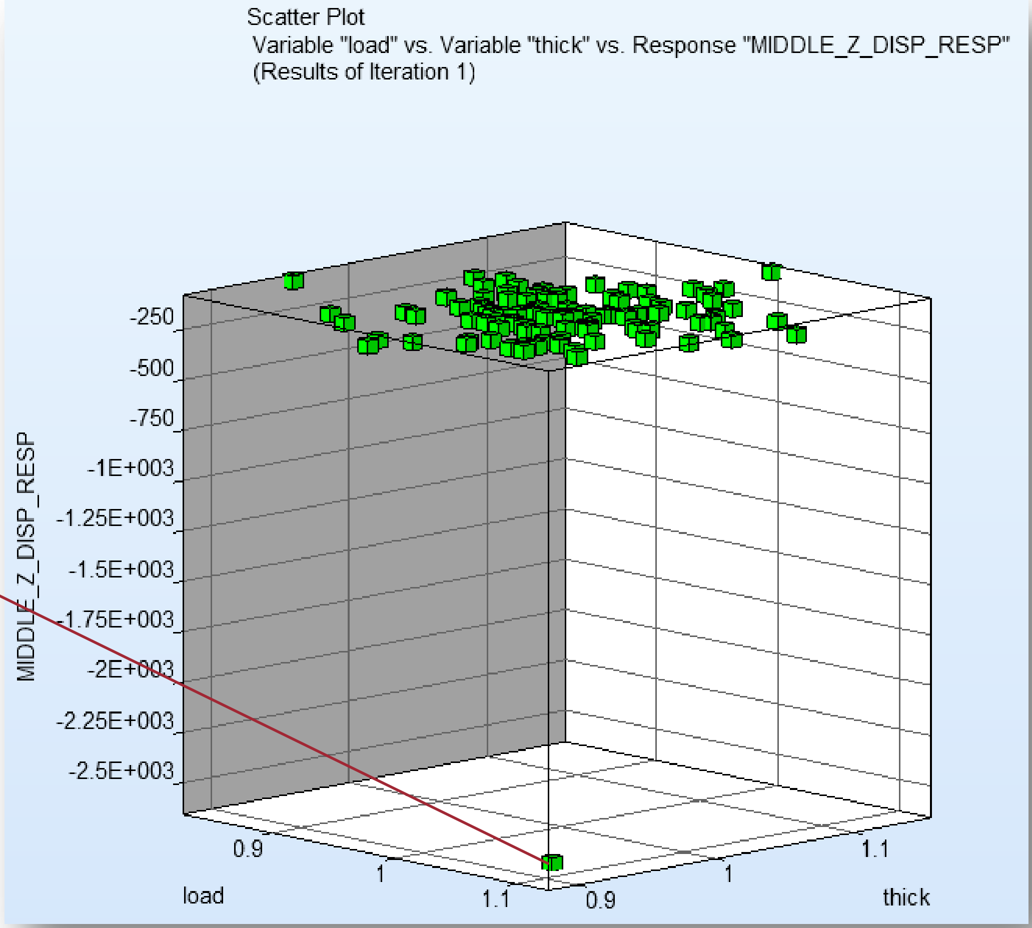
- Recall from last example:
- Probability of z-displacement exceeding -140 is 10.8%



Problem

- After adding variability in load 5% one out of a 100 samples was leading to collapse of the structure!



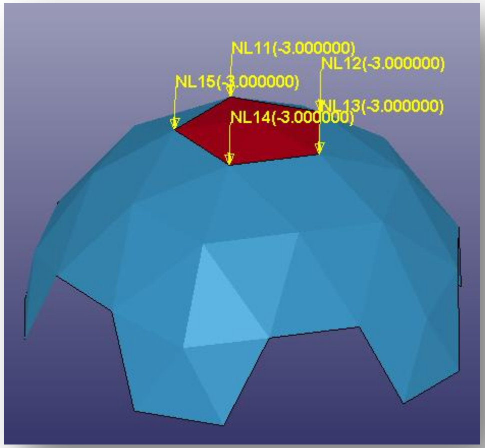


Problem

- The system can be redesign to reduce the probability of the failure.
- RBDO tasks can be defined accordingly:
  - Find ranges for design variables that will assure that the probability of occurrence of unwanted event will remain below specified limit.
  - Here: Find ranges for design variables that will assure that the probability of z-displacement being greater than 140 units is below 2.5%

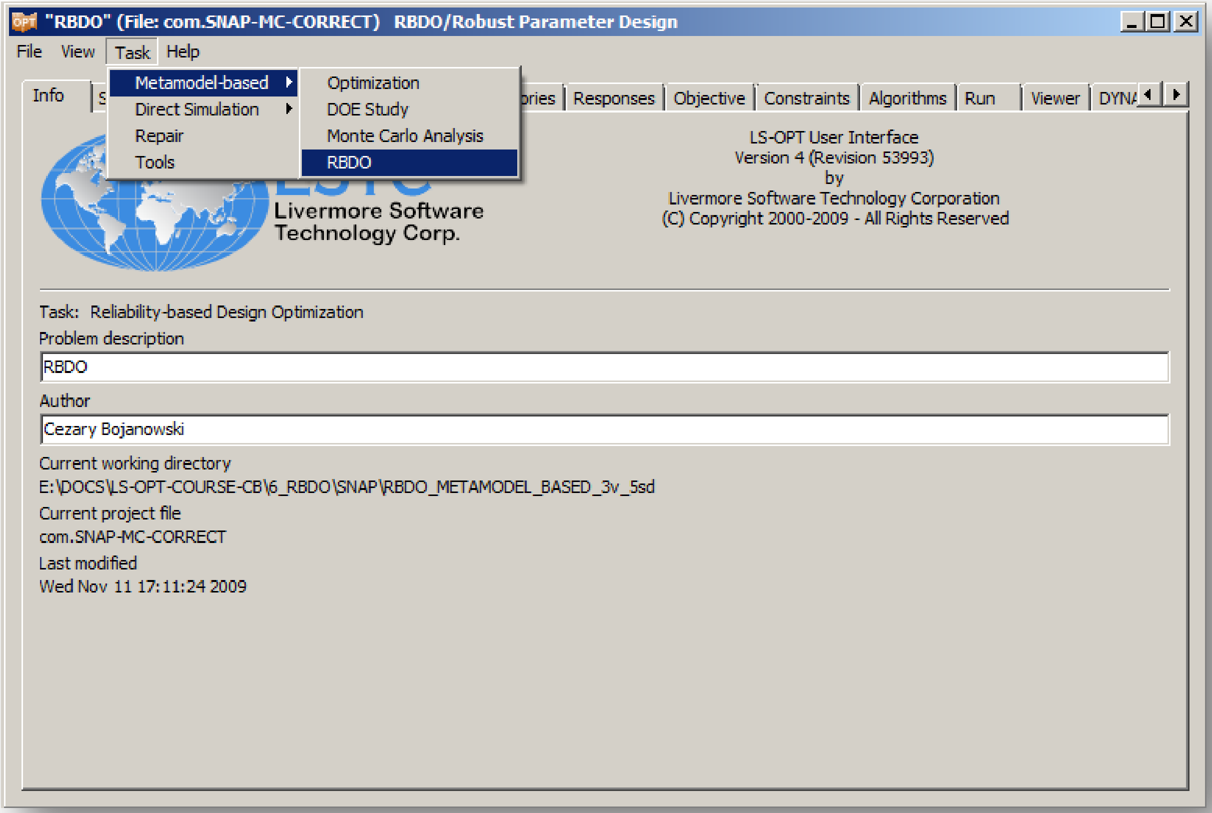
New K-file

- Two parts created
- New variables:
  - Design variable `thick1`
  - Design variable `thick2`
  - Design variable `eb`
  - Noise variable `load`
- Objective: minimize mass of the structure.
- Constraint: z-displacement of node `51` less than `-140` with probability not greater than `2.5%`



