

# Introductory Course: Using LS-OPT<sup>®</sup> on the TRACC Cluster

## 2.5a - Probabilistic Analysis

By: Cezary Bojanowski, PhD

# Introduction

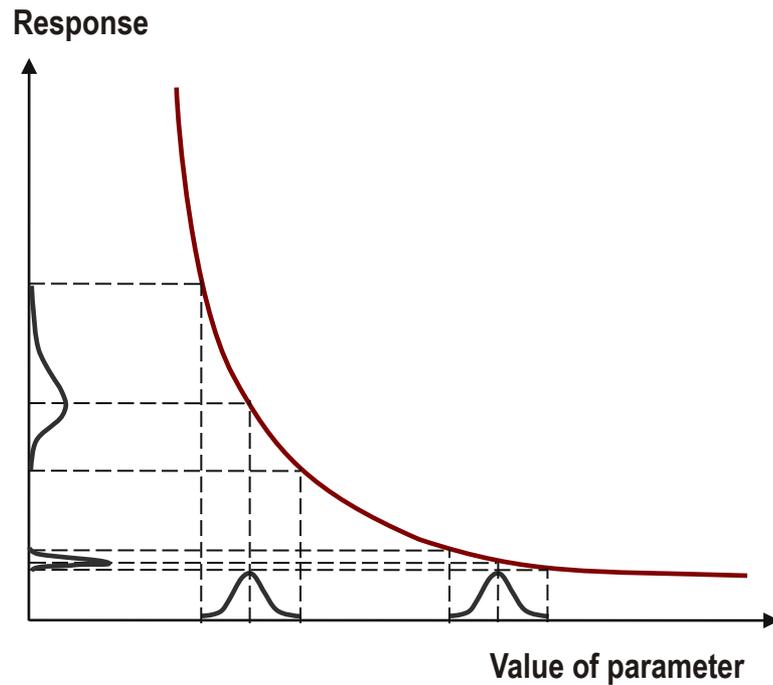
- No two structural events will be exactly similar, nor will structural event occur exactly as designed or analyzed.
- Adverse combinations of design and loading variation may lead to undesirable behavior or failure.
- Therefore if significant variation is expected – a probabilistic evaluation is required.
- Differences in structural performances can be attributed to deterministic and random effects. Understanding their sources is crucial for successful analysis, yet it is very challenging.

# Response Variation

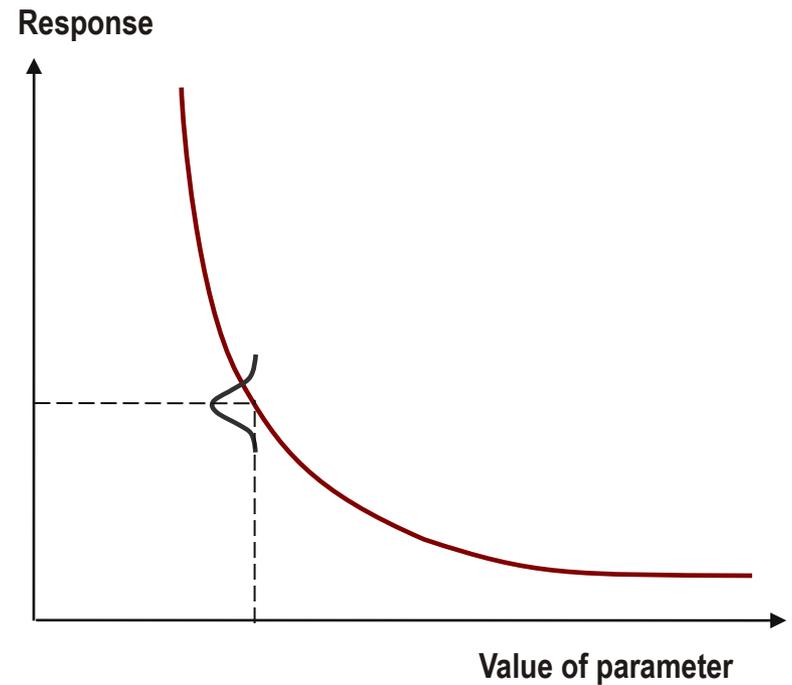
- Deterministic variation
  - Expected, predictable and repeatable variation in a response associated with a variation in a parameter.
  - Can be controllable or uncontrollable
- Random variation
  - Variation that cannot be associated with a change in system parameters
    - Regular random variation – not associated with the physics of the system
    - Chaotic random variation – noise caused by bifurcation behavior in the structure

# Response Variation

- Deterministic variation



- Random variation (buckling, contact etc.)

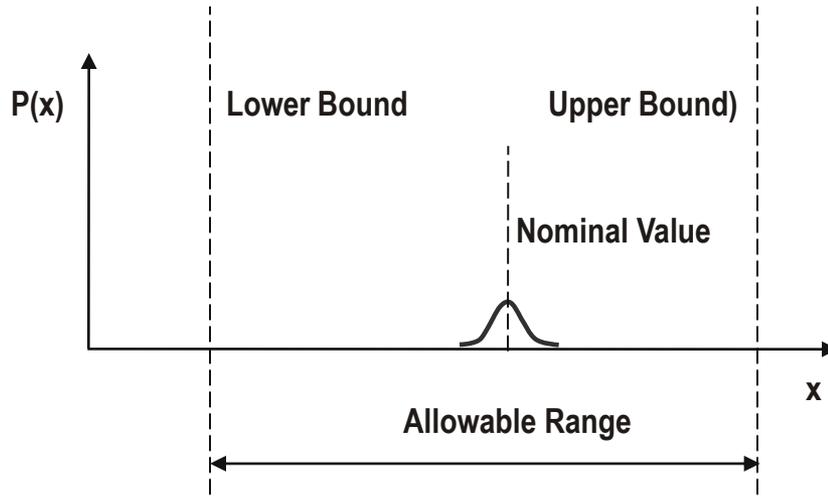


# Sources of Variation

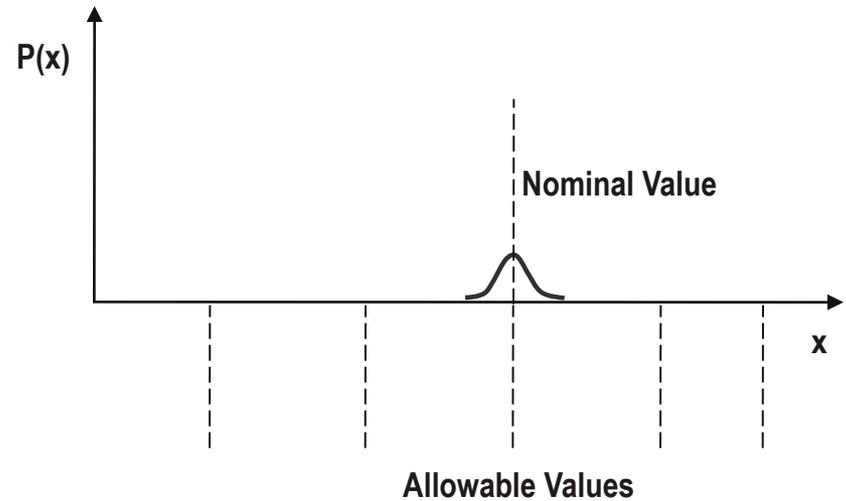
- Design Parameter Variation
  - Control variables
    - can be controlled in the design, analysis and production level.
    - it can be assigned a nominal value and will have variation around it.
    - example: shell thickness
  - Noise variables
    - are difficult or impossible to control and the design and production level.
    - but can be controlled on the analysis level.
    - will have the nominal value and will follow exactly the distribution.
    - example: variations in loads and material properties

# Sources of Variation

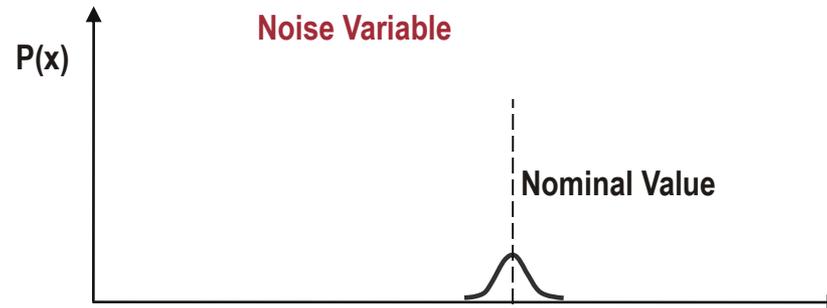
**Design Variable**



**Discrete Variable**

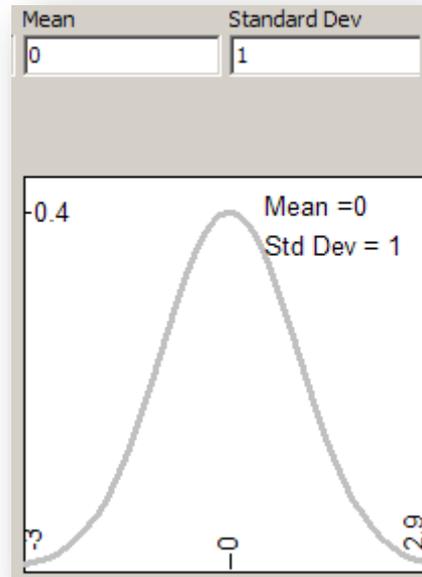


**Noise Variable**



# Distributions in LS-OPT

- Normal
- Uniform
- Lognormal
- Weibull
- Beta
- Binomial
- Truncated normal
- User defined



$$f(x) = ae^{-\frac{(x-b)^2}{2c^2}}$$

- a* – Height of the peak
- b* – Position of center (mean)
- c* – Controls the width of the “bell” (standard deviation)

# Distributions

- Probability density function:

$$\int_{-\infty}^{+\infty} f(x)dx = 1 \quad f(x) \geq 0$$

- Distribution function of the random variable:

$$F(x) = \int_{-\infty}^x f(t)dt$$

- Mean of a probability density:

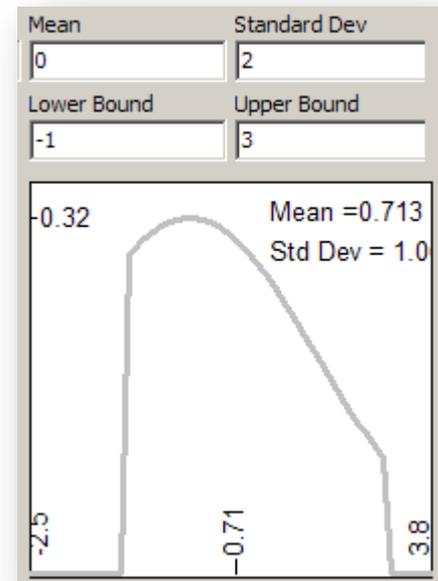
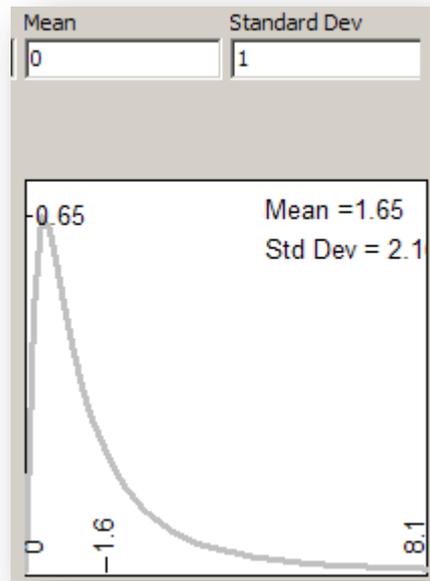
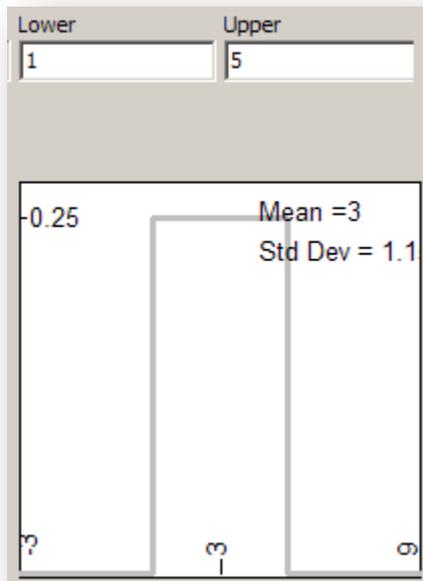
$$\mu = \int_{-\infty}^{+\infty} x \cdot f(x)dx$$

- Variance - the second moment about the mean:

$$\sigma^2 = \int_{-\infty}^{+\infty} (x - \mu)^2 \cdot f(x)dx$$

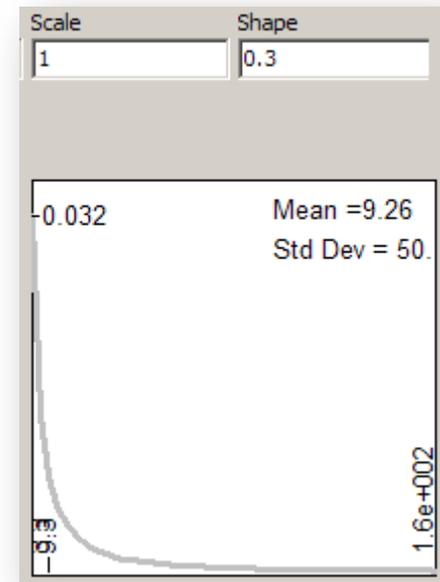
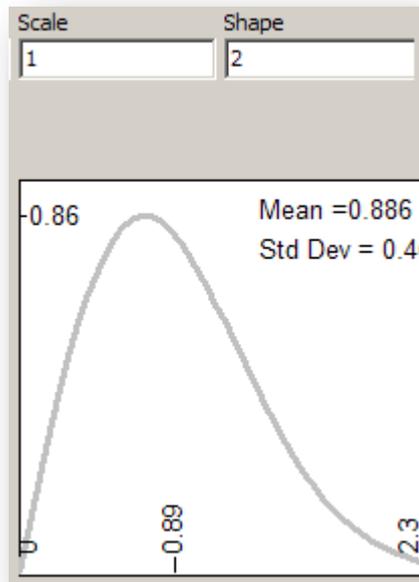
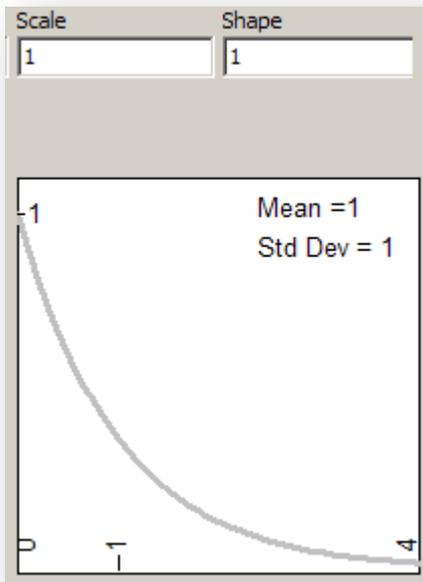
# Distributions in LS-OPT

- Uniform distribution
- Lognormal distribution
- Truncated normal distribution



# Distributions in LS-OPT

- Weibull distribution



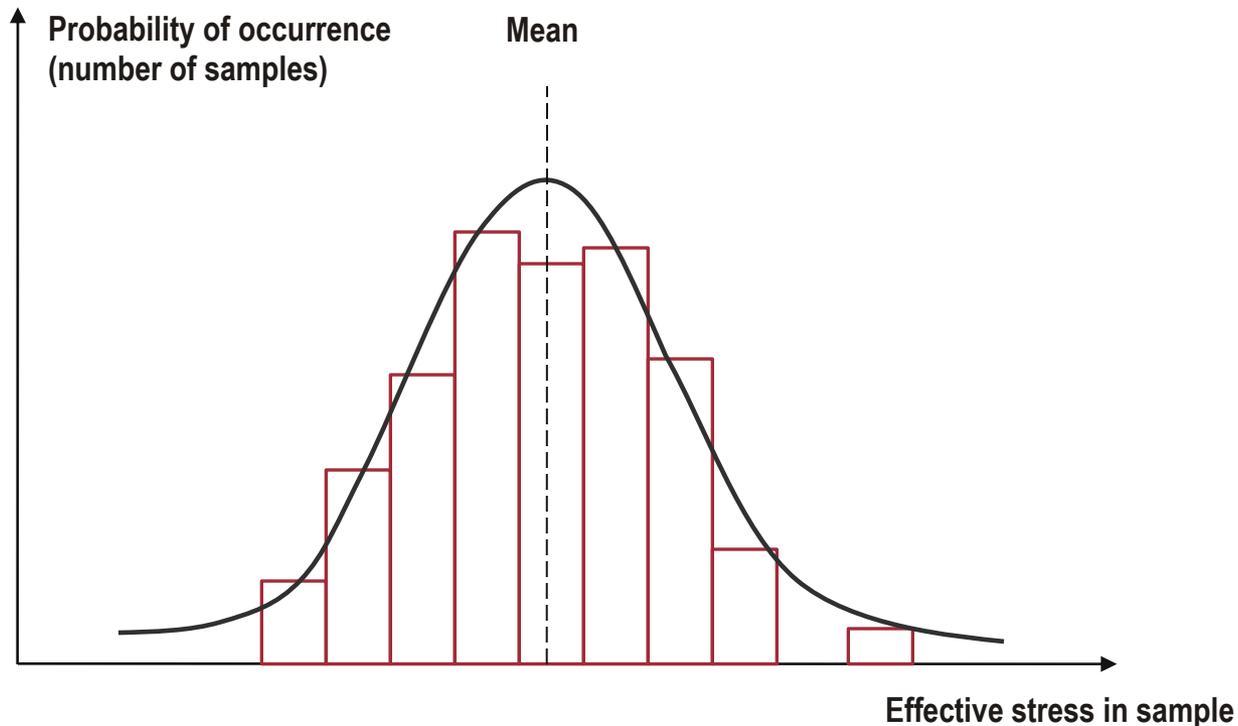
# Sources of Variation

- Modeling variation
  - Differences in modeling will give different results as well as introduce noise to the results.
    - mesh density
    - choice between FEA, SPH, MMALE
    - resolution of output data
  
