Introductory Course: Using LS-OPT® on the TRACC Cluster

2.5a - Probabilistic Analysis

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Introduction

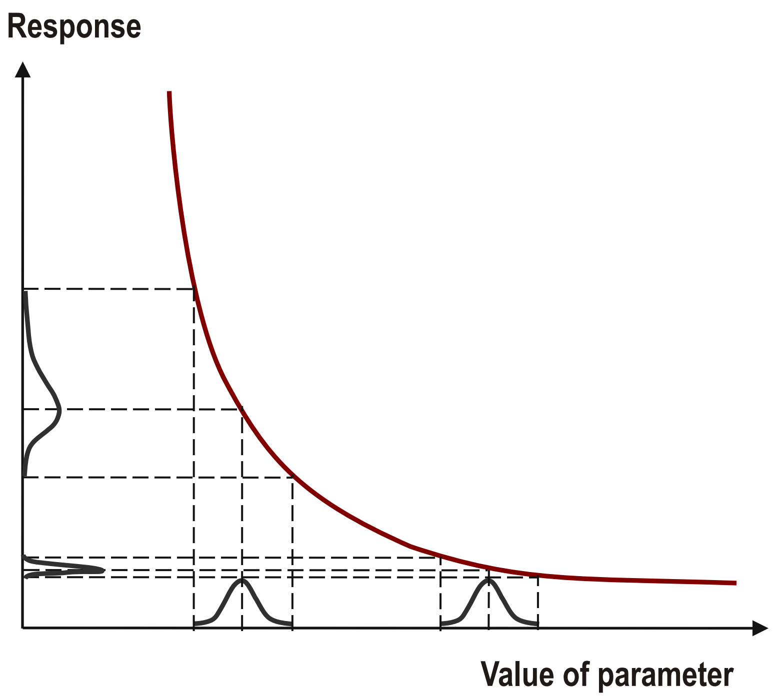
- No two structural events will be exactly similar, nor will structural event occur exactly as designed or analyzed.
- Adverse combinations of design and loading variation may lead to undesirable behavior or failure.
- Therefore if significant variation is expected – a probabilistic evaluation is required.
- Differences in structural performances can be attributed to deterministic and random effects. Understanding their sources is crucial for successful analysis, yet it is very challenging.

Response Variation

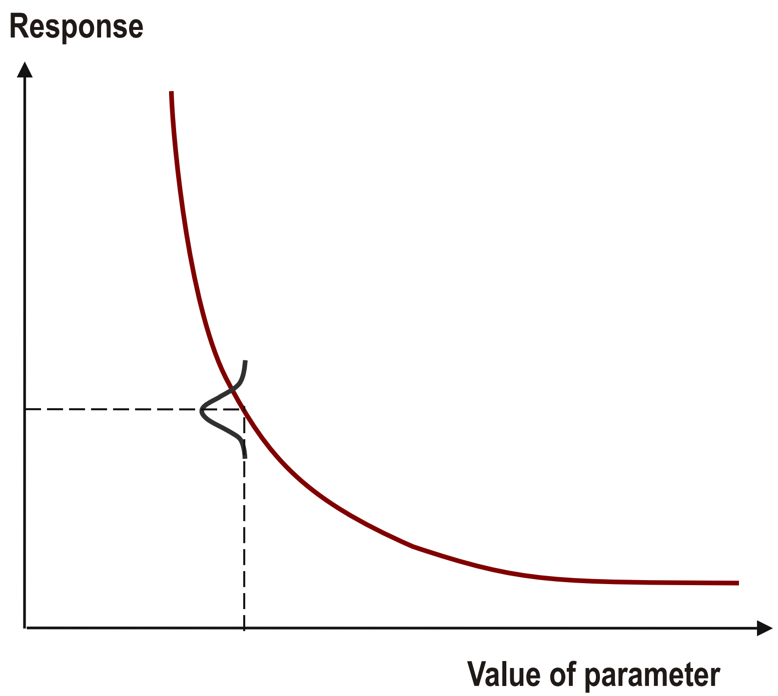
- Deterministic variation
  - Expected, predictable and repeatable variation in a response associated with a variation in a parameter.
  - Can be controllable or uncontrollable
- Random variation
  - Variation that cannot be associated with a change in system parameters
    - Regular random variation – not associated with the physics of the system
    - Chaotic random variation – noise caused by bifurcation behavior in the structure

Response Variation

- Deterministic variation



- Random variation (buckling, contact etc.)