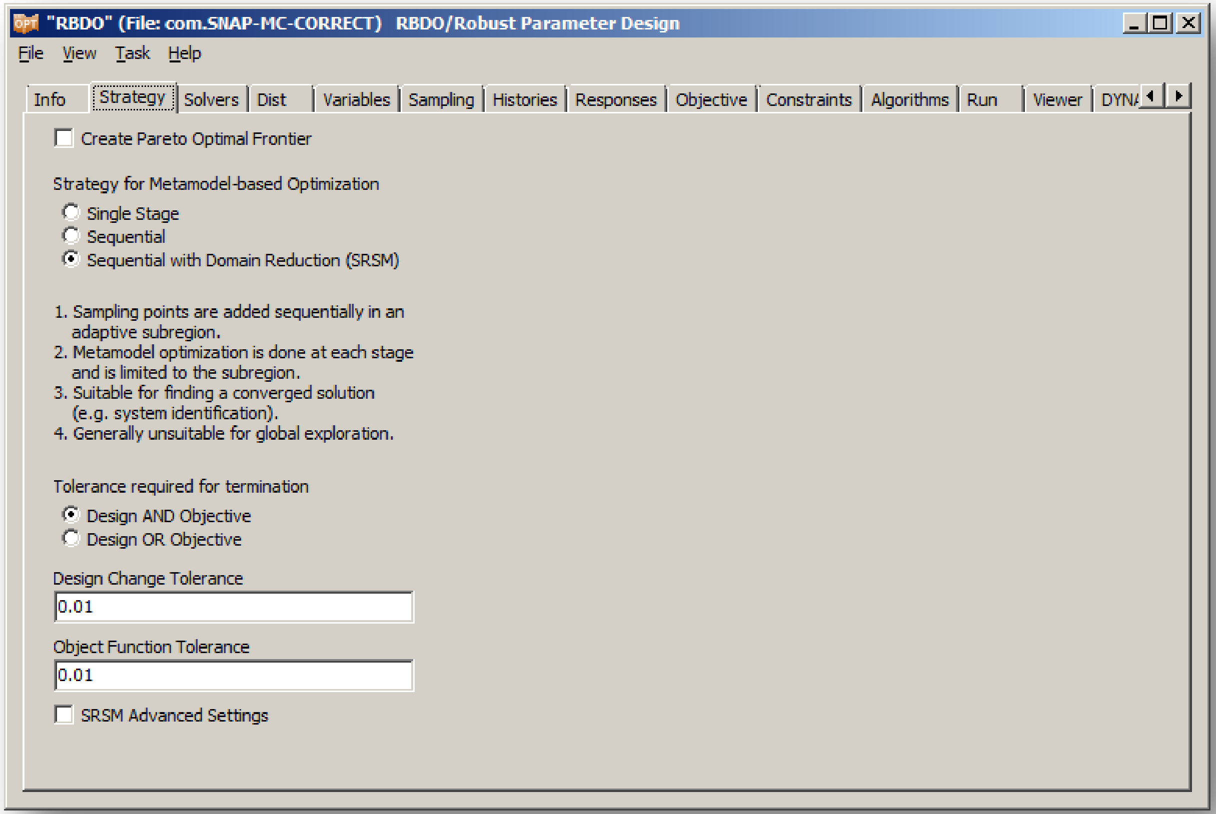
Task Tab

- Go to Task tab
- Select RBDO from Metamodel based group



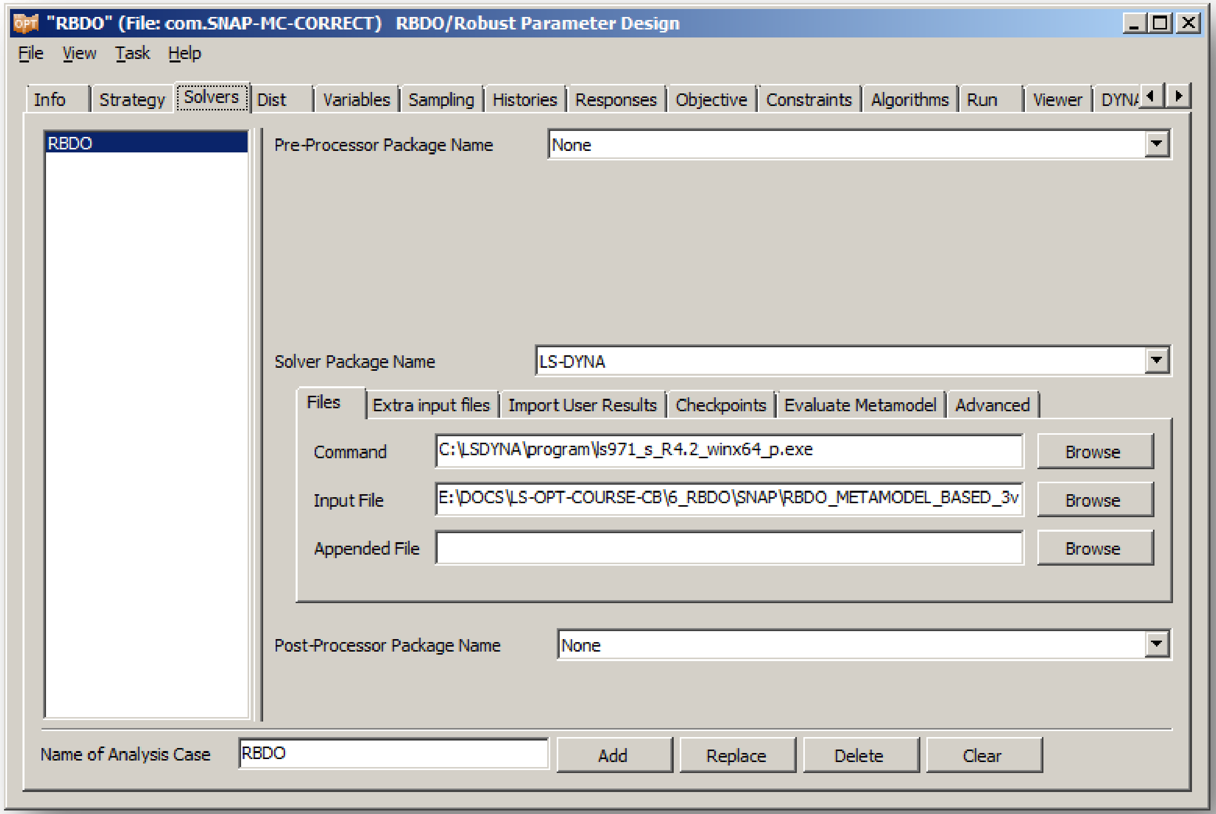
Strategy Tab

- Go to Strategy tab
- Select Sequential with Domain Reduction SRSM as an Optimization Strategy



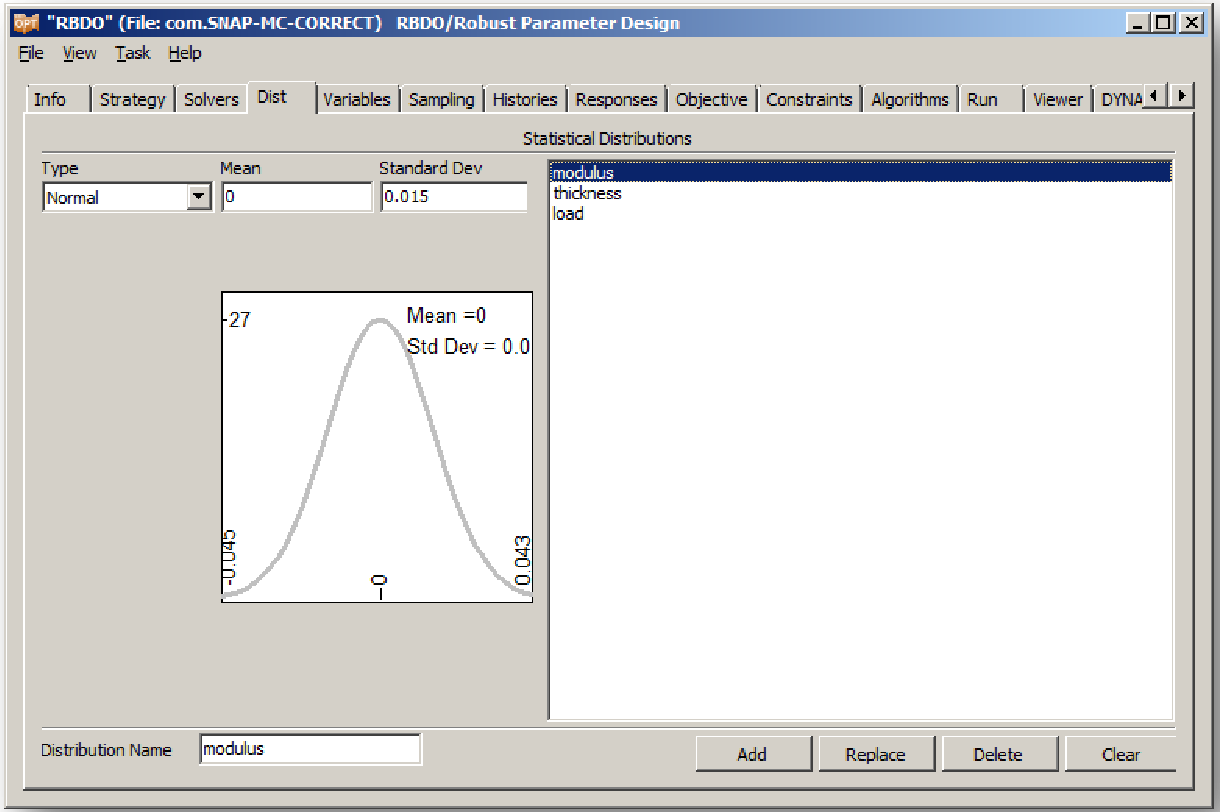
Solver Tab

- Navigate to appropriate `lsoptscript` in Command field.
- Find correct k-file in Input File field
- Enter RBDO as a Name of Analysis Case and press Add