- Analysis variation
  - Variations in initial conditions can lead to noticeable differences in responses.
    - Physical: bifurcation events, sequence of impact (contact) due to changes in the design variables
    - Algorithmic: due to discretization a node can come into contact with an element or adjacent to it

# Sources of Variation

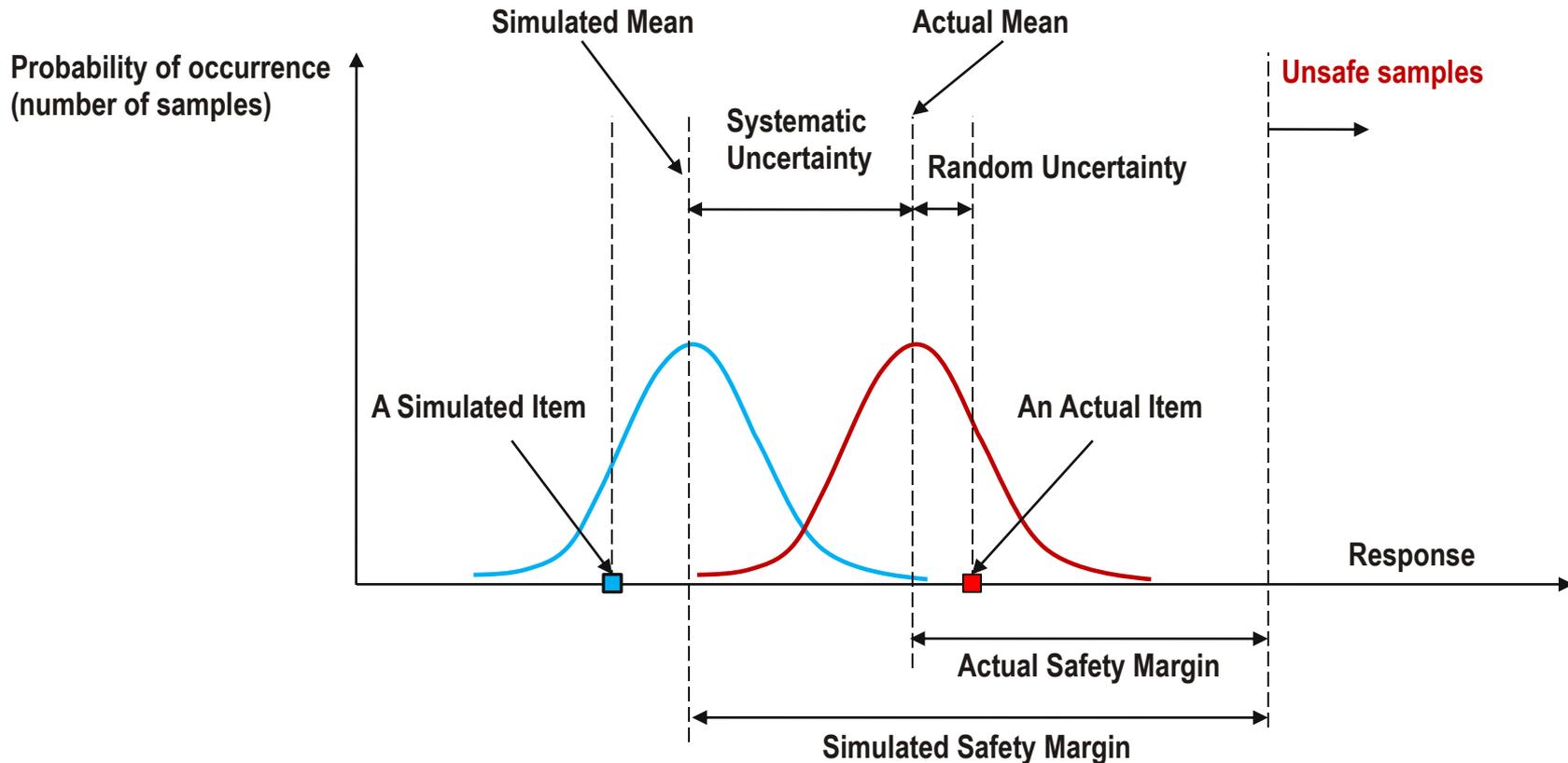
- Pure Variation
  - change in the results nonrelated to the input
    - different versions of solver
    - machine precision
    - round-off error

# Response Variation



- What is the distribution of the response, given the distribution of parameters?
- What is the probability of structure failure?
- What are efficient redesign strategies?

# Safety Margin

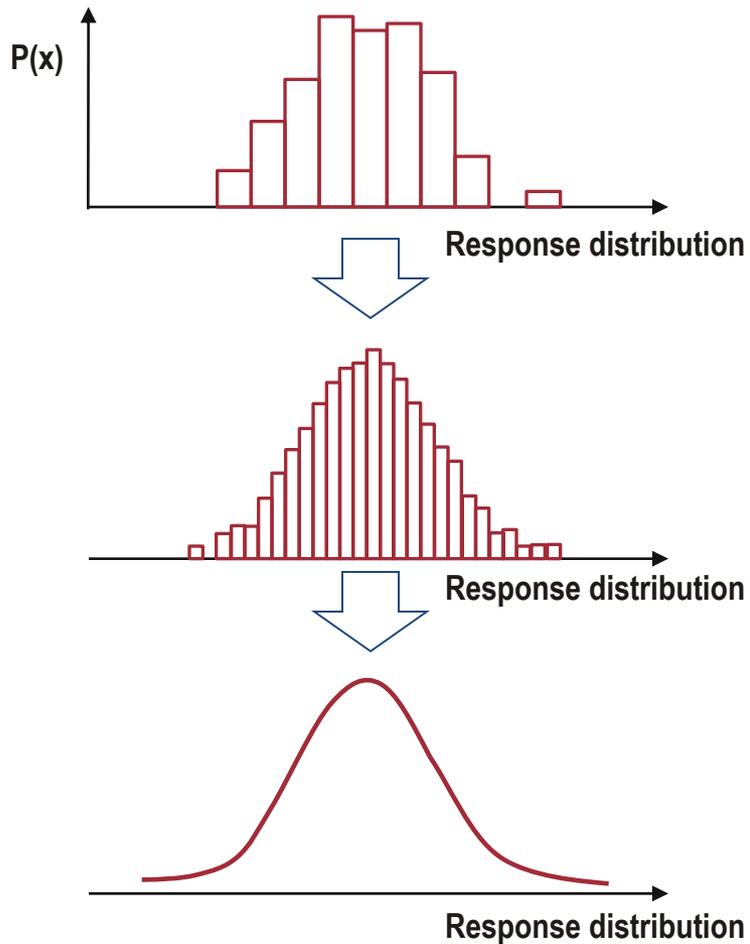


- What is the distribution of the response, given the distribution of parameters?
- What is the probability of structure failure?
- What are efficient redesign strategies?

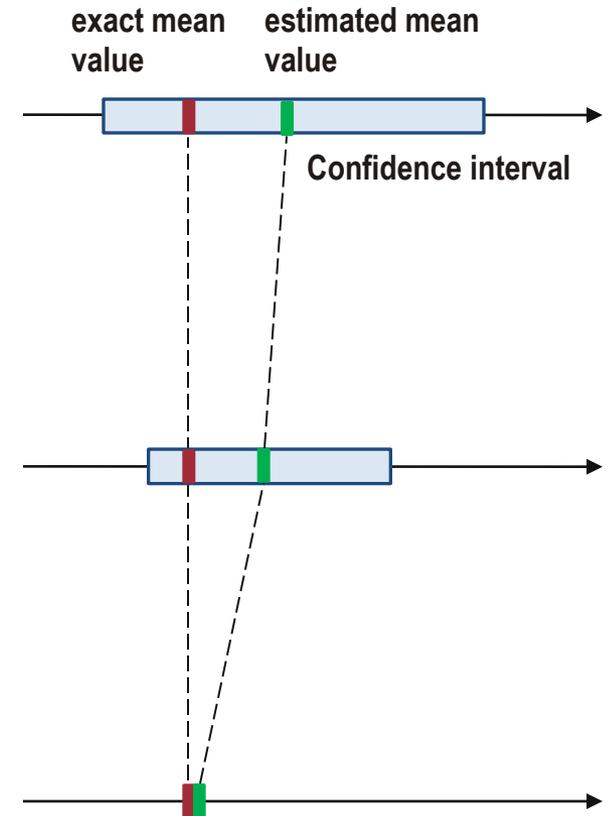
# Monte Carlo Analysis

- Monte Carlo simulation aims to compute results with the same scatter as what will occur in practice.
- Multiple analyses are conducted using values of the input variables selected considering their probability density function.
- The results from these analyses should have the scatter expected in practice.
- Requires large number of runs and random sampling.

# Monte Carlo Analysis



Number of experiments  
accuracy increases

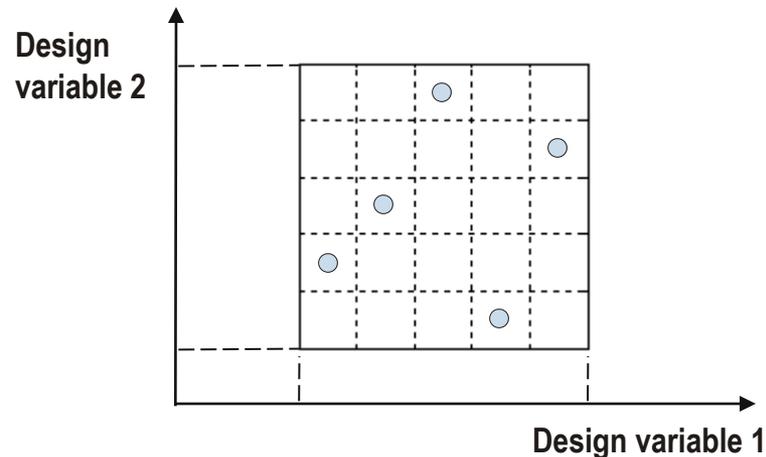


# Latin Hypercube sampling

- The Latin Hypercube method is a random experimental design process. Was developed to generate a reasonable collection of parameter values from a multidimensional distribution.
- A square grid containing sample points is a Latin square iff there is only one sample in each row and each column.
- A Latin hypercube sampling is the generalization of this concept to an arbitrary number of dimensions.
- LH designs are independent of the mathematical model of the approximation and allow estimation of the main effects of all factors in the design in an unbiased manner

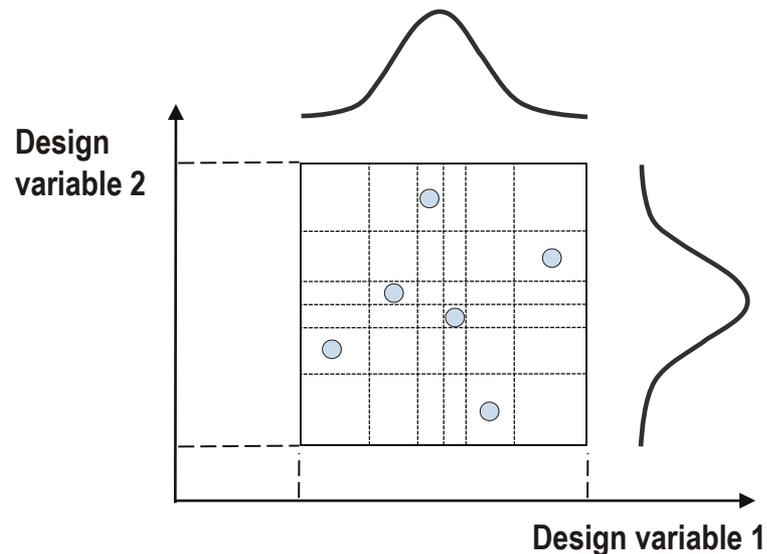
# Latin Hypercube sampling

- When sampling a function of  $n$  variables, the range of each of them is divided into  $P$  equally probable intervals.  $P$  sample points are then placed to satisfy the LH requirements.
- This forces the number of divisions to be equal for each variable.
- LH sampling does not require more samples for more dimensions (variables)



# Latin Hypercube sampling

- If the variable has assigned distribution, the partitions for LH sampling will be subdivided as to have equal probability

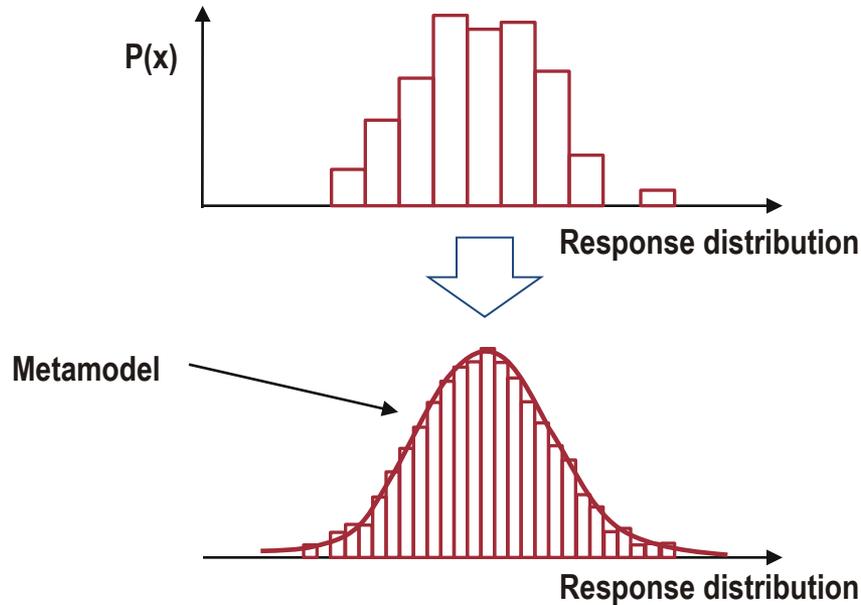


- LH sampling may be used further for space filling designs

# Metamodel-based Monte Carlo

- Problem: Monte Carlo analysis requires large number of runs. May be prohibitive for large models (100+ runs).
- Solution: Use Monte Carlo analysis with reduced number of runs and construct metamodel (10-30+ runs). Very large number of function evaluations are possible when using metamodels ( $10^6$ ).
- The results are exact for linear or quadratic responses approximated with linear or quadratic response surfaces.
- For random shapes of response use Neural Networks (NN) or Radial Basis Function (RBF) Networks