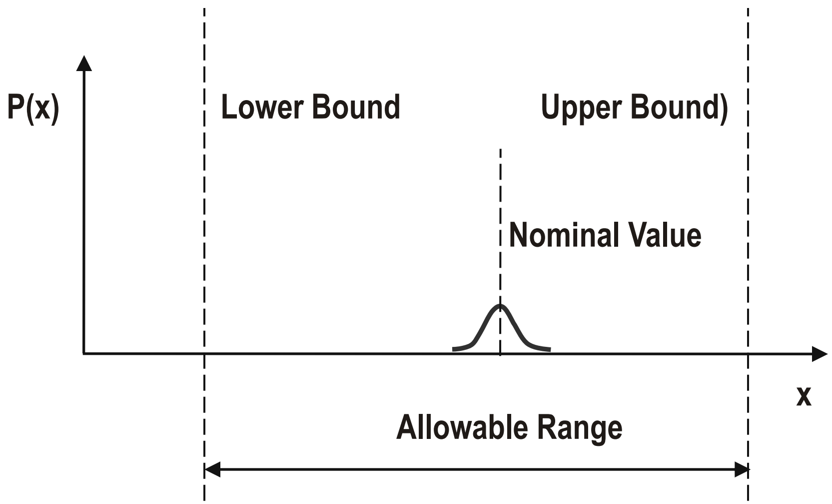


Sources of Variation

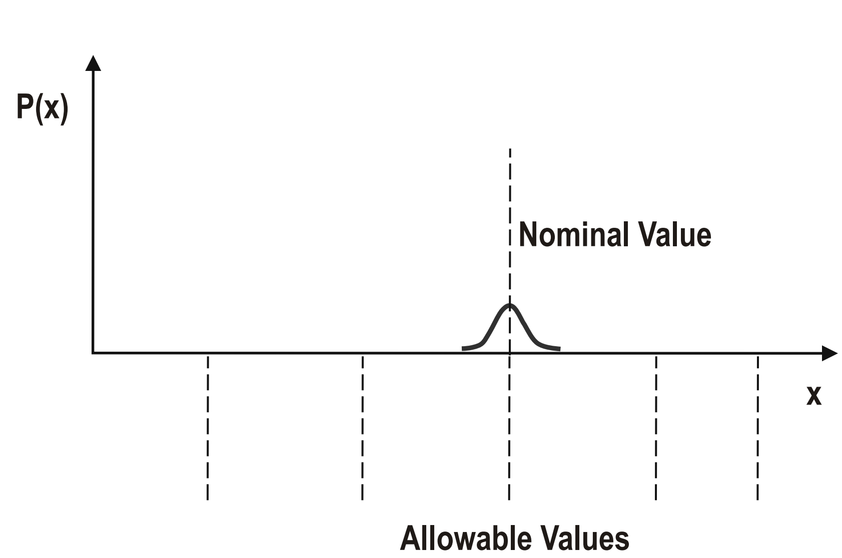
- Design Parameter Variation
  - Control variables
    - can be controlled in the design, analysis and production level.
    - it can be assigned a nominal value and will have variation around it.
    - example: shell thickness
  - Noise variables
    - are difficult or impossible to control and the design and production level.
    - but can be controlled on the analysis level.
    - will have the nominal value and will follow exactly the distribution.
    - example: variations in loads and material properties