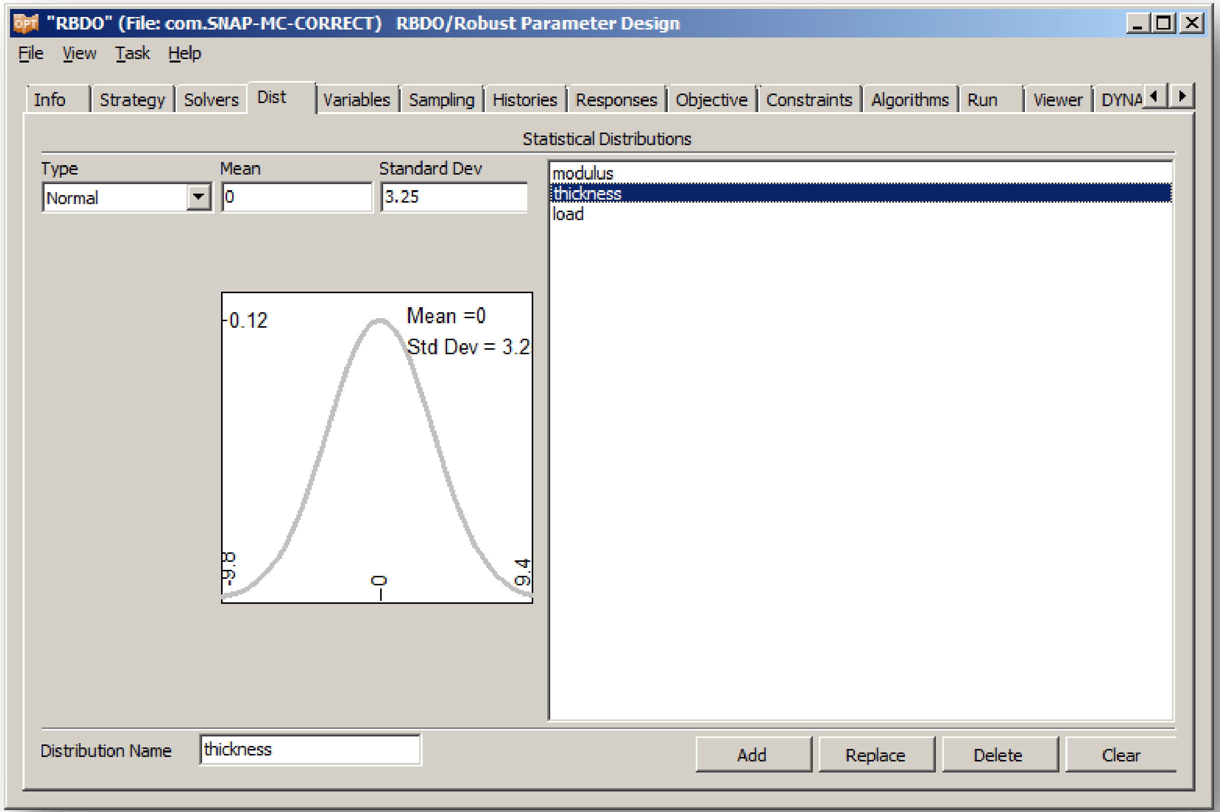
Distributions Tab

- Modify distribution `modulus` to: Mean `0` and Standard Deviation `0.015` (5 % of initial value 0.3)



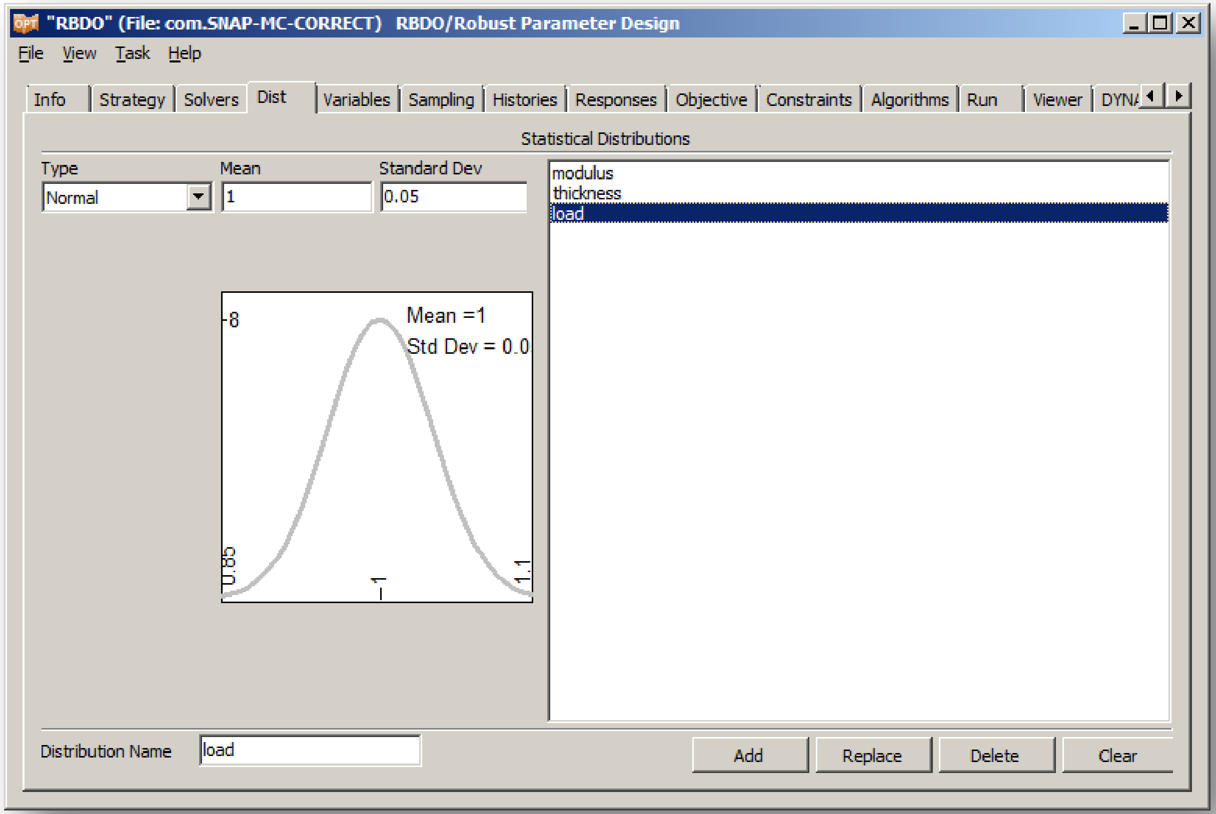
Distributions Tab

- Modify distribution `thickness` to: Mean `0` and Standard Deviation `3.25` (5 % of initial value 65)