# Metamodel Based Monte Carlo Analysis

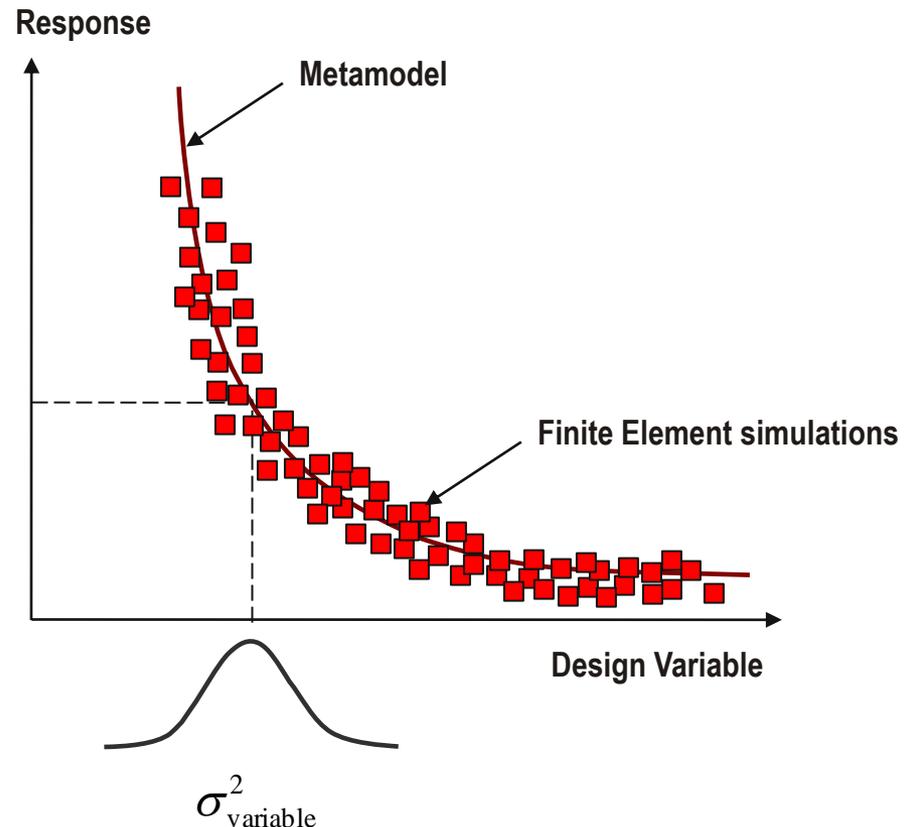


**Build a metamodel  
on medium number  
of simulation points**

**Perform hundreds  
of function evaluations  
on cheap metamodel**

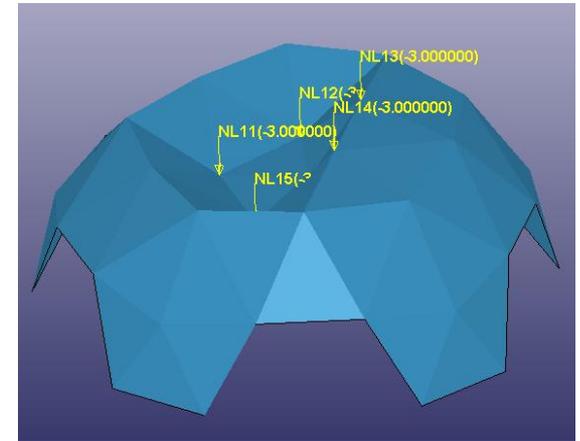
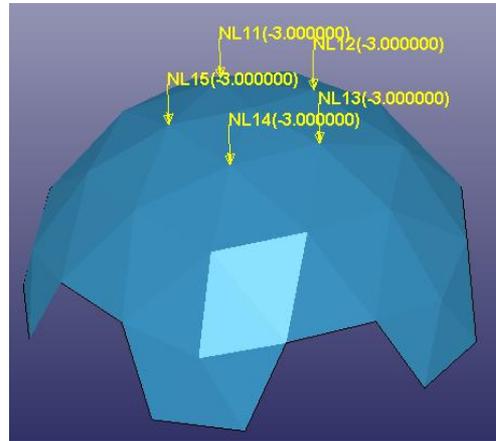
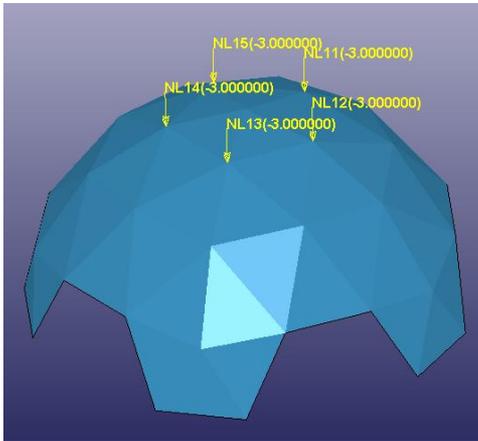
# Stochastic Contribution in Metamodel based Monte Carlo Simulations

A diagram illustrating the decomposition of total variance. On the left, a green bell-shaped curve represents the total variance, labeled  $\sigma^2_{\text{total}}$ . This is followed by an equals sign, then a red bell-shaped curve representing residual variance, labeled  $\sigma^2_{\text{residual}}$ , followed by a plus sign, and finally a blue bell-shaped curve representing deterministic variance, labeled  $\sigma^2_{\text{deterministic}}$ .



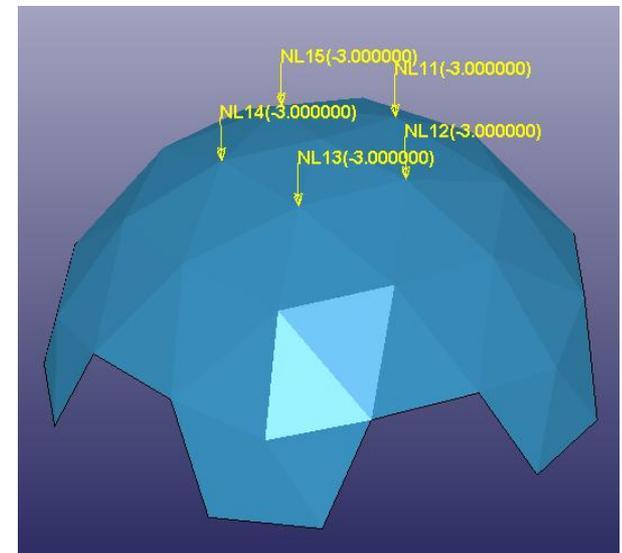
# Monte Carlo Analysis with LS-OPT Direct Simulations

Example: Deflection of the dome structure.



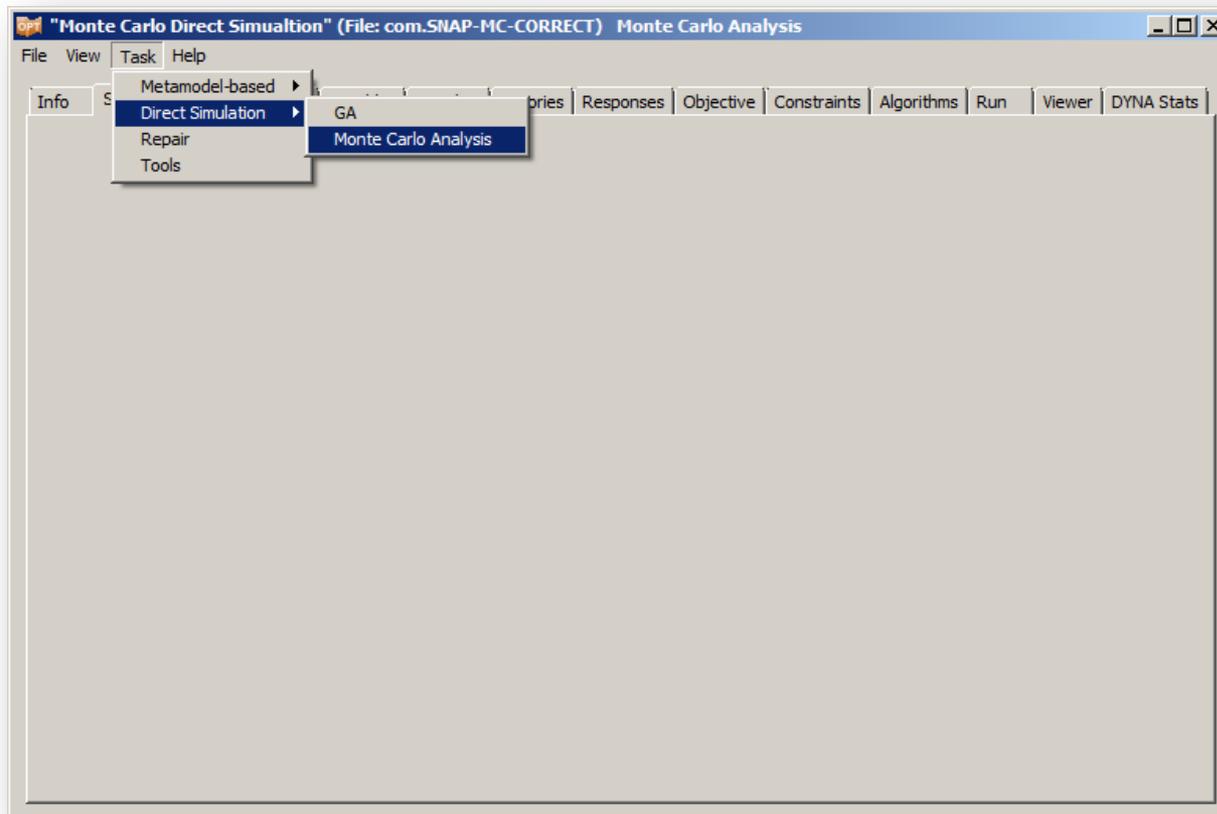
# Problem description

- This example is a Monte Carlo analysis of a dome structure being loaded from the top. The effect of variations in:
  - material properties,
  - thickness,are investigated.
- The geometry is shown in Figure below.
- The x, y and z – displacements are restrained at the bottom of the structure.
- It is loaded from the top in five nodes.
- Maximum z – displacement of the central node is used as the response variable



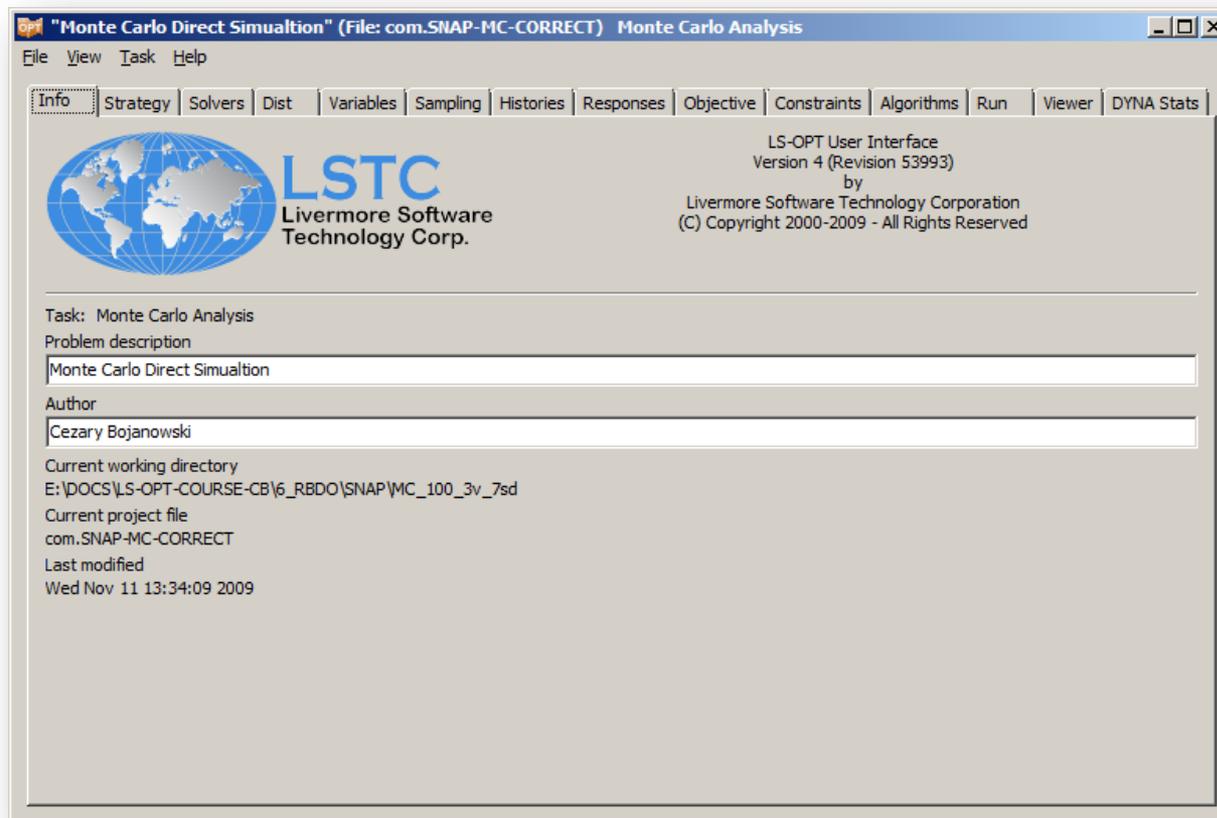
# Approach 1 - Monte Carlo Direct Analysis

- Go to Task tab
- Select Monte Carlo Analysis from Direct Simulations group



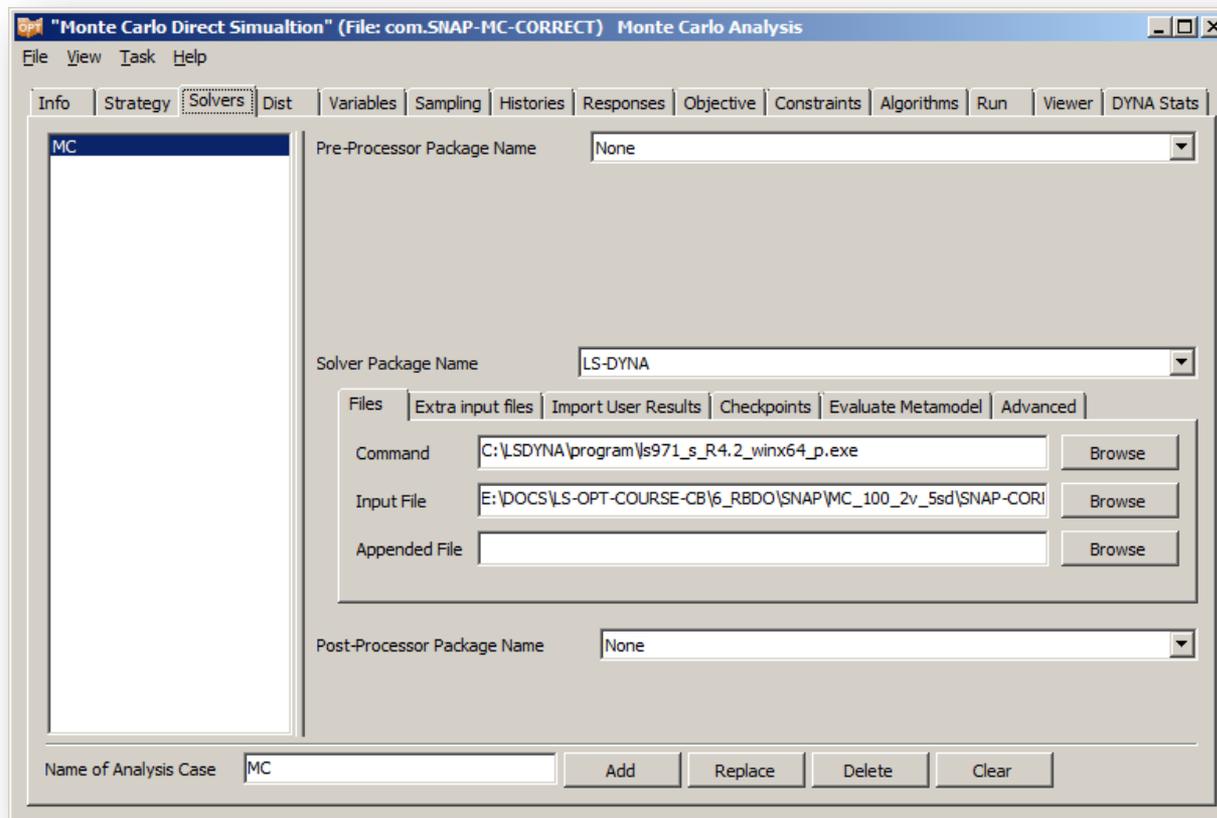
# Info Panel

- Describe the problem
- Give author's name



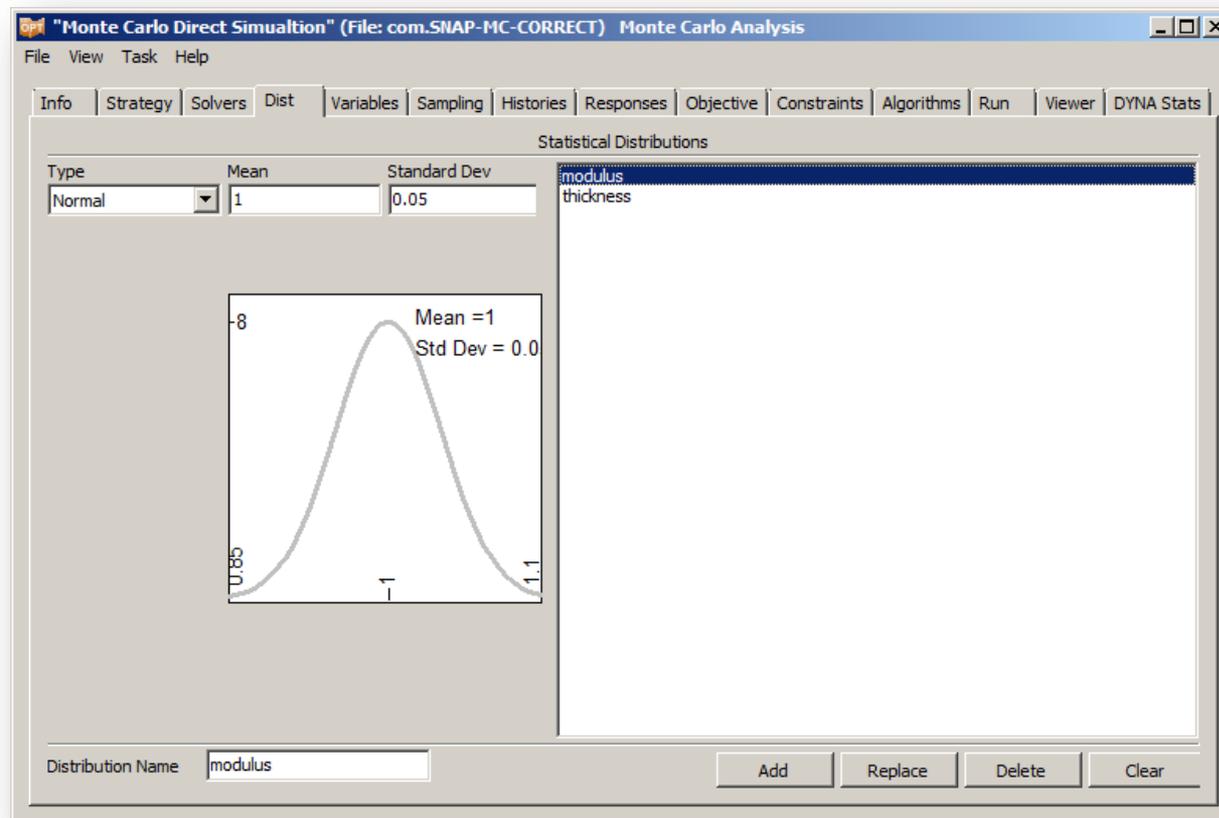
# Solvers Panel

- On TRACC cluster specify the command as full path to **lsoptscript**
- Navigate to the input file
- Name the analysis **MC** and hit Add