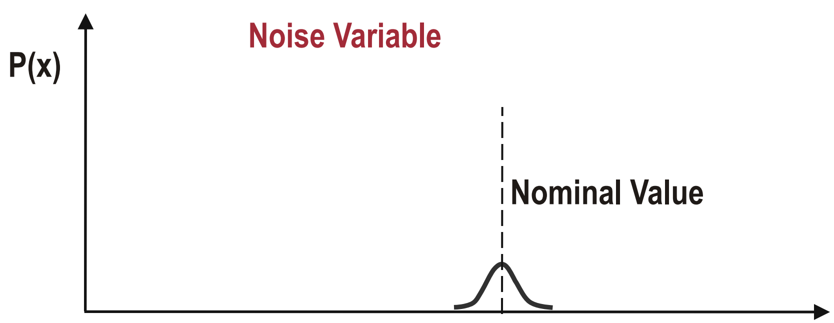
Sources of Variation

Design Variable



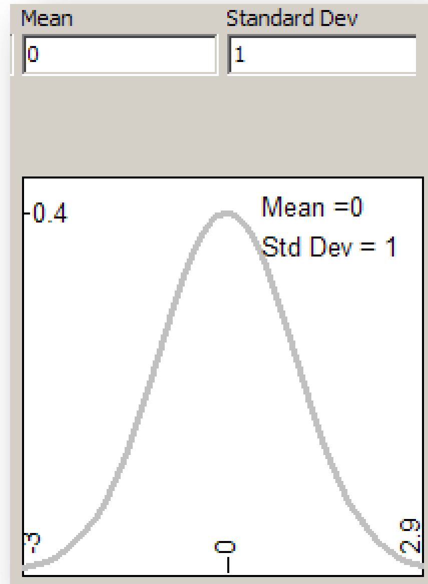
Discrete Variable





Distributions in LS-OPT

- **Normal**
- Uniform
- Lognormal
- Weibull
- Beta
- Binomial
- Truncated normal
- User defined



$$f(x) = ae^{-\frac{(x-b)^2}{2c^2}}$$

a – Height of the peak

b – Position of center (mean)

c – Controls the width of the “bell” (standard deviation)

Distributions

- Probability density function:

$$\int_{-\infty}^{+\infty} f(x)dx = 1 \qquad f(x) \geq 0$$

- Distribution function of the random variable:

$$F(x)\int_{-\infty}^{x} f(t)dt$$

- Mean of a probability density:

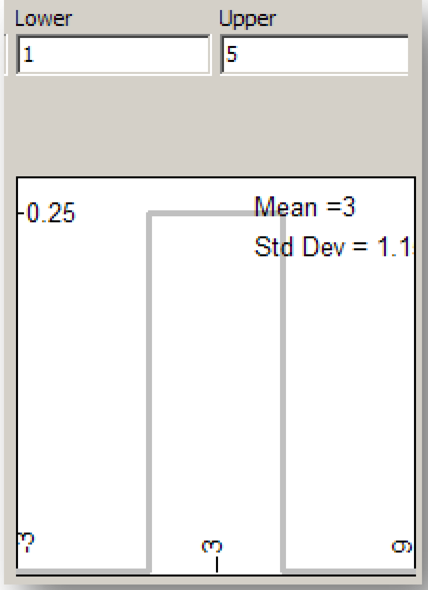
$$\mu = \int_{-\infty}^{+\infty} x \cdot f(x)dx$$

- Variance - the second moment about the mean:

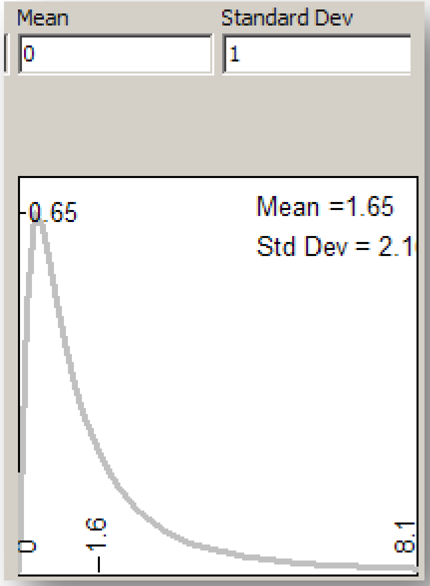
$$\sigma^2 = \int_{-\infty}^{+\infty} (x-\mu)^2 \cdot f(x)dx$$