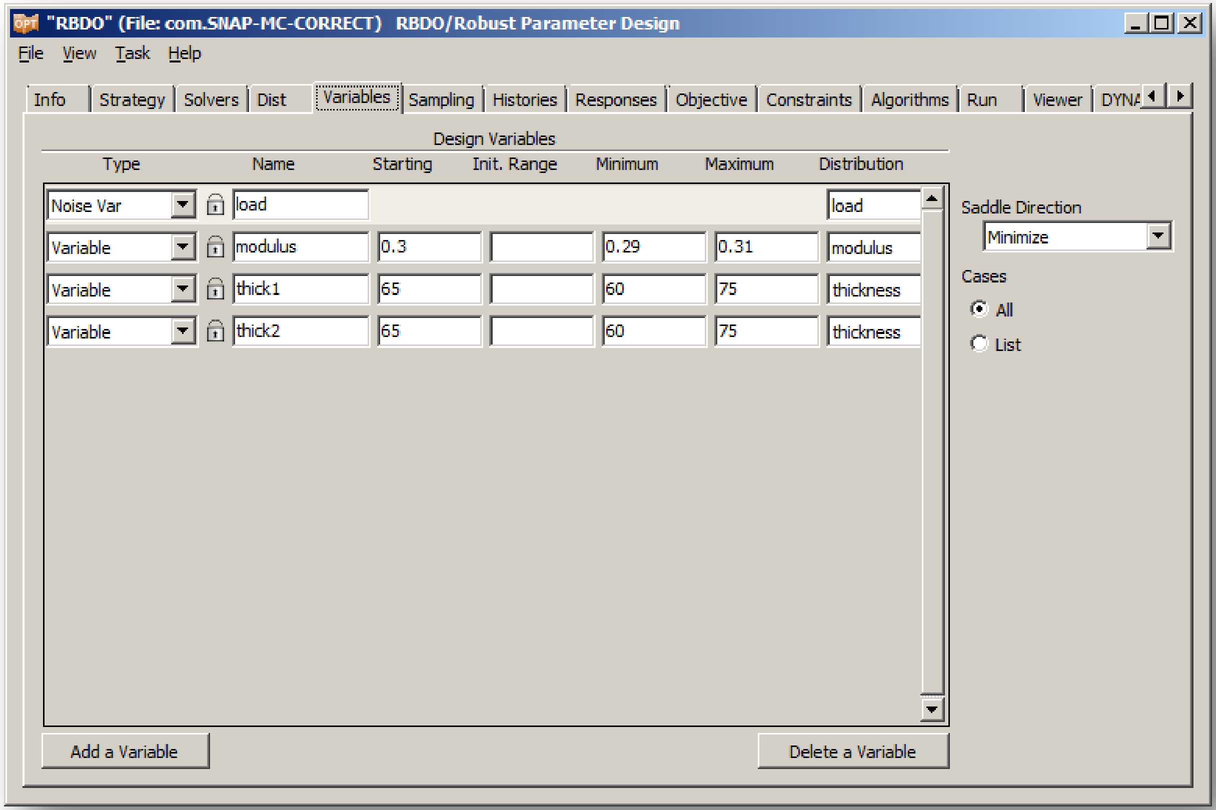
Distributions Tab

- Create additional Normal distribution `load` with Mean `1` and Standard Deviation `0.05`



Variables Panel

- Create variable `load` with distribution `load`
- Create variable `modulus` with starting value `0.3` min `0.29` and max `0.31`
- Assign to it distribution `modulus`
- Create variables `thick1` and `thick2` with starting value `65` min `60` and max `75`
- Assign to them distribution `thickness`



K-file Modification

Previously:

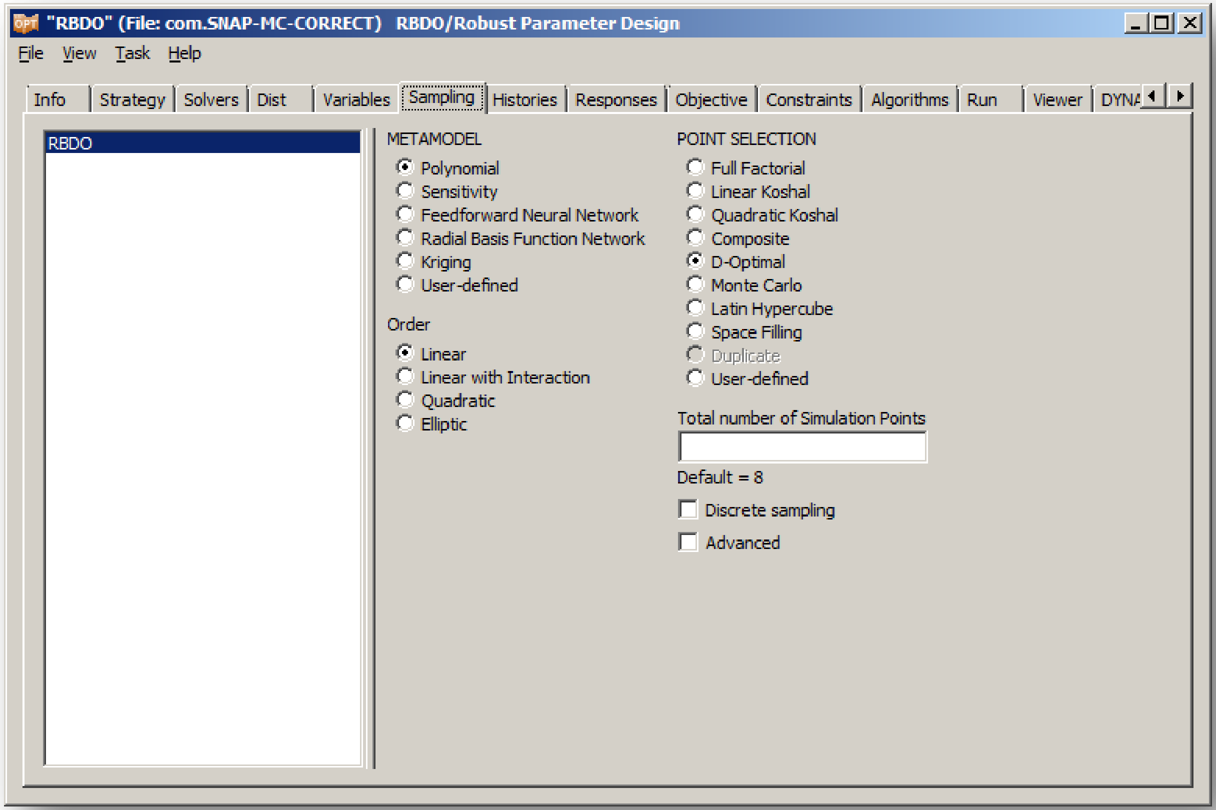
```
*SECTION_SHELL
$#   secid    elform      shrf       nip     propt   qr/irid     icomp     setyp
         2         4     0.000         3         0         0         0         0
$#      t1        t2        t3        t4      nloc     marea      idof    edgset
<<65*thick>>,<<65*thick>>,<<65*thick>>,<<65*thick>>
```

Now:

```
*PARAMETER
Rthick,65
*SECTION_SHELL
$#   secid    elform      shrf       nip     propt   qr/irid     icomp     setyp
         2         4     0.000         3         0         0         0         0
$#      t1        t2        t3        t4      nloc     marea      idof    edgset
&thick2,&thick2,&thick2,&thick2
```