# Distributions Panel

- In distributions panel select Type Normal, type **1** for Mean and **0.05** for Std. dev.
- Name the distribution – **modulus** and hit Add button
- Repeat the same procedure for **thickness** distribution



# LS-DYNA k-file

- Defined noise variables will be the multipliers of the design parameters – thickness and elastic modulus

```
*SECTION_SHELL
```

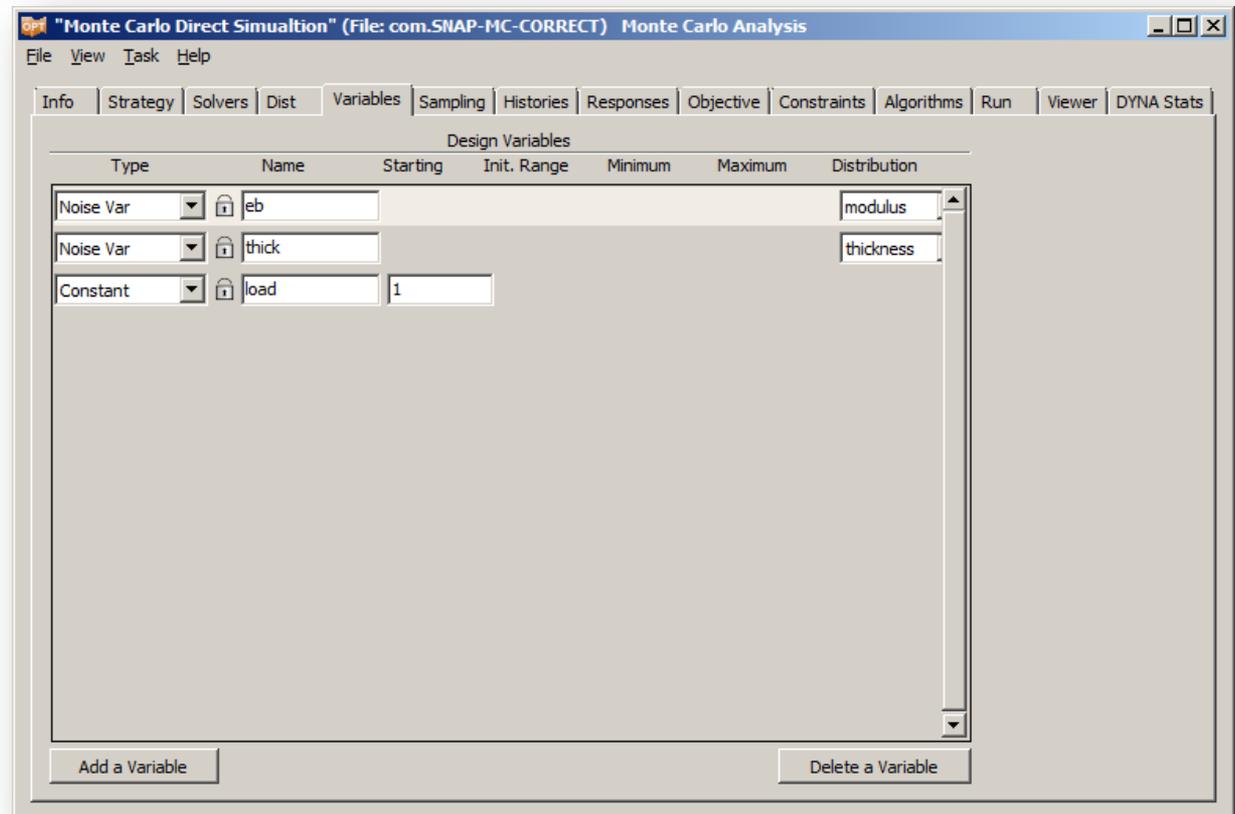
```
$#   secid   elform   shrf     nip     propt   qr/irid   icomp   setyp
      2      4      0.000     3       0       0       0       0
$#   t1      t2      t3      t4     nloc     marea     idof     edgset
<<65*thick>>,<<65*thick>>,<<65*thick>>,<<65*thick>>
```

```
*MAT_ORTHOTROPIC_ELASTIC
```

```
$#   mid     ro      ea      eb      ec      prba     prca     prcb
      2  2.1500E-6  10.000000 <<0.3*eb>>  1.0000E-5  0.050000  1.0000E-5  1.0000E-5
$#   gab     gbc     gca     aopt     g      sigf
      0.500000  0.001000  0.001000  0.000  0.000  0.000
$#   xp      yp      zp      a1      a2      a3      macf
      0.000  0.000  0.000  0.000  0.000  0.000  1
$#   v1      v2      v3      d1      d2      d3      beta     ref
      0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
```

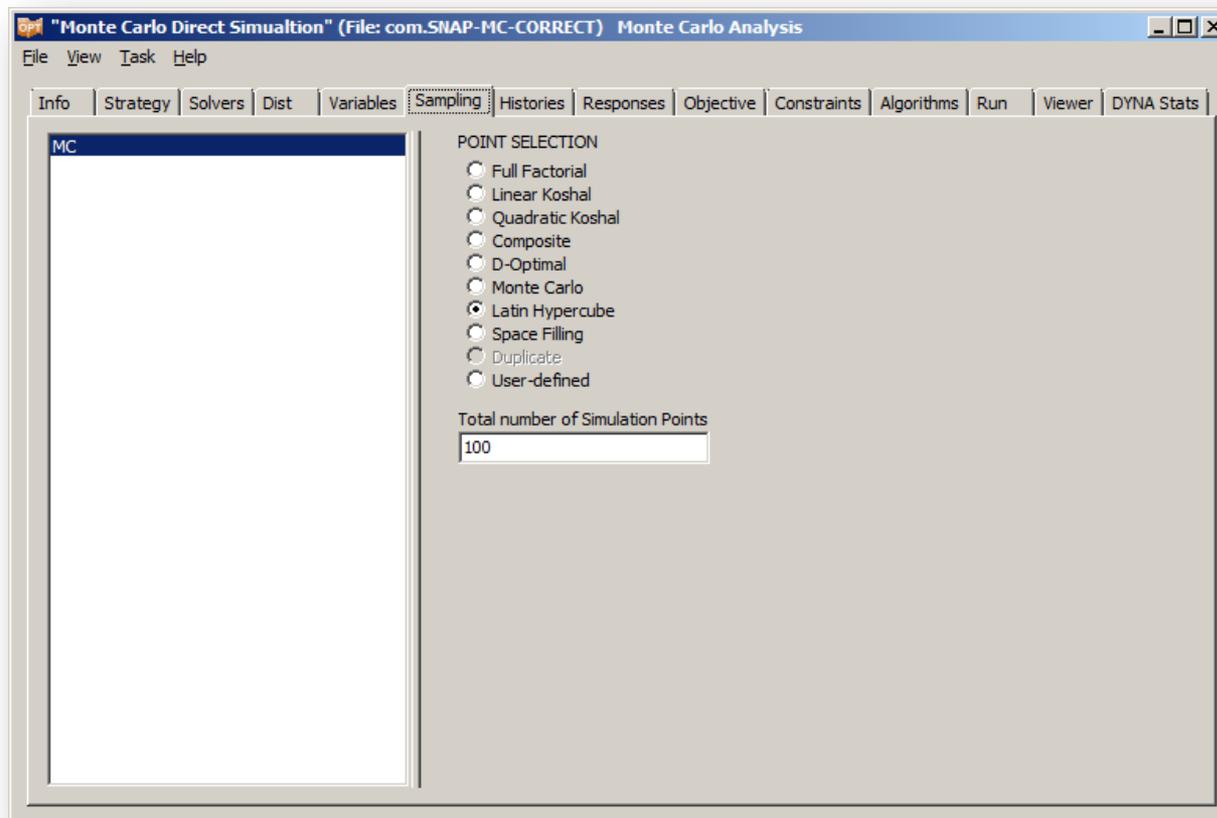
# Variables Panel

- After reading the k-file, variables should be defined already.
- Change type for variable **eb** to Noise variable
- Select distribution type **modulus**
- For **thick** variable change type to Noise variable and select **thickness** distribution
- For Constant load type **1** as starting value



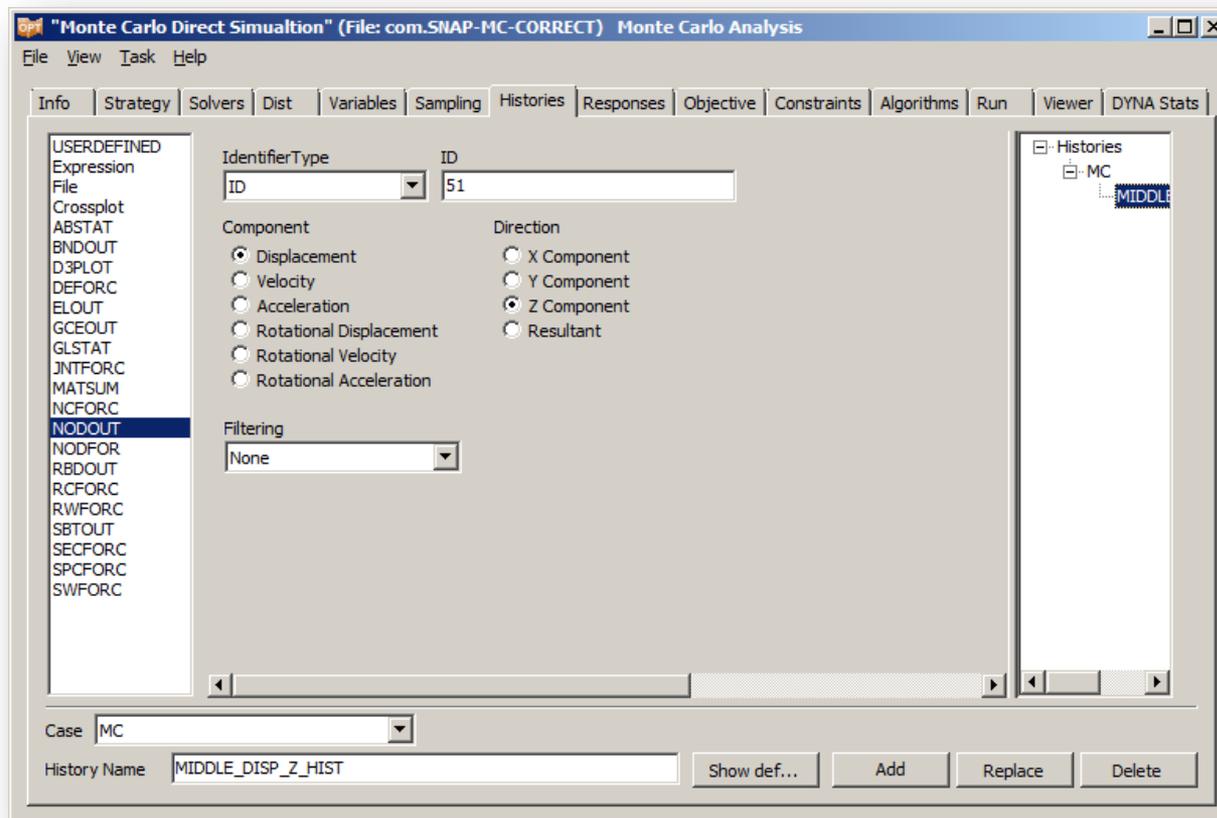
# Sampling Panel

- In Sampling Panel pick Latin Hypercube as Point selection method
- Type **100** for Total number of Simulation Points



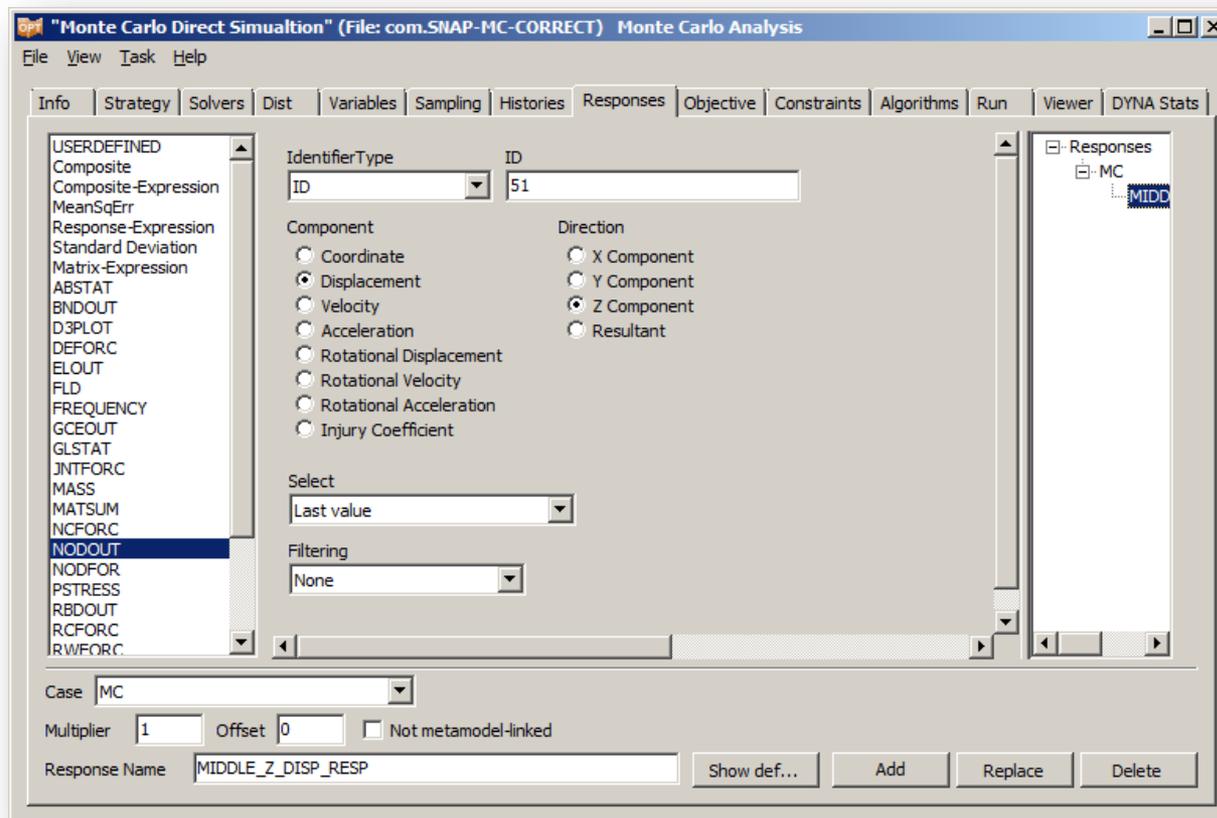
# Histories Panel

- In Histories Panel pick **NODOUT** from left window
- Choose ID for Identifier Type
- Type **51** for ID and select Z Component as Direction of Displacement
- Give name to the history **MIDDLE\_DISP\_Z\_HIST** and press Add



# Responses Panel

- In Histories Panel pick **NODOUT** from left window
- Choose ID for Identifier Type
- Type **51** for ID and select Z Component as Direction of Displacement
- Give name to the history **MIDDLE\_DISP\_Z\_RESP** and press Add



# Run Panel

- In Run Panel select **PBS** for Queuing system if run on TRACC cluster
- Leave **none** if run locally on Windows machine
- Type **8** for concurrent jobs and press Run button (one core is enough for this job)

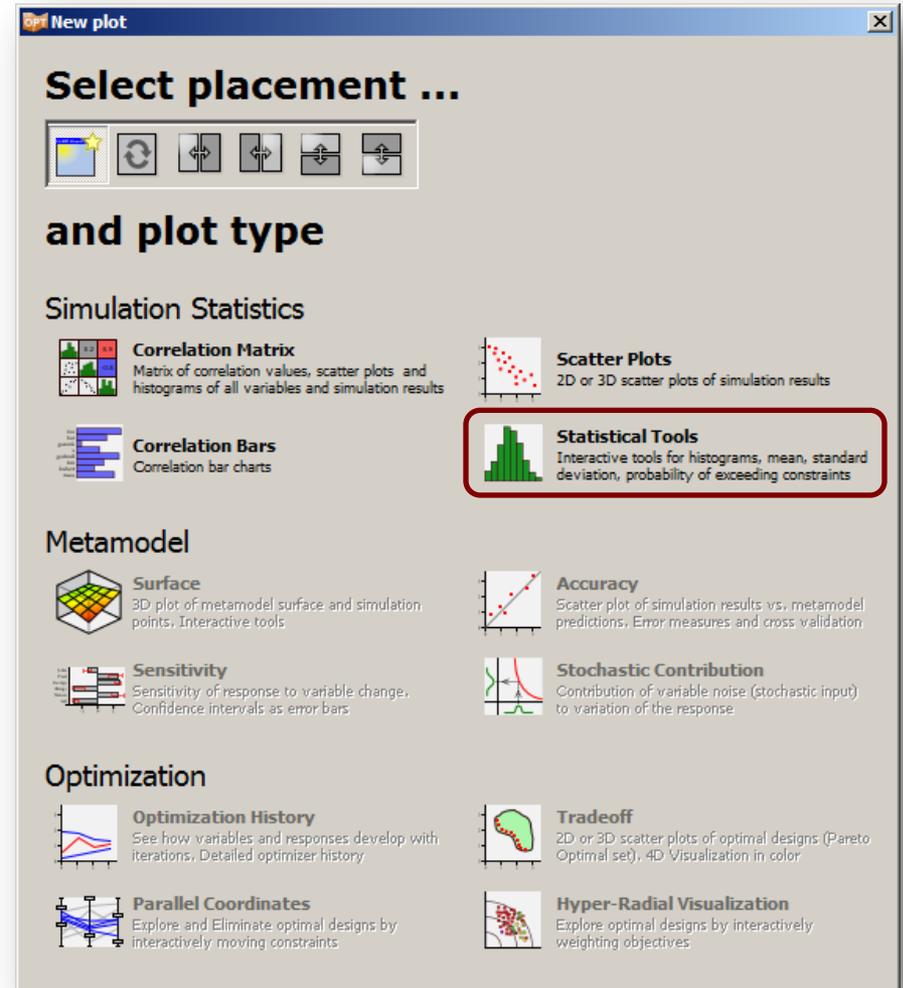
The screenshot shows the "Monte Carlo Direct Simulation" software interface. The title bar indicates the file is "com.SNAP-MC-CORRECT". The menu bar includes File, View, Task, and Help. The toolbar contains various options: Info, Strategy, Solvers, Dist, Variables, Sampling, Histories, Responses, Objective, Constraints, Algorithms, Run, Viewer, and DYNA Stats. The "Run" button is highlighted.