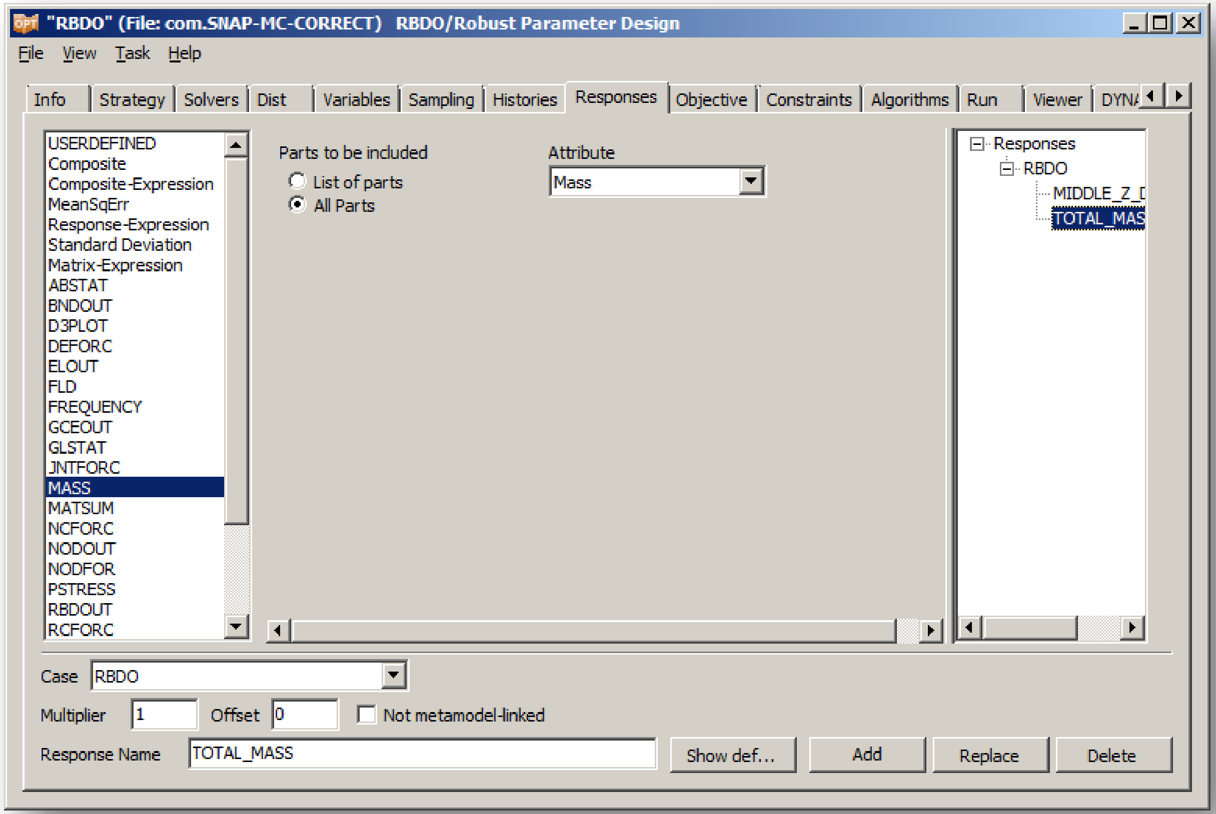
Sampling Tab

- Go to Sampling Tab
- Select Polynomial Metamodel with Linear Order
- Use D-Optimal Point Selection method and leave default 8 Simulation Points



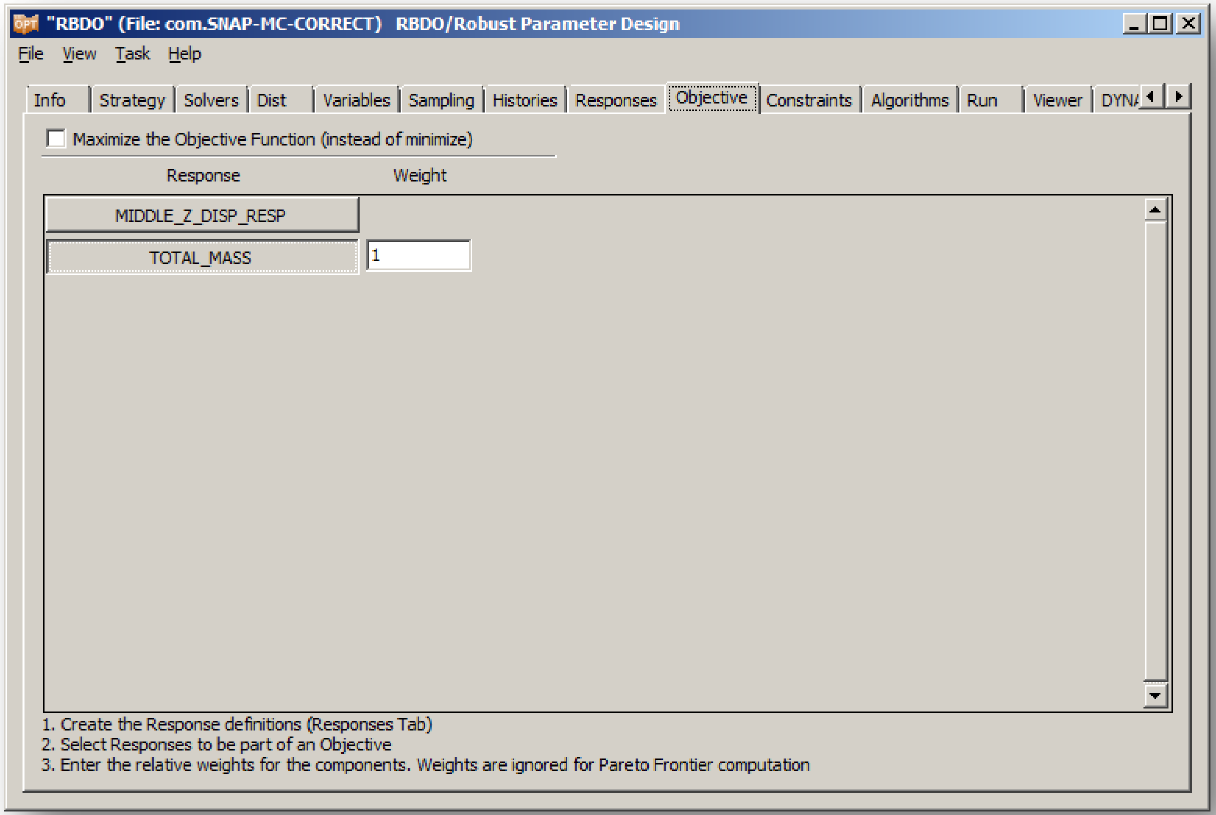
Responses Tab

- Go to Responses Tab
- From left window select MASS and pick All Parts to be included in the response
- Enter TOTAL_MASS for response name and press Add



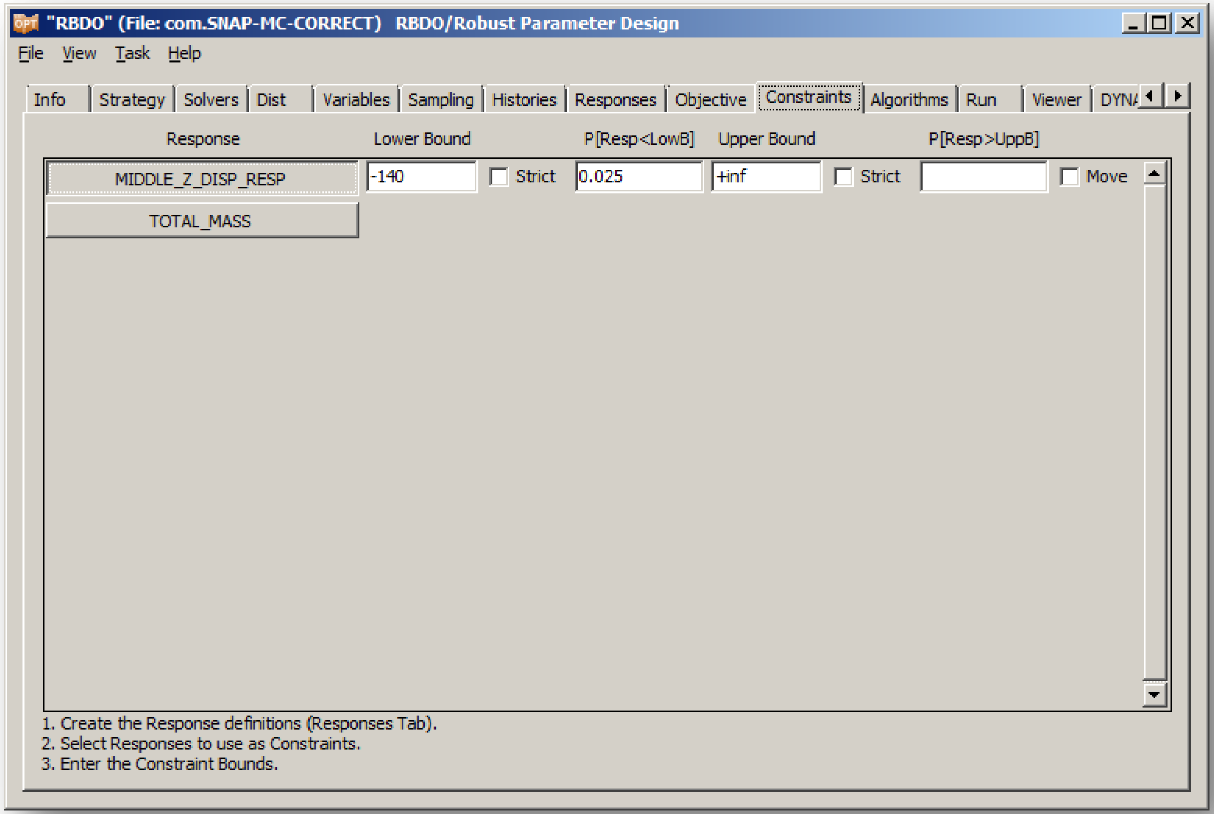
Objective Tab

- Go to Objective Tab
- Select TOTAL_MASS and leave default Weight 1.0



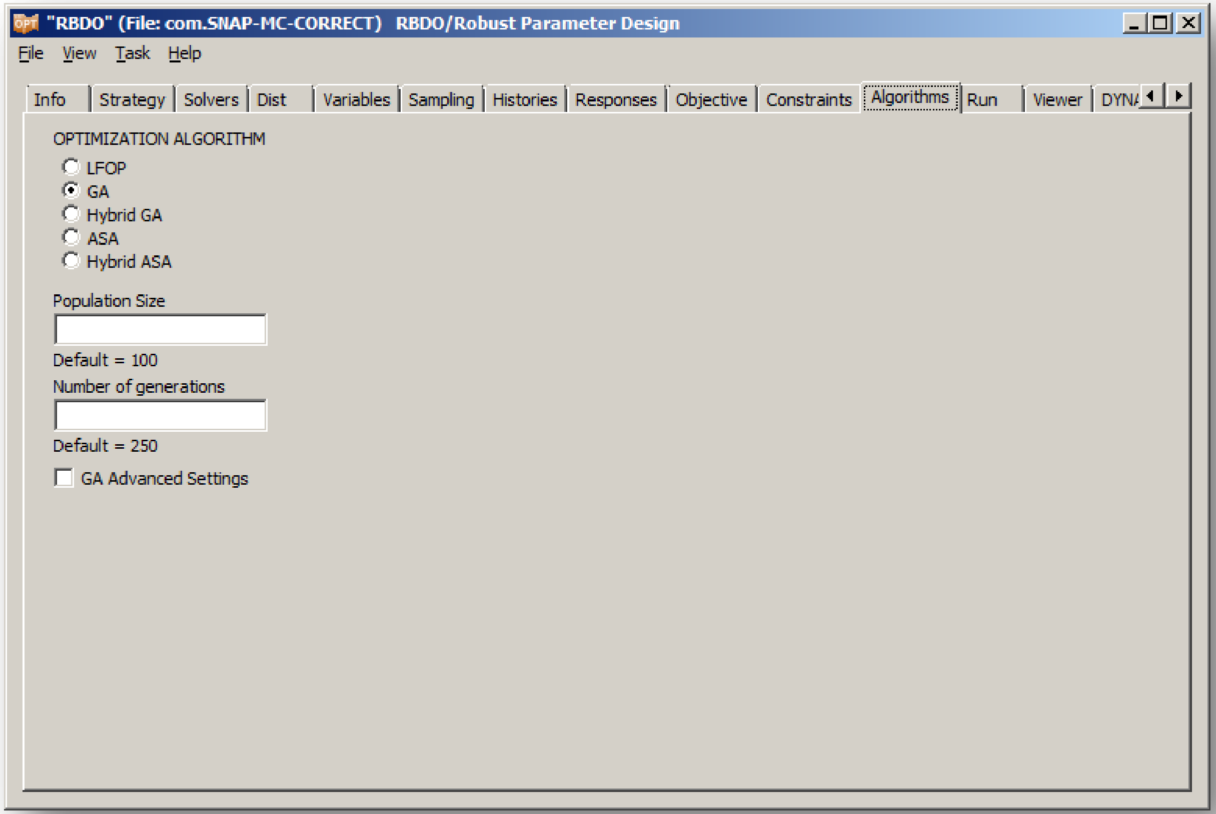
Constraints Tab

- Go to Constraints Tab
- Select `MIDDLE_Z_DISP_RESP`
- Enter `-140` for lower bound and `0.025` for probability of response being lower than that lower bound