The main window is divided into several sections:

- Job ID Table:** A table with columns for Job ID, PID, and Progress. All six jobs listed (Job IDs 1-6) show "Normal Termination" in green text.
- QUEUING Panel:** A panel with a dropdown menu set to "None", a "Concurrent Jobs" input field set to "8", and a "Case" dropdown menu set to "MC".
- MONTE CARLO ANALYSIS Panel:** A panel with "Run" and "Stop" buttons.
- Time History Plot:** A plot showing "Internal Energy (x10<sup>-20</sup>)" on the y-axis (ranging from 0.5 to 1.5) versus "Simulation Time" on the x-axis (ranging from -0.8 to 1.0). The plot area is currently empty.
- Time Step List:** A list of variables including Time Step, Kinetic Energy, Internal Energy (highlighted), Total Energy, Energy Ratio, Global X Velocity, Global Y Velocity, Global Z Velocity, Total CPU Time, and Time to Completion.

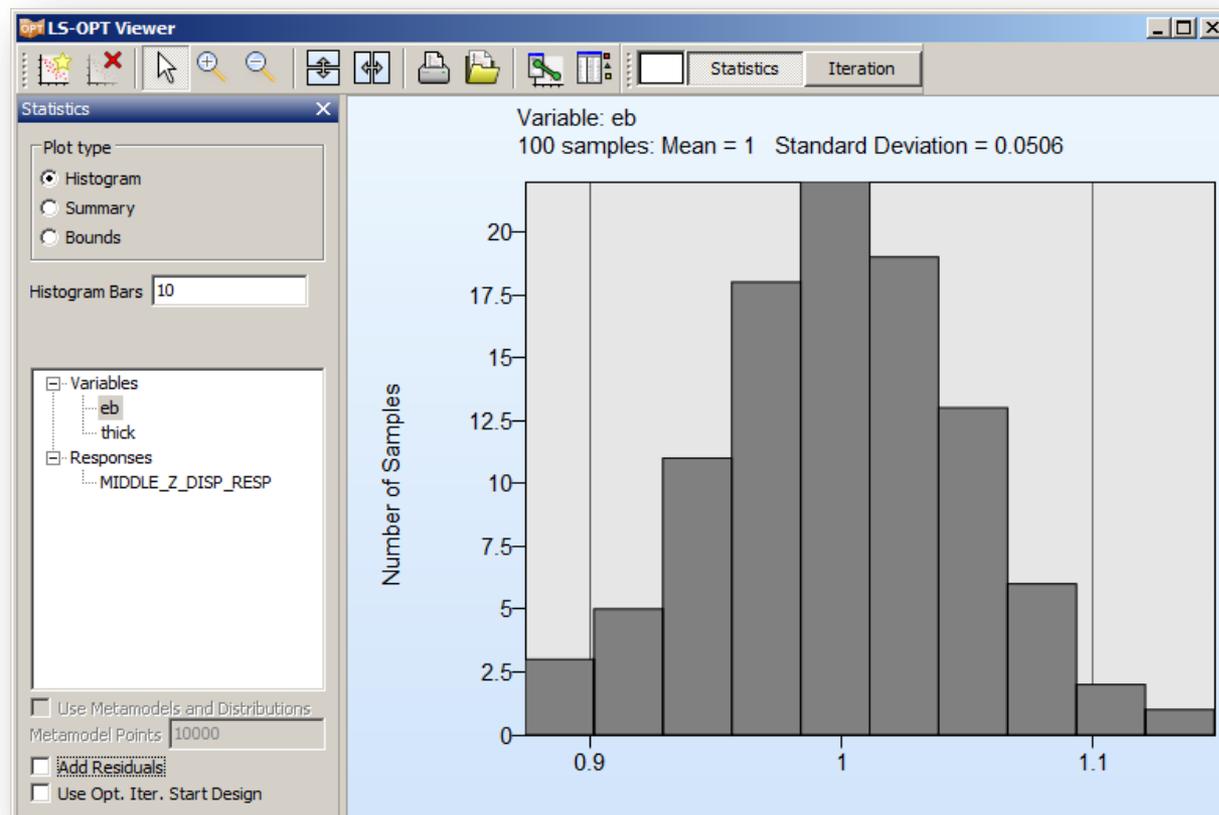
# Viewer

- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select “Statistical Tools”



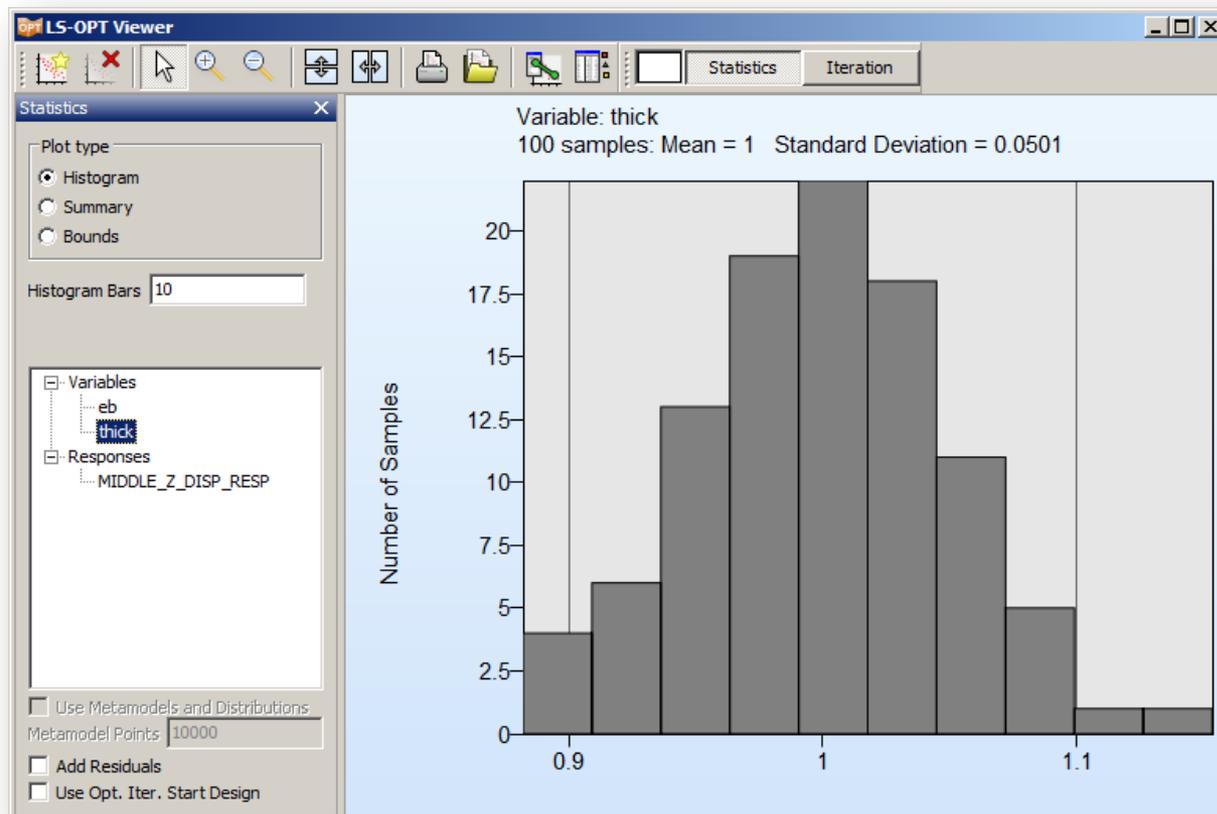
# Statistical Tools

- From Histogram Plot type select **eb** Variable statistics to display
- As requested mean is **1** and the std. deviation is **~0.05**



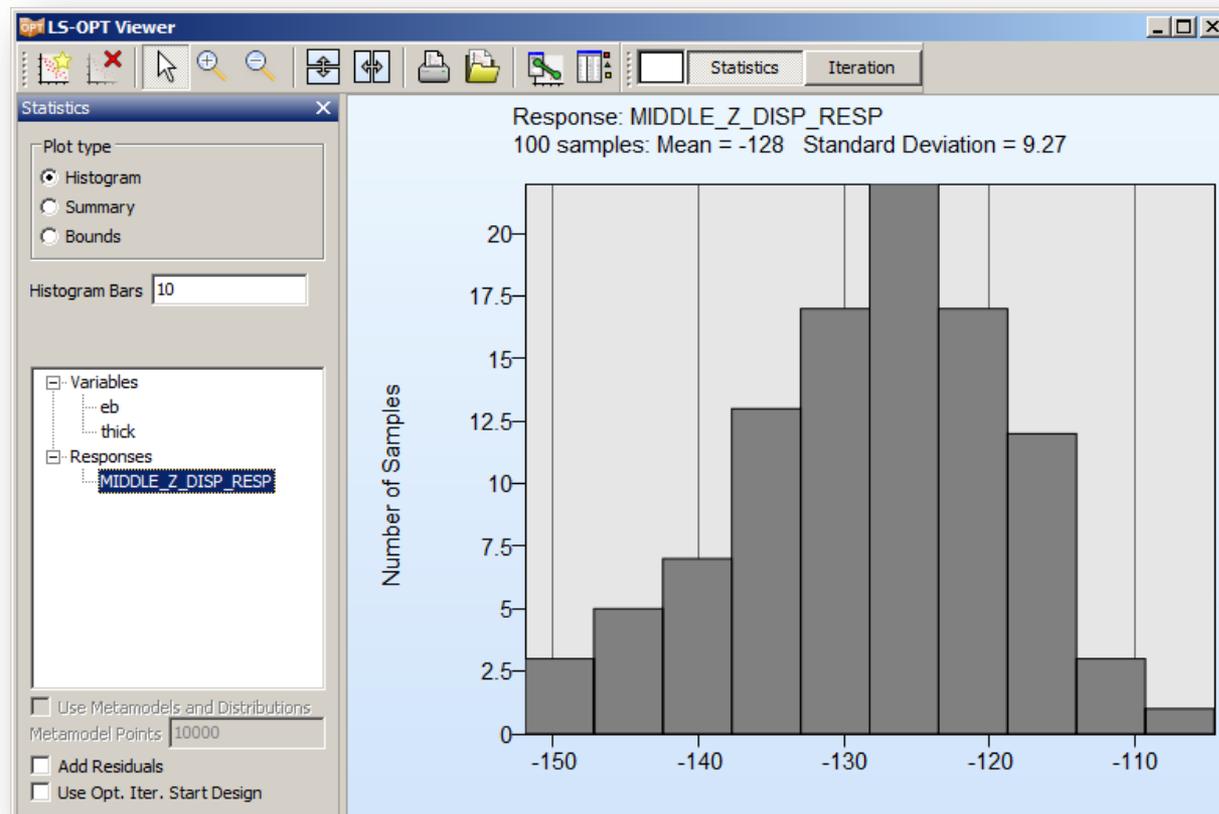
# Statistical Tools

- From Histogram Plot type select **thick** Variable statistics to display
- As requested mean is **1** and the std. deviation is **~0.05**



# Statistical Tools

- From Histogram Plot type select **MIDDLE\_Z\_DISP\_RESP** Response statistics to display
- The mean of the responses is **-128** with standard deviation of **9.27**



# Correlation of Responses

- The mean of a set of responses is:

$$\mu = \frac{1}{n} \sum_{i=1}^n y_i$$

- Its variance is defined as:

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^n (y_i - \mu)^2$$

- The standard deviation is:

$$\sigma = \sqrt{\sigma^2}$$

- The covariance of two variables indicates whether a change in the one is associated with a change in the other:

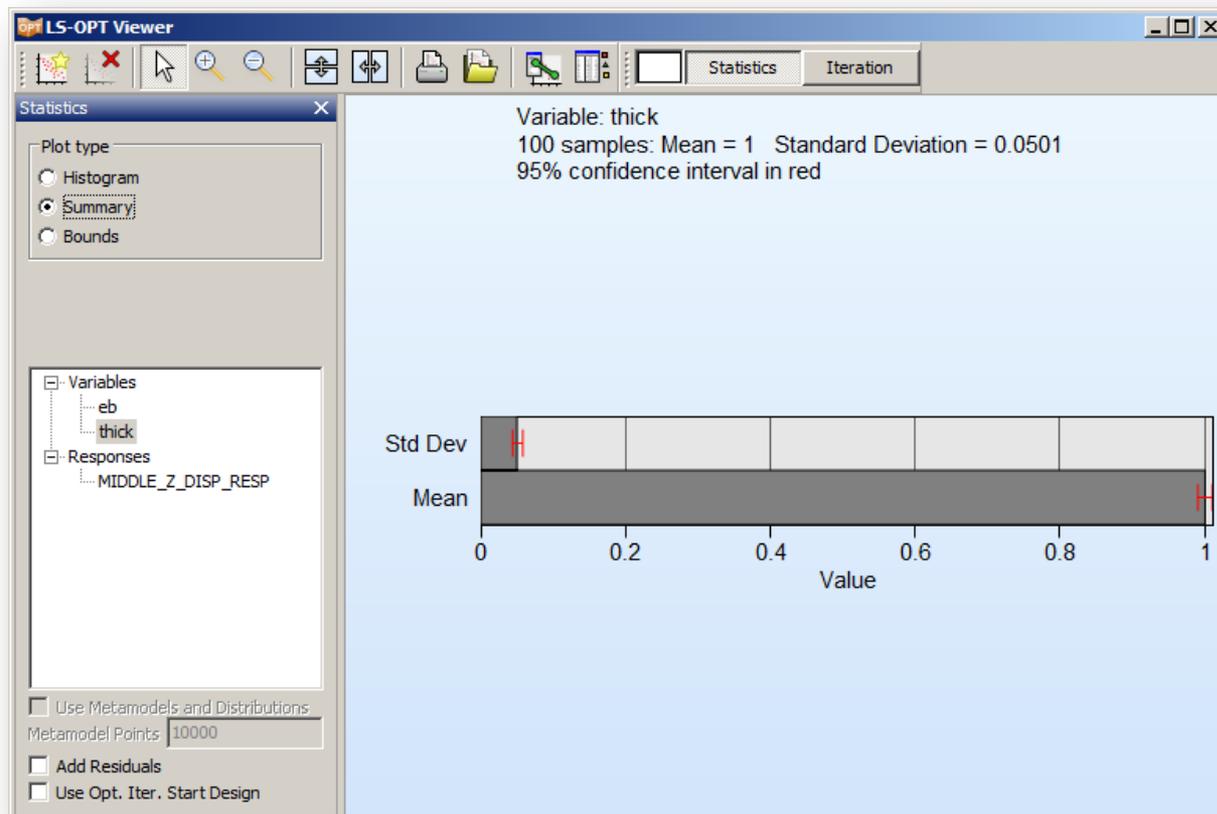
$$Cov(y_1, y_2) = E[(y_1 - \mu_1)(y_2 - \mu_2)]$$