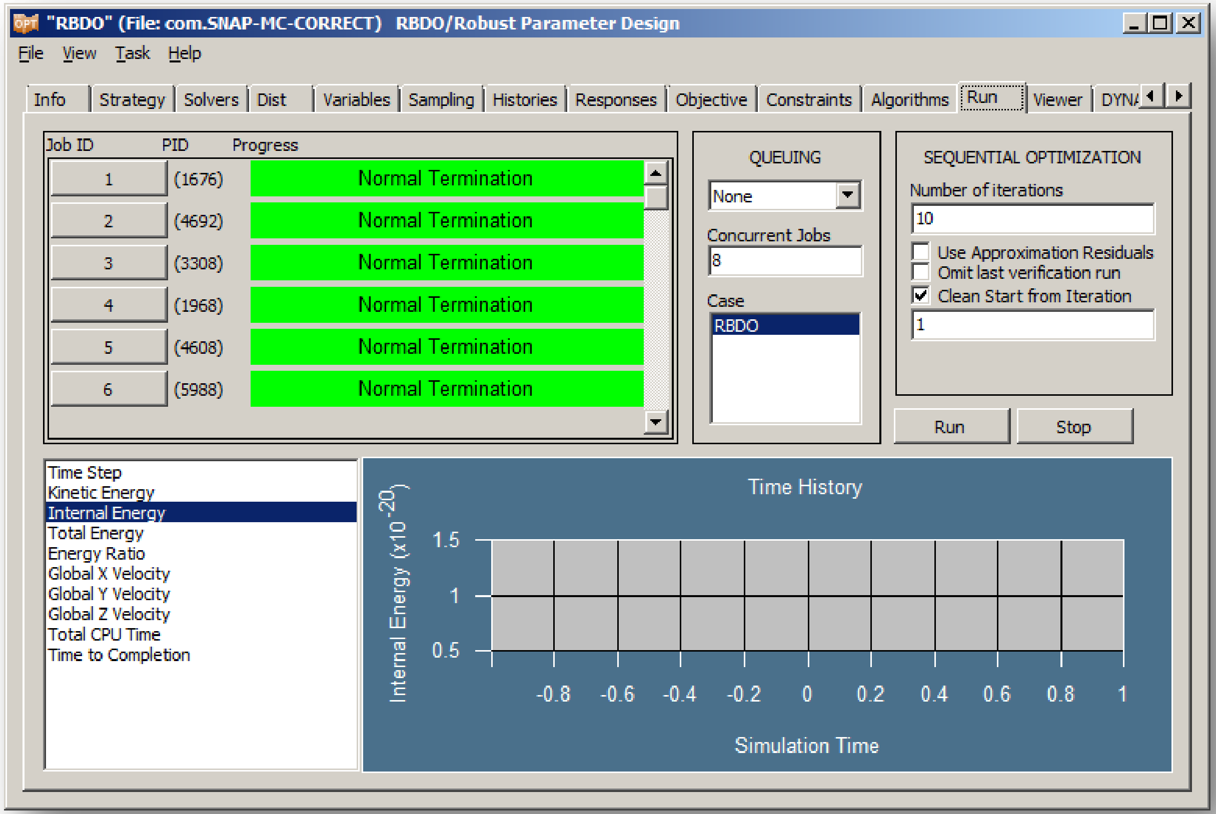
Algorithms Tab

- Go to Algorithms Tab
- Select `GA (Genetic Algorithm)`



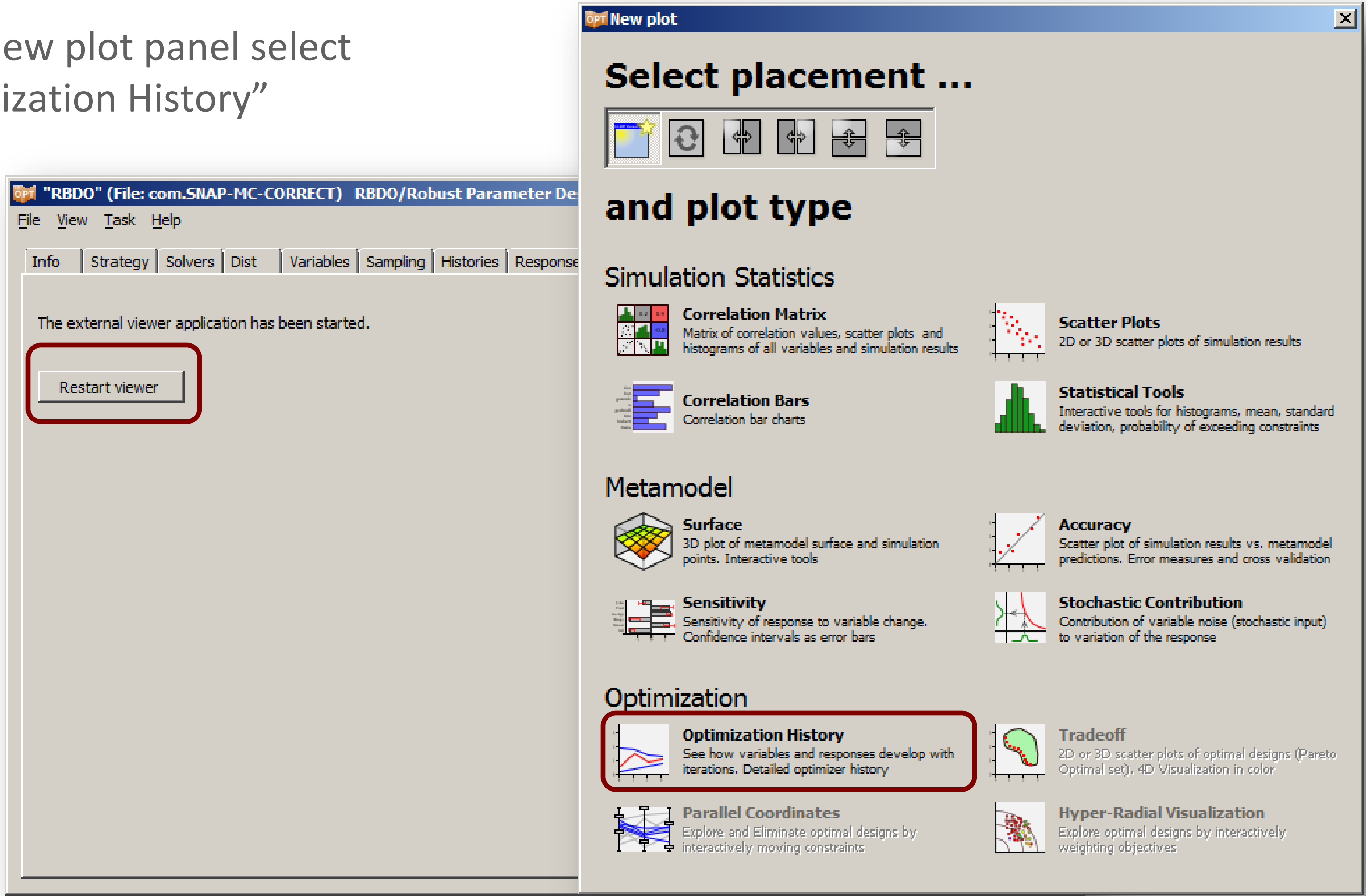
Run Tab

- Go to Run tab
- Select PBS for your Queuing system (if on TRACC cluster) or leave none
- Set the number of concurrent jobs 8 and number of iterations to 10
- Press Run button

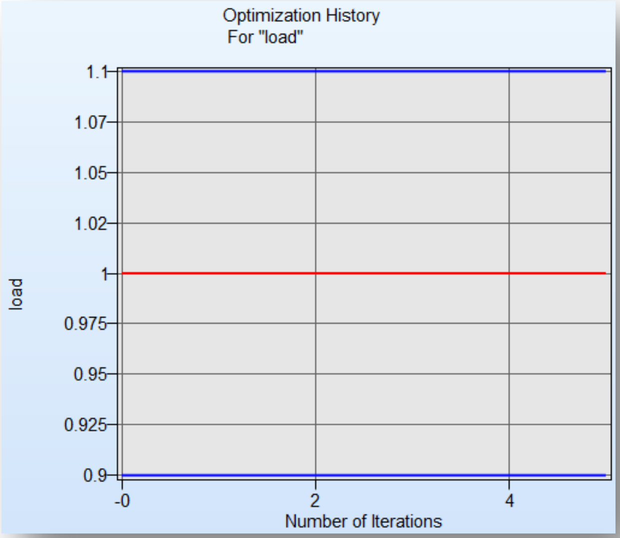


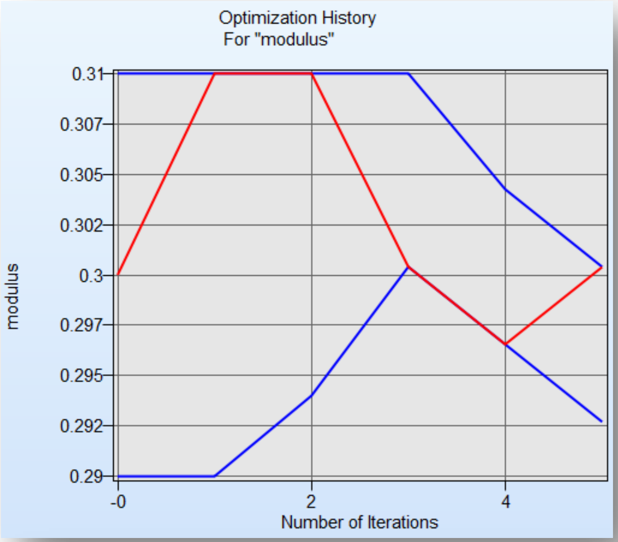
Viewer

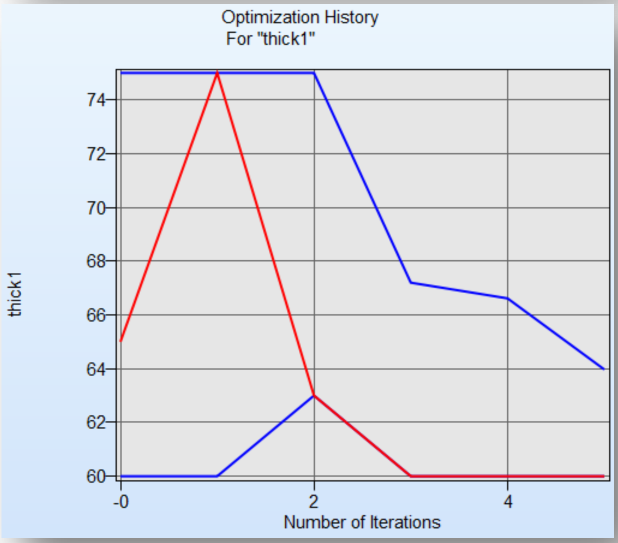
- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select "Optimization History"