- Coefficient of correlation is used instead as scaled quantity (always between -1 and 1)

$$\rho = \frac{Cov(y_1, y_2)}{\sigma_1 \sigma_2}$$

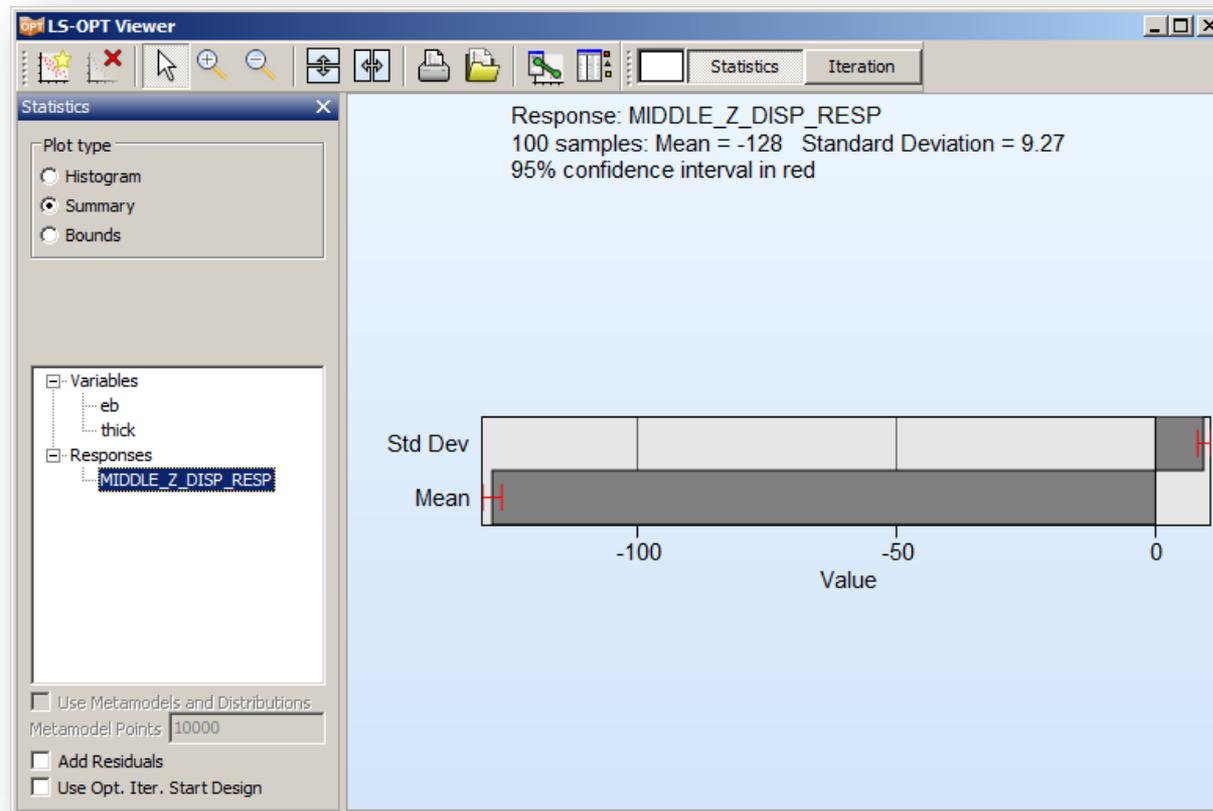
# Statistical Tools

- From Summary Plot type select **thick** Variable



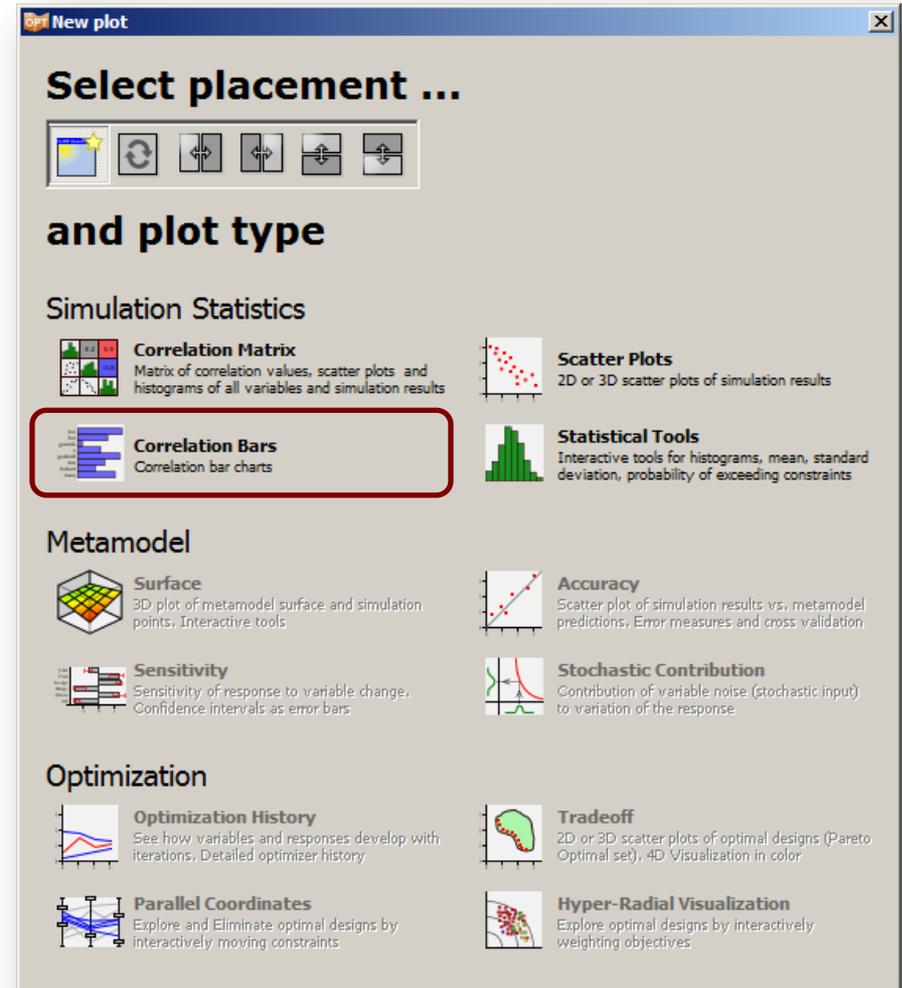
# Statistical Tools

- From Summary Plot type select **MIDDLE\_Z\_DISP\_RESP** Response to see as bars mean and standard deviation of the item



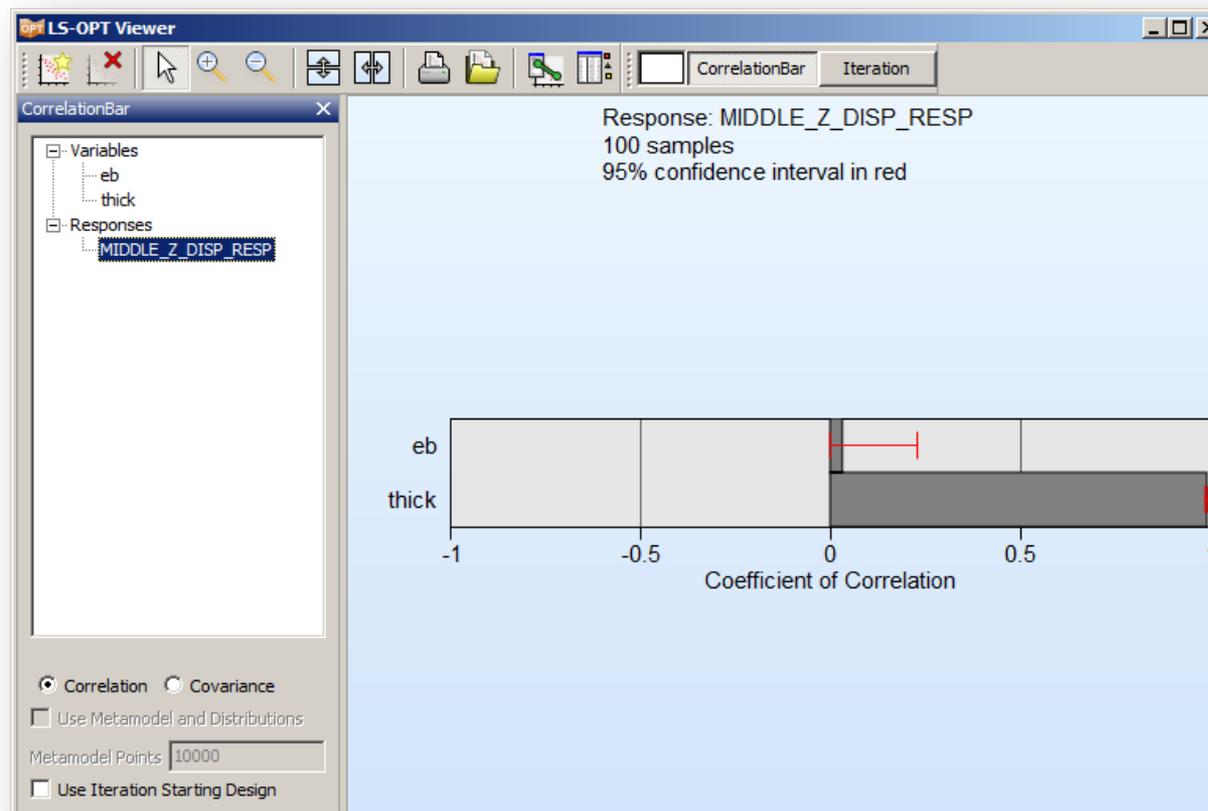
# Viewer

- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select “Correlation Bars”



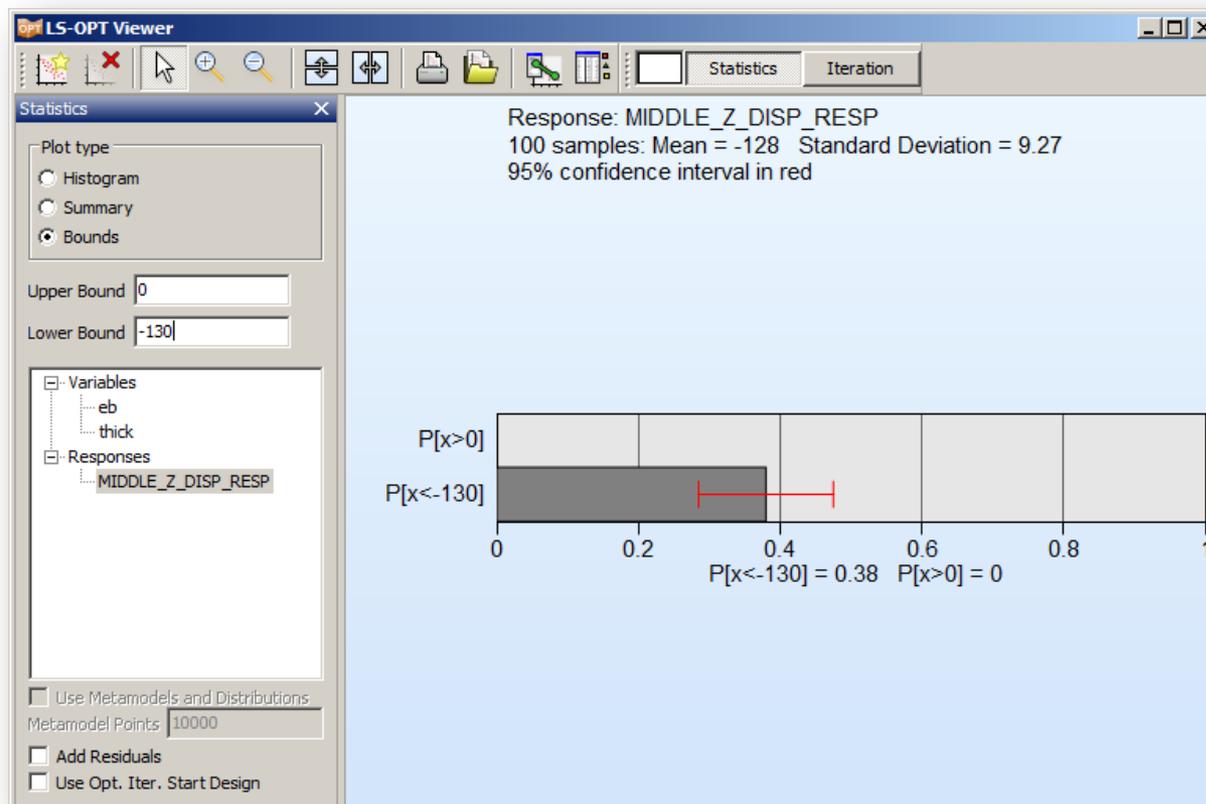
# Correlation Bars

- From Correlation Bar menu select **MIDDLE\_Z\_DISP\_RESP** Response to see correlation bars between the response and design variables



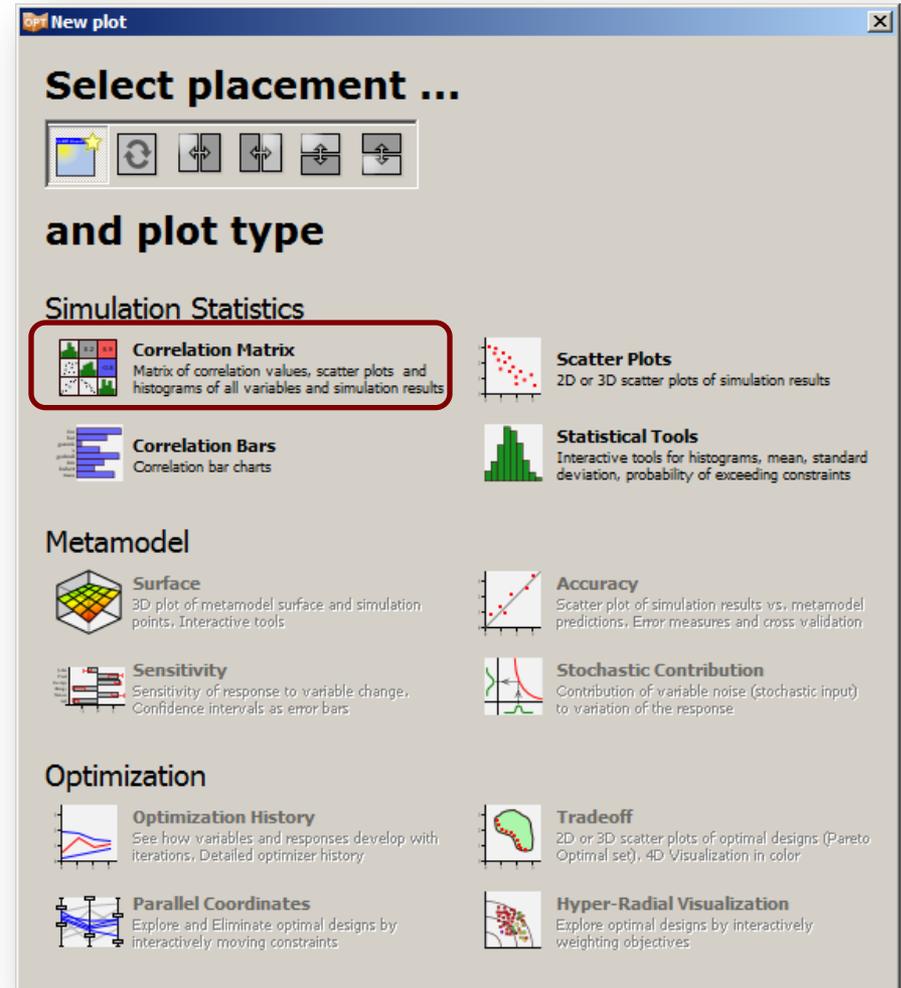
# Statistical Tools

- Go back to Statistical Tools
- Pick Bounds and type **-130** as Lower bound for **MIDDLE\_Z\_DISP\_RESP** Response
- Probability of z-displacement exceeding **-130** is **38%**