Optimization History

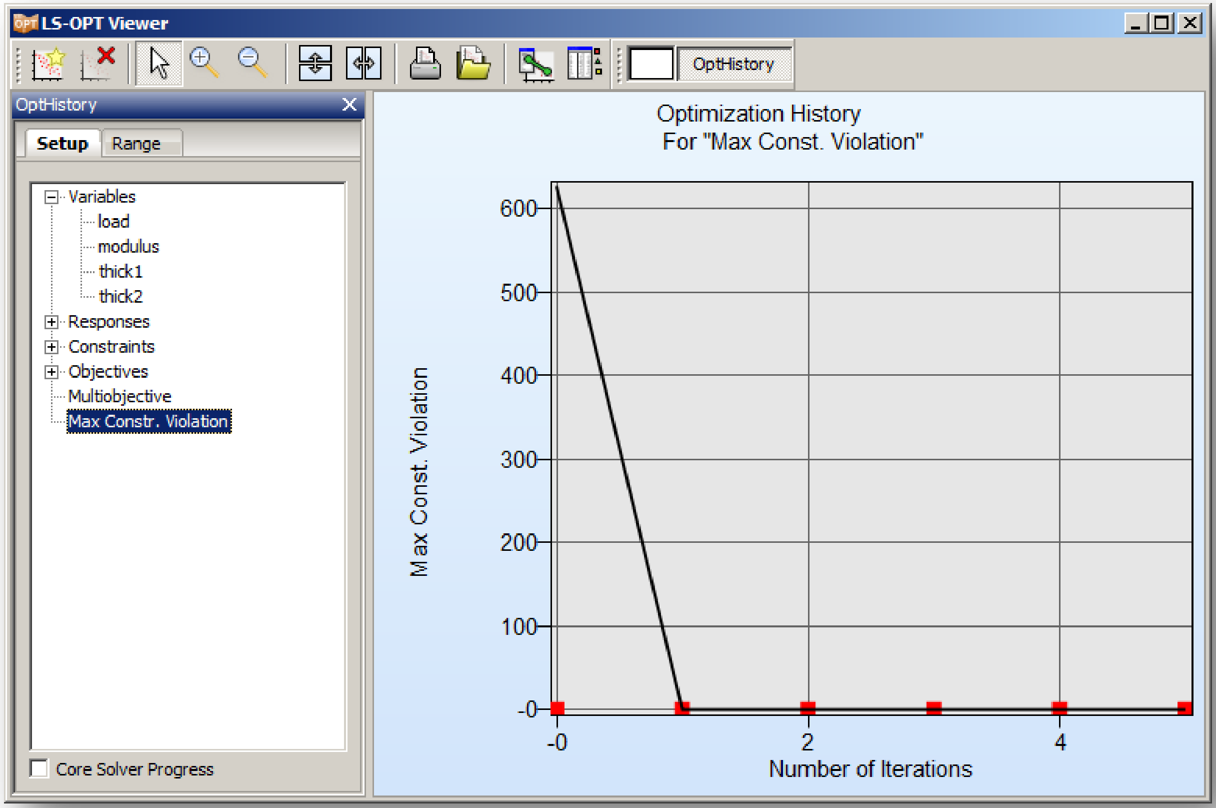






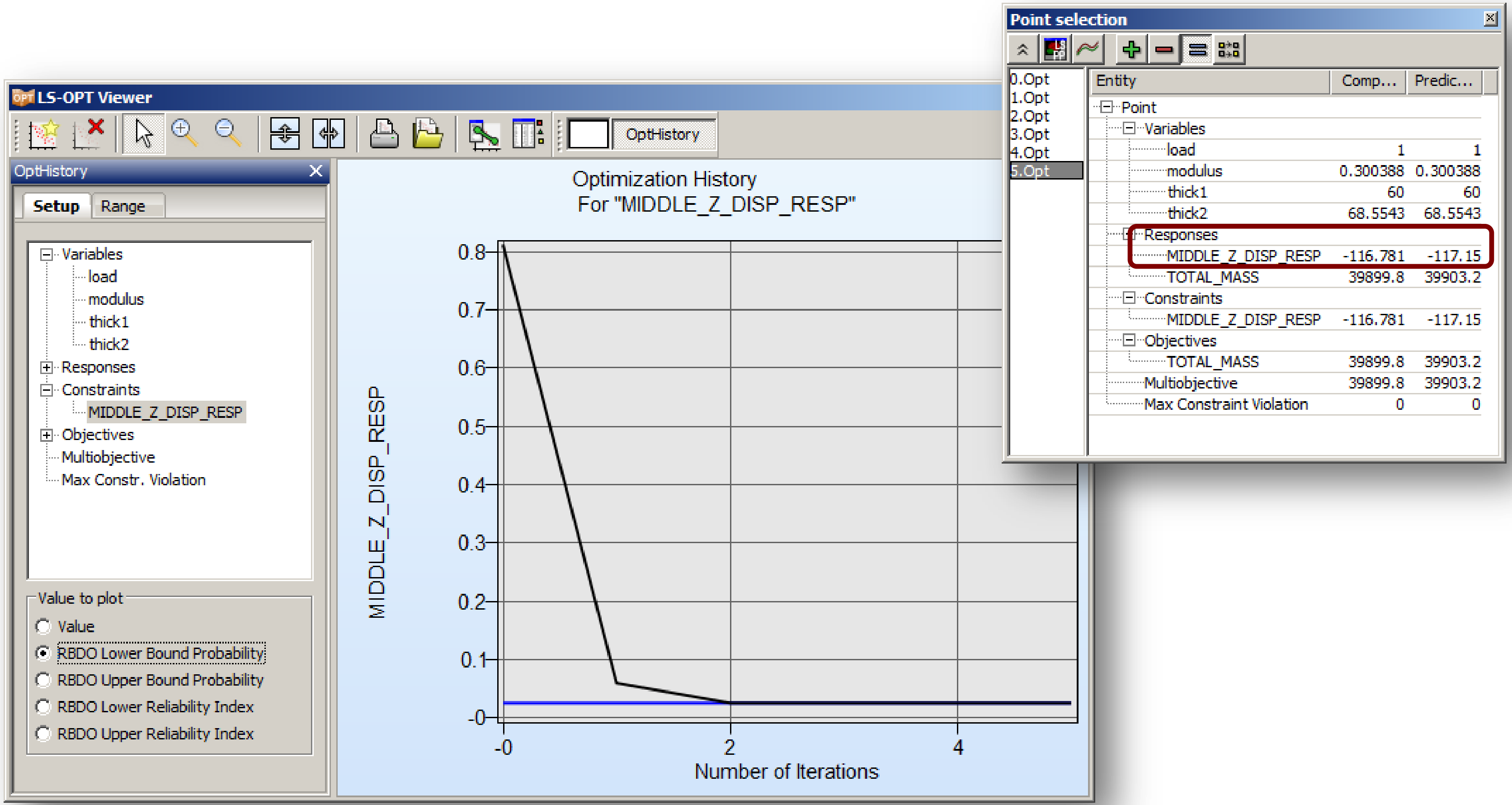
Optimization History

- Go to Max Constraint Violation
- In 1st iteration the constraints are dealt



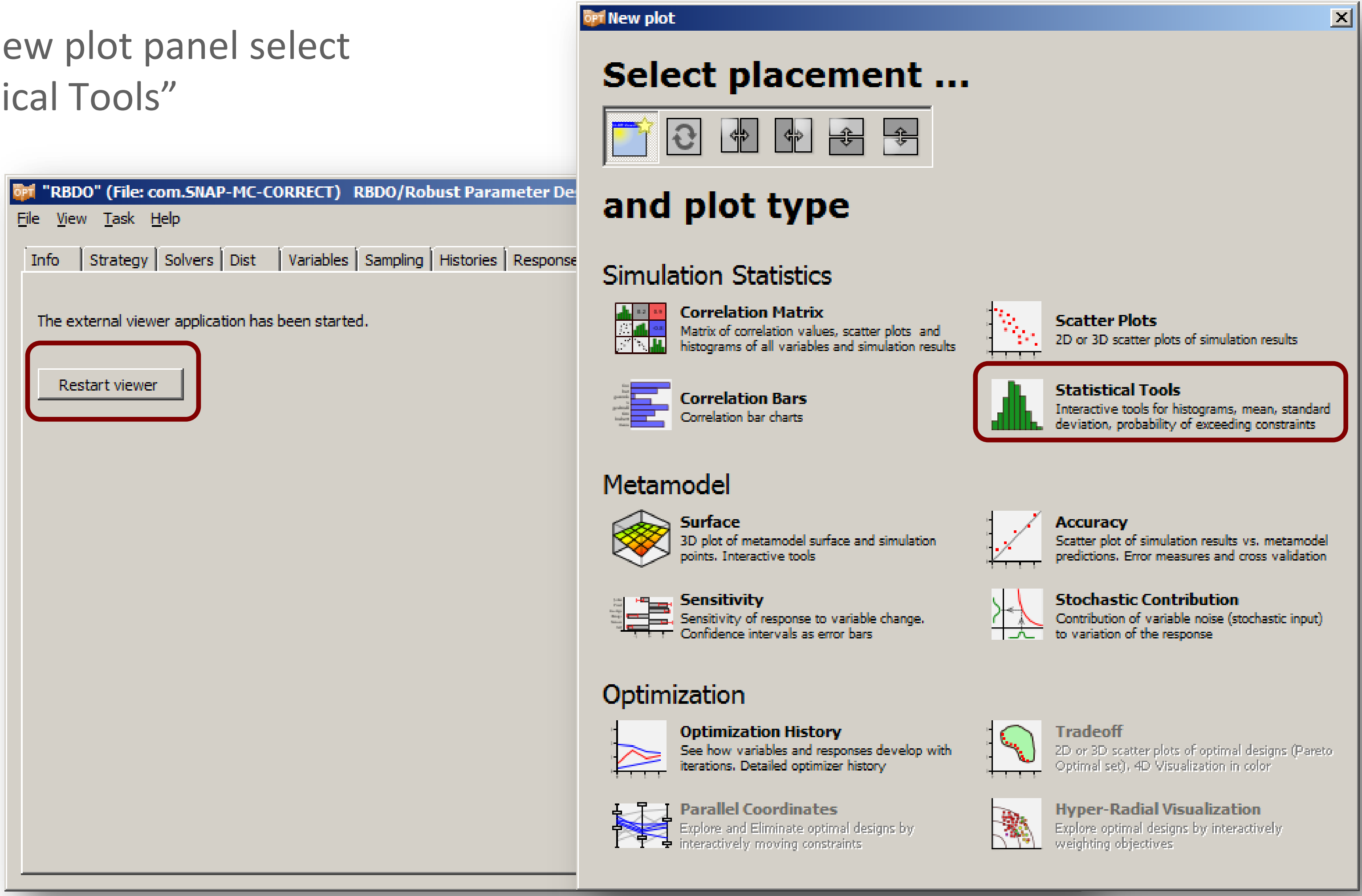
Optimization History

- Go to Constraints and select `MIDDLE_Z_DISP_RESP`
- Select RBDO Lower Bound Probability as a Value to plot
- Click with mouse close to the right end of the plot



Viewer

- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select "Statistical Tools"



Statistical Tools

- Go to Statistical Tools
- Pick Bounds and type `-140` as Lower bound for `MIDDLE_Z_DISP_RESP` Response
- Probability of z-displacement exceeding `-140` is `2.5%`