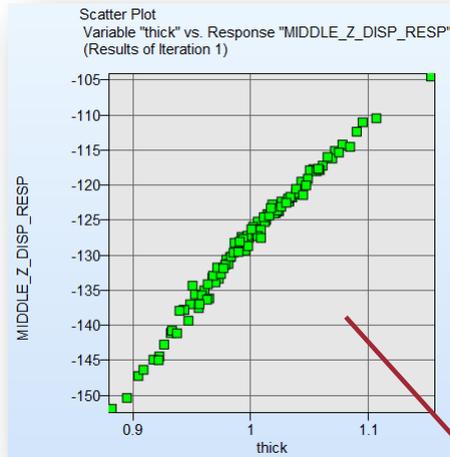


# Viewer

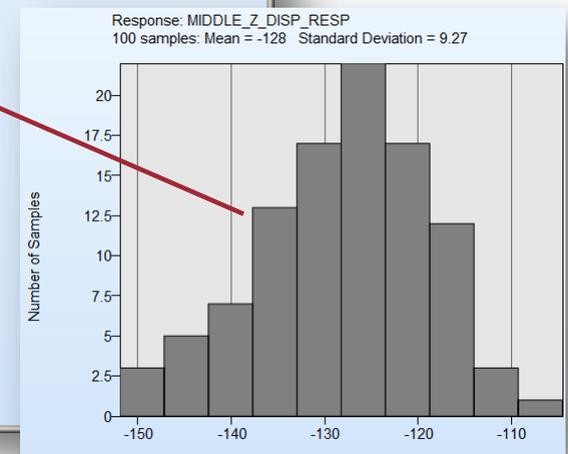
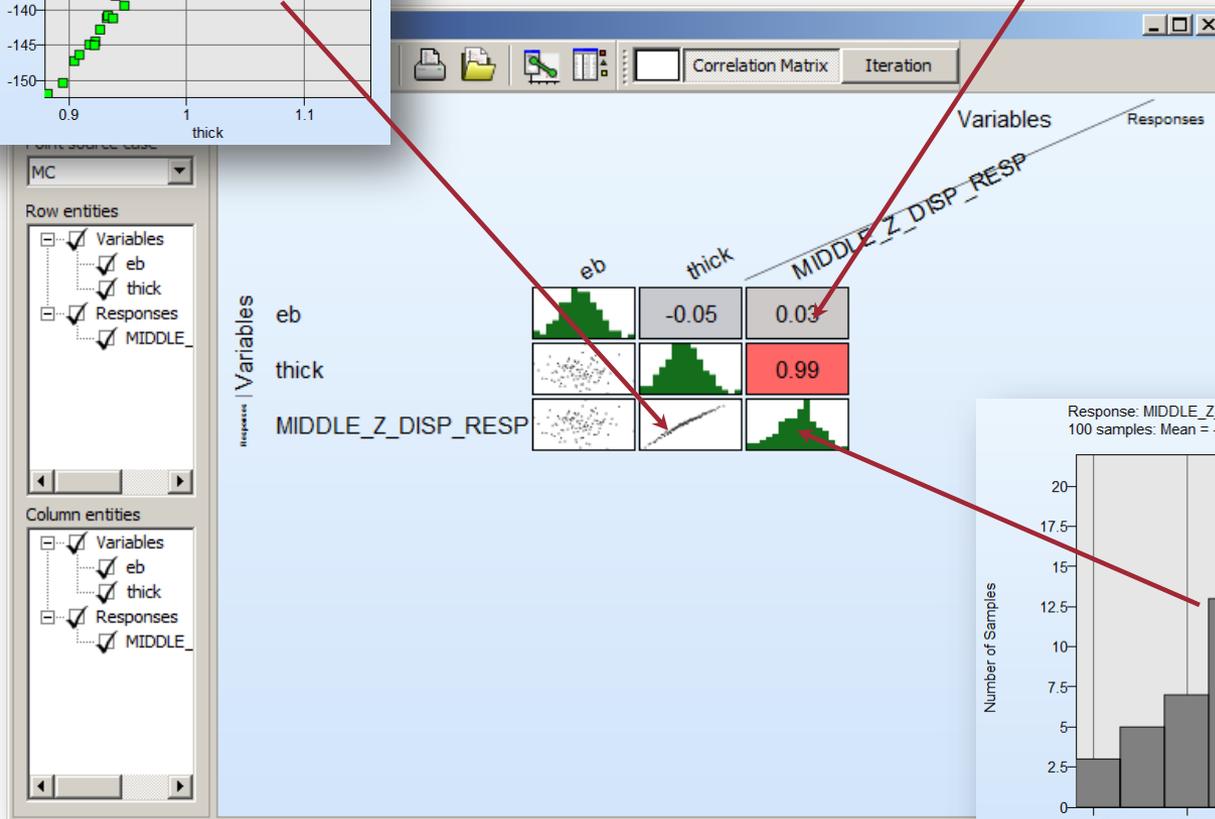
- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select “Correlation Matrix”



# Correlation Matrix

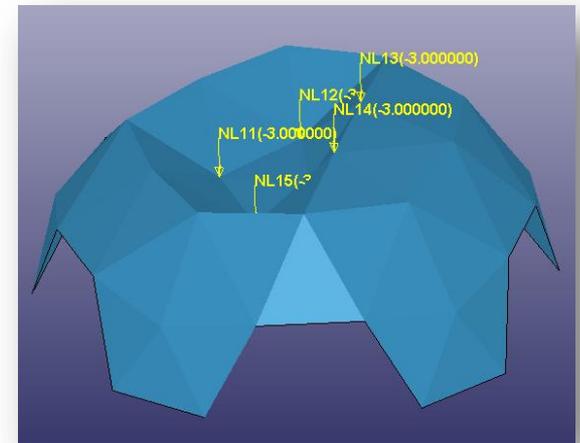
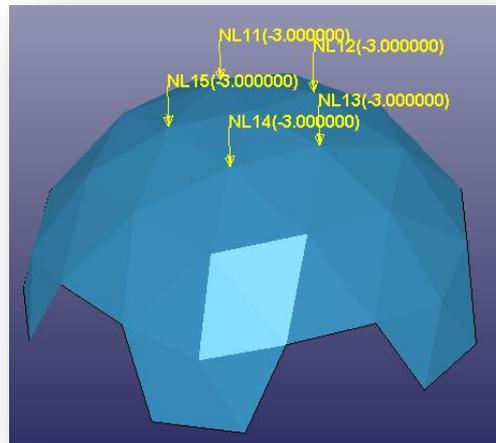
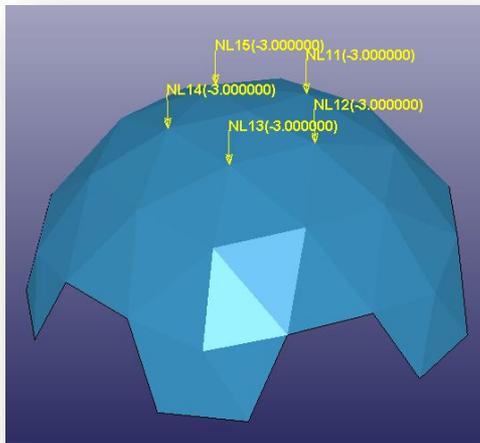


$$\rho = \frac{\text{Cov}(y_1, y_2)}{\sigma_1 \sigma_2}$$



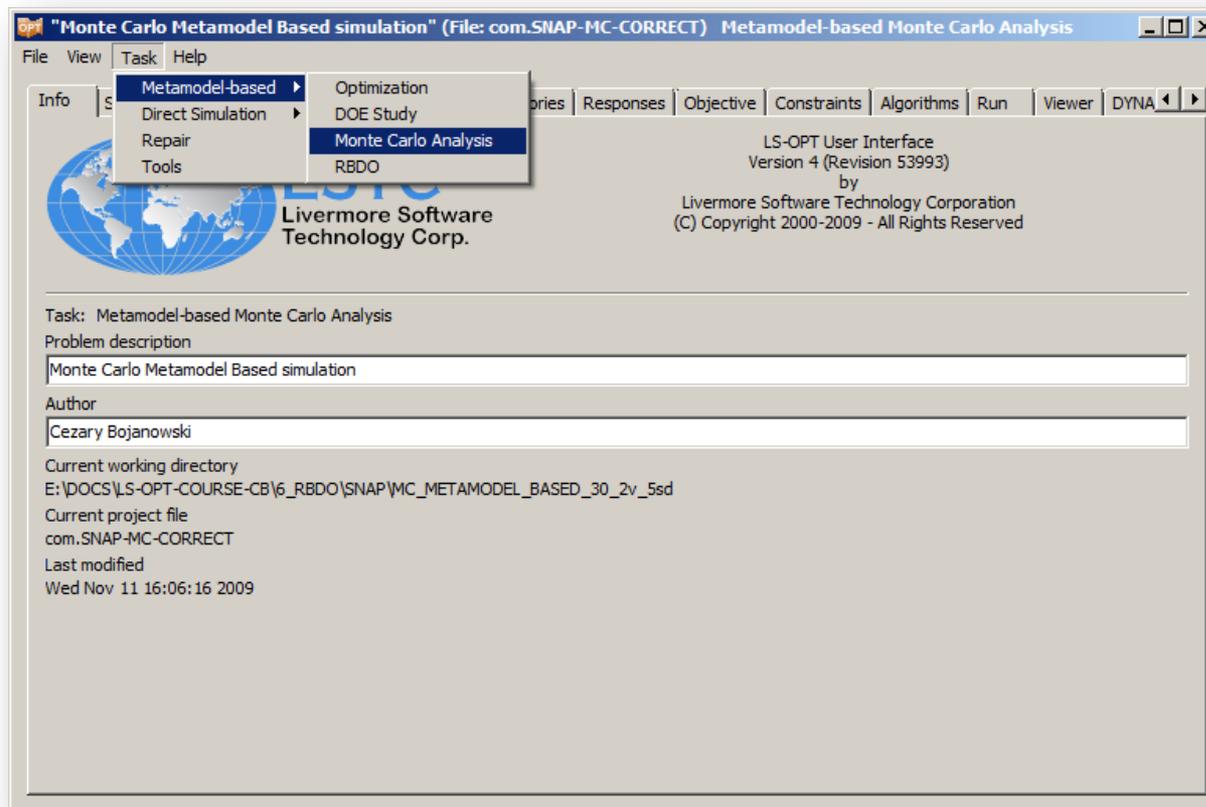
# Monte Carlo Analysis with LS-OPT Metamodel Based Simulation

Example: Deflection of the dome structure.



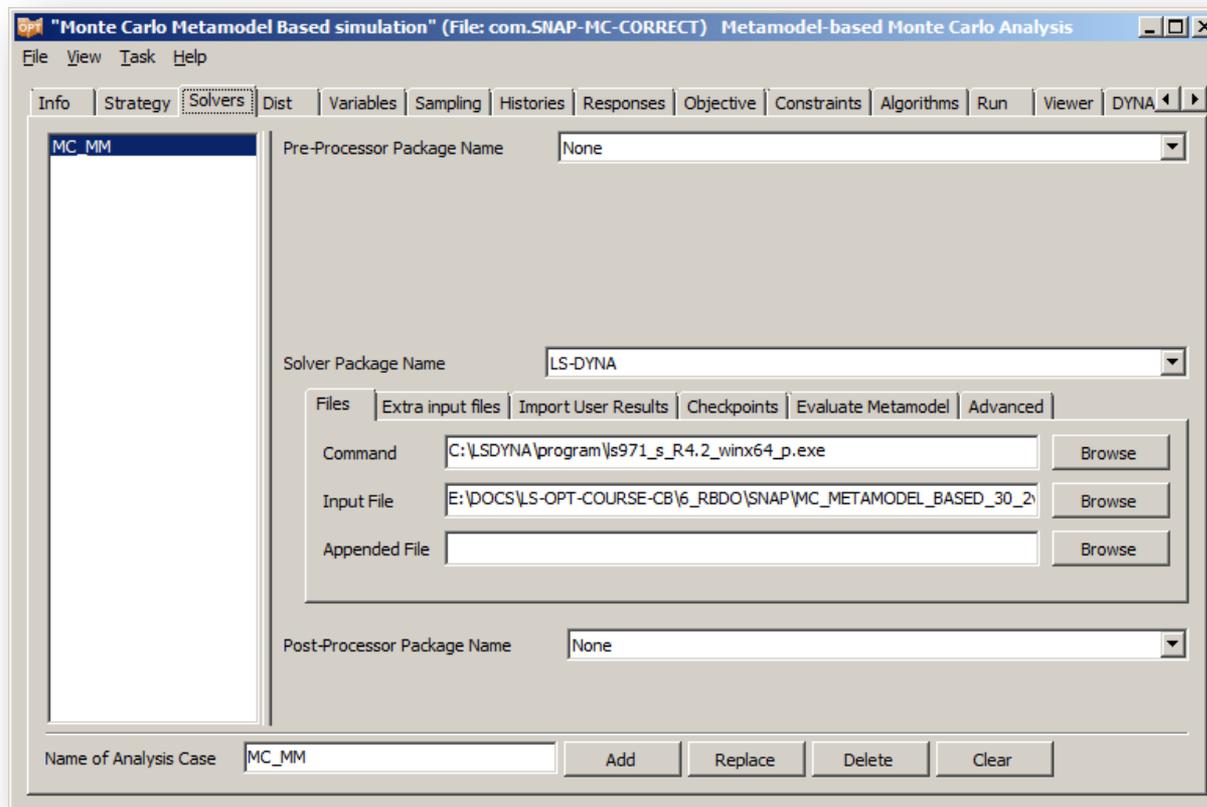
# Approach 2 - Metamodel based Monte Carlo Analysis

- Go to Task tab
- Select Monte Carlo Analysis from Metamodel based group



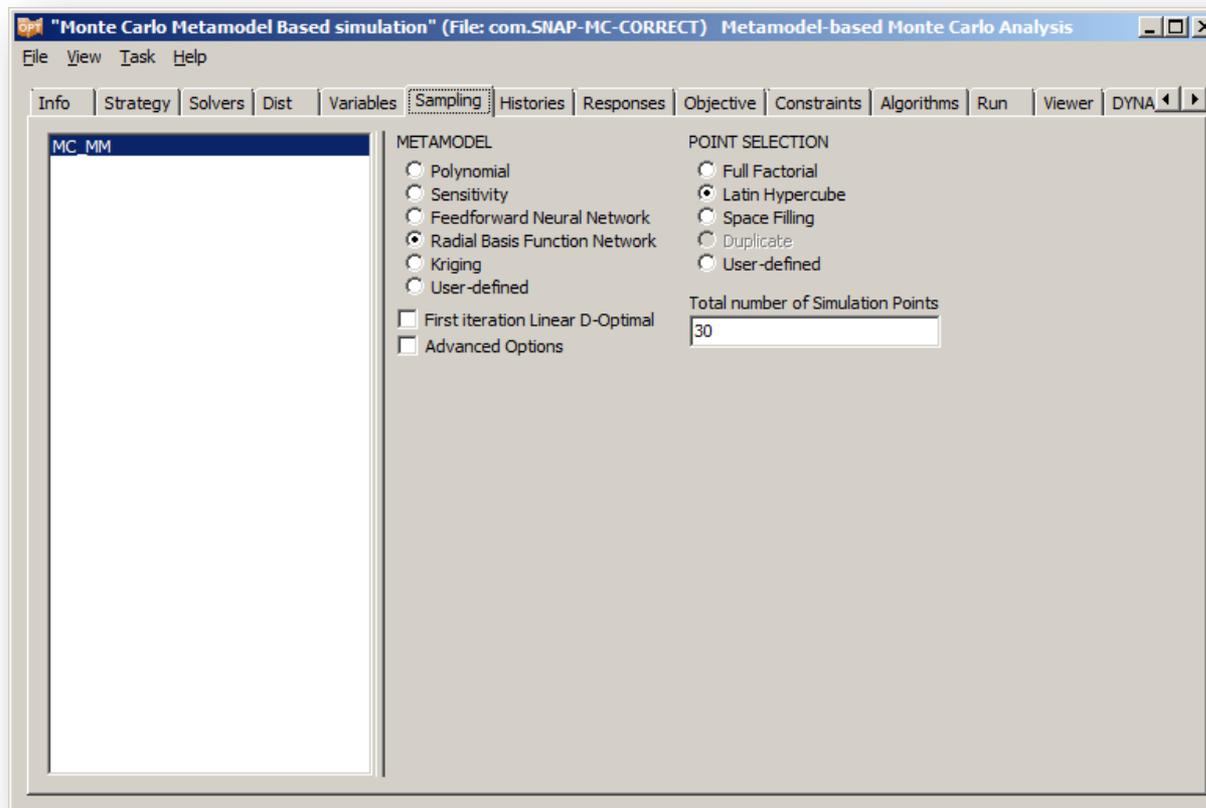
# Solvers Tab

- Go to Solvers tab
- Navigate to the **lsopscript** (or ls-dyna executable) in Command field
- Find the k-file in Input file tab
- For Name of Analysis Case type **MC\_MM** and press Add



# Sampling Tab

- Go to Sampling tab
- Select Radial basis Function Network for Metamodel type and
- Choose 30 for Total number of Simulation Points



# Run Tab

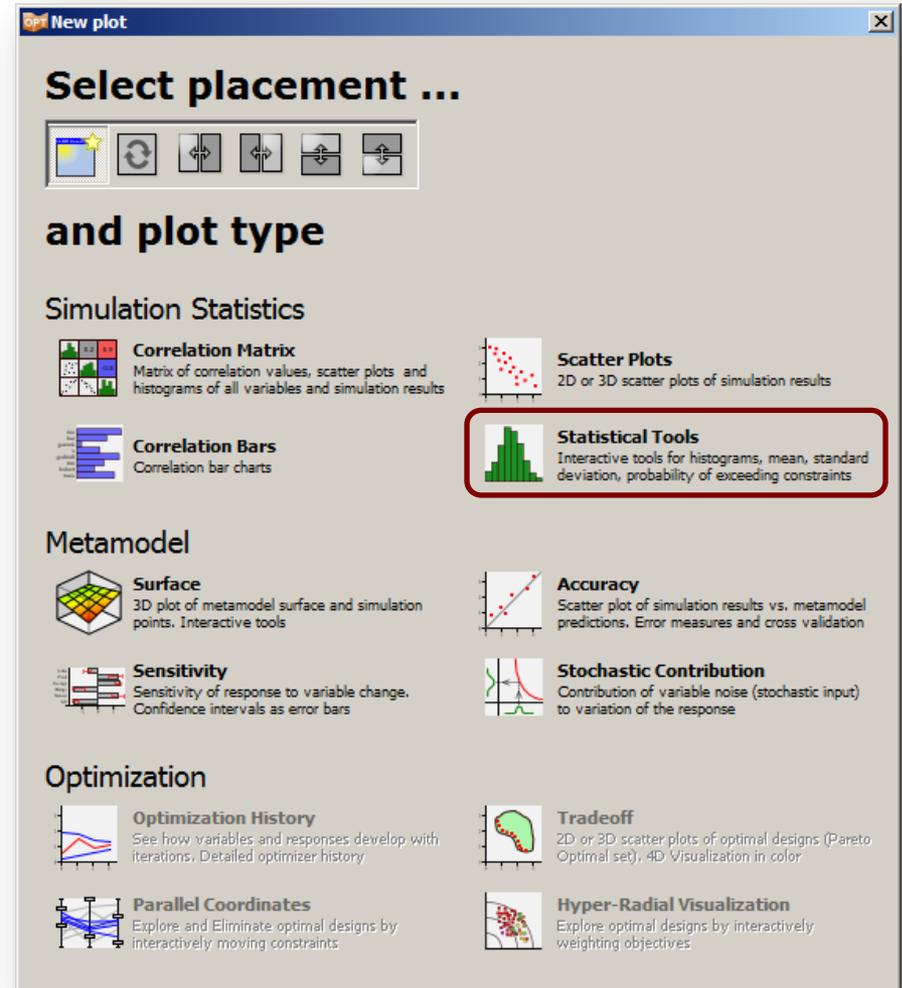
- Go to Run tab
- Select PBS for your Queuing system (if on TRACC cluster) or leave none
- Press Run button

The screenshot displays the 'Monte Carlo Metamodel Based simulation' software interface. The title bar indicates the file path: "com.SNAP-MC-CORRECT". The interface includes a menu bar (File, View, Task, Help) and a tabbed interface with the following tabs: Info, Strategy, Solvers, Dist, Variables, Sampling, Histories, Responses, Objective, Constraints, Algorithms, Run (selected), Viewer, and DYNA. The main workspace is divided into several sections:

- Job ID Table:** A table showing the status of six jobs (IDs 21-26). All jobs are in a 'Normal Termination' state, indicated by green progress bars.
- QUEUING Section:** A dropdown menu is set to 'None'. The 'Concurrent Jobs' input field is set to '8'. The 'Case' dropdown is set to 'MC\_MM'.
- METAMODEL-BASED MONTE CARLO ANALYSIS Section:** The 'Noise Variable Subregion Size (in Standard Deviations)' input field is set to '2'. The 'Use Approximation Residuals' checkbox is unchecked.
- Buttons:** 'Run' and 'Stop' buttons are located below the analysis section.
- Plot Area:** A plot area at the bottom right shows 'No Processes Selected' with a y-axis from 0 to 1 and an x-axis from 0 to 1.
- Variable List:** A list of variables is visible on the left side of the plot area, including Time Step, Kinetic Energy, Internal Energy, Total Energy, Energy Ratio, Global X Velocity, Global Y Velocity, Global Z Velocity, Total CPU Time, and Time to Completion.

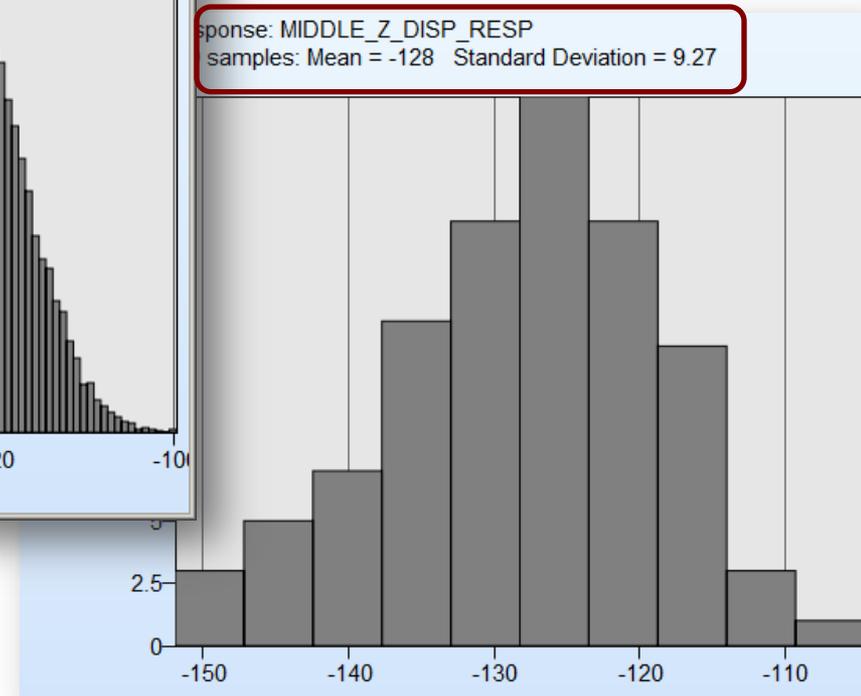
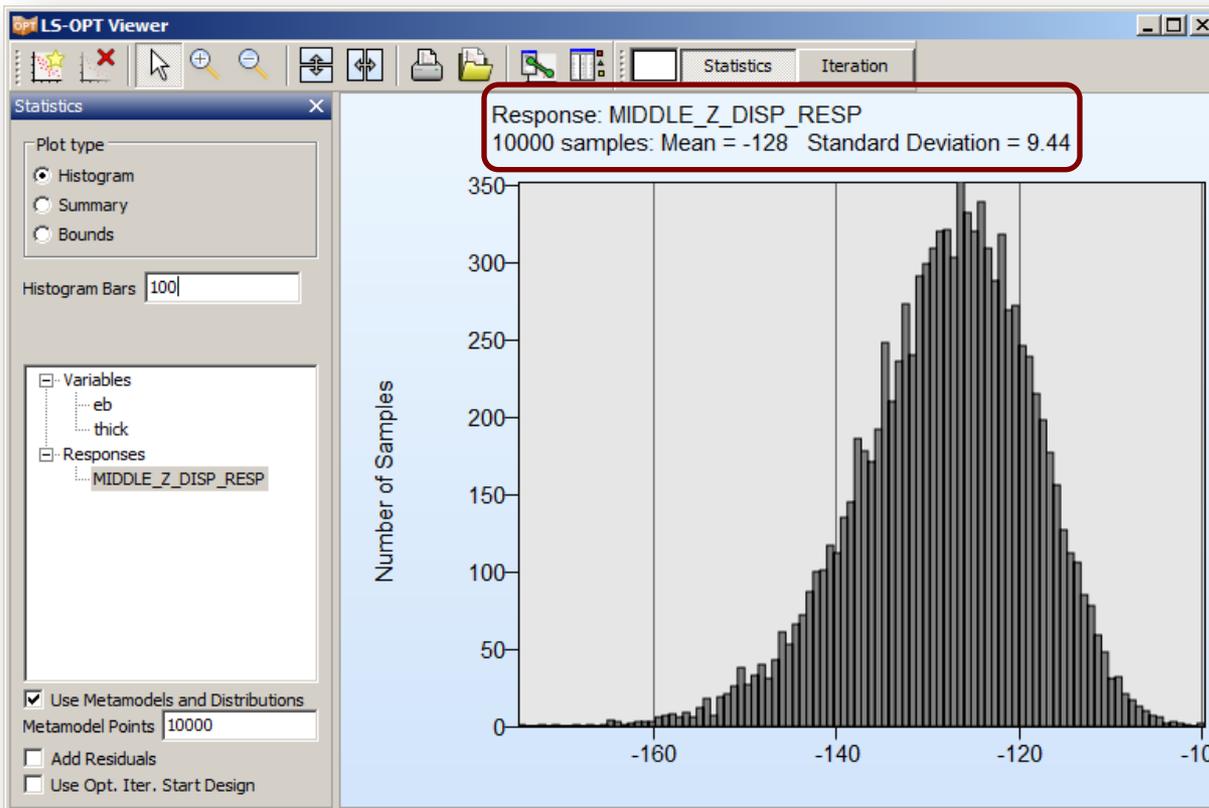
# Viewer

- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select “Statistical Tools”



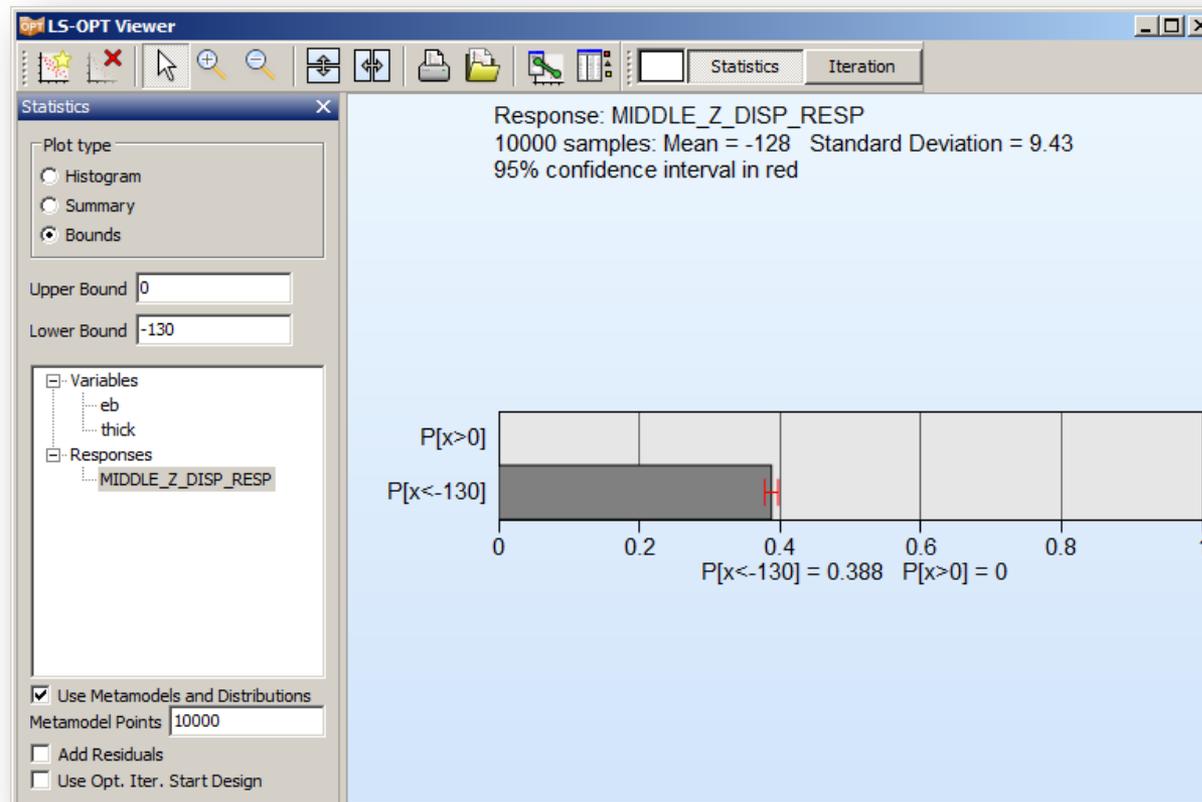
# Statistical Tools

- Plot histograms for Variables and for Response **MIDDLE\_Z\_DISP\_RESP**
- In Histogram Bars field type **100**



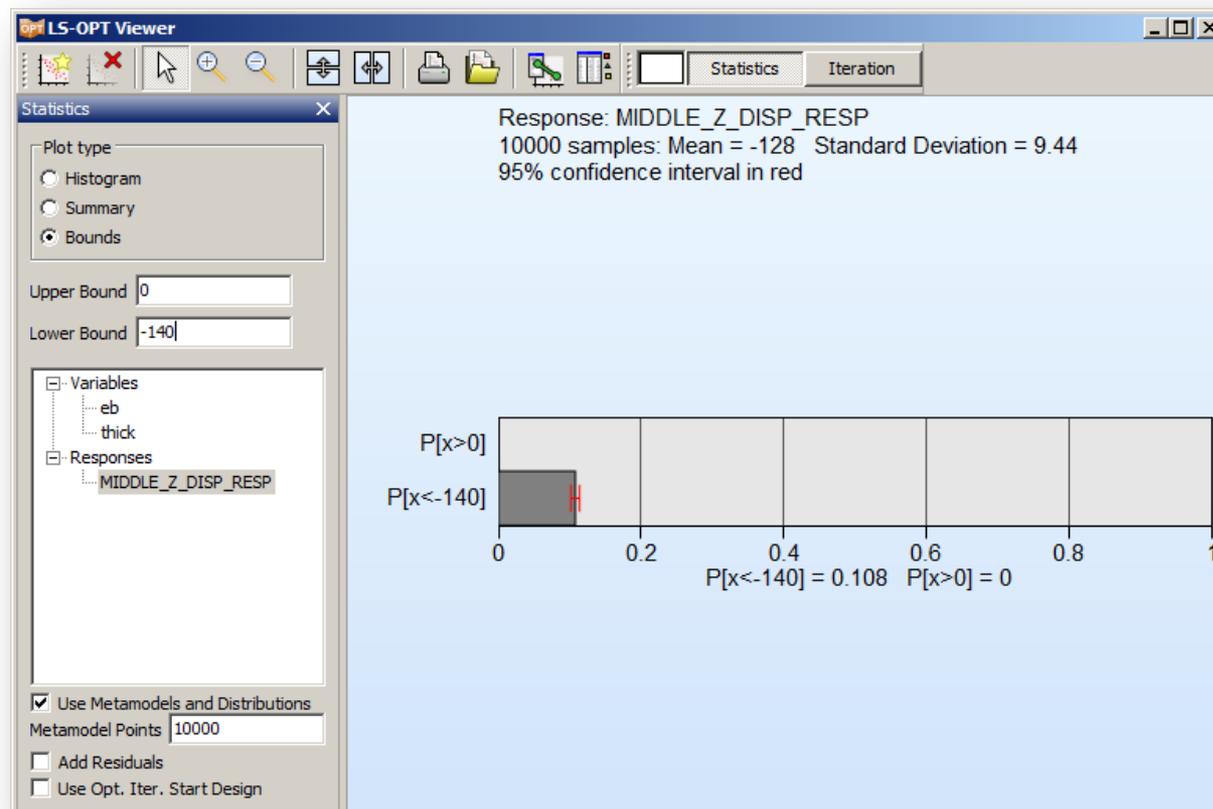
# Statistical Tools

- Go back to Statistical Tools
- Pick Bounds and type **-130** as Lower bound for **MIDDLE\_Z\_DISP\_RESP** Response
- Probability of z-displacement exceeding **-130** is **38.8%** (previously **38.0%**)



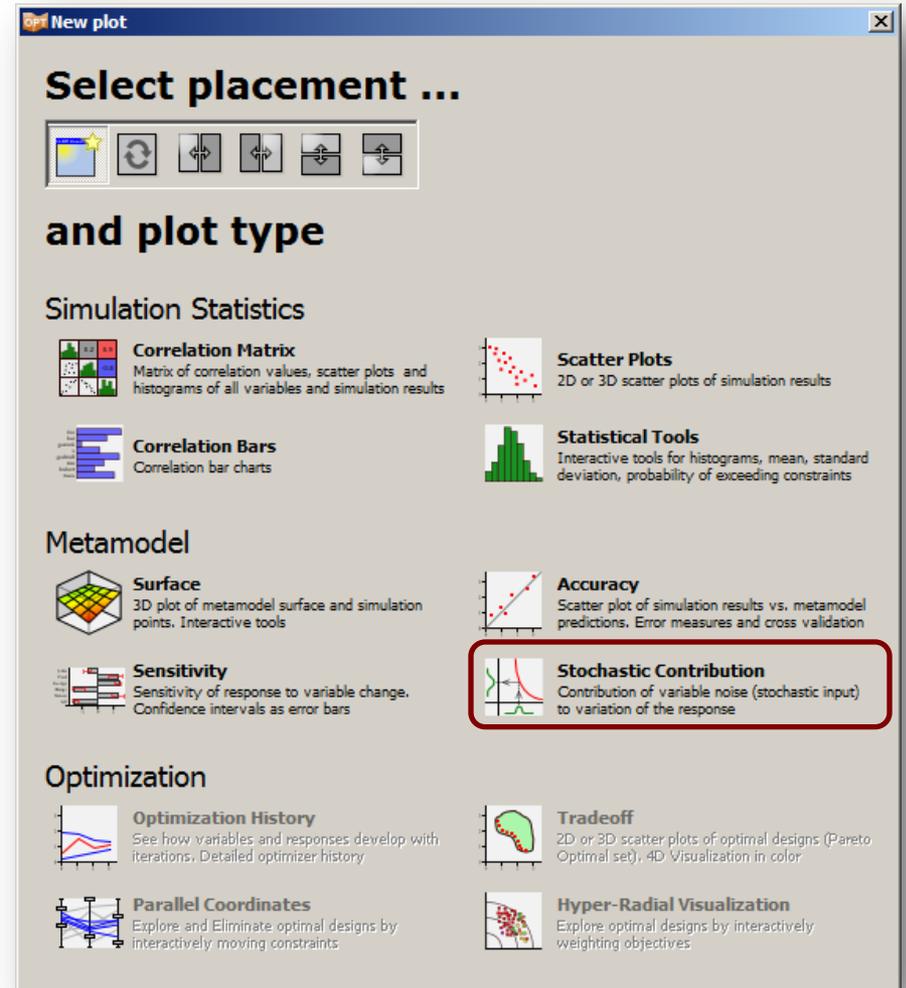
# Statistical Tools

- Go back to Statistical Tools
- Pick Bounds and type **-140** as Lower bound for **MIDDLE\_Z\_DISP\_RESP** Response
- Probability of z-displacement exceeding **-140** is **10.8%**



# Viewer

- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select “Stochastic Contribution”



# Stochastic Contribution

- Standard deviation in response due to distributions of variables should be shown