Distributions in LS-OPT

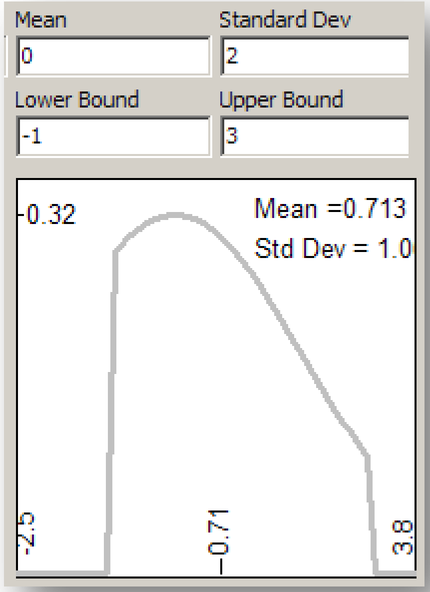
- Uniform distribution



- Lognormal distribution

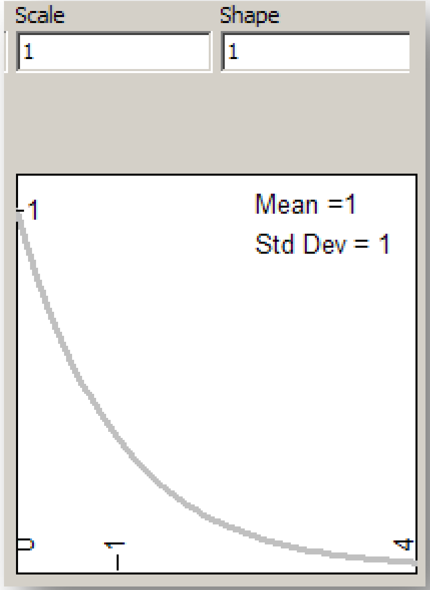


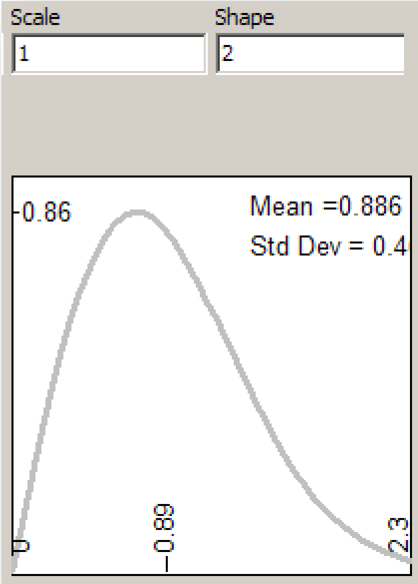
- Truncated normal distribution

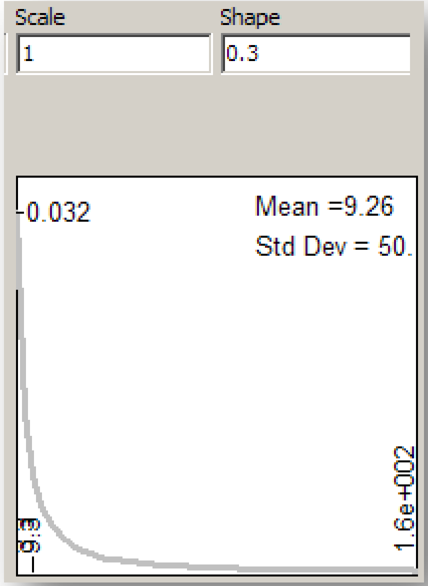


Distributions in LS-OPT

- Weibull distribution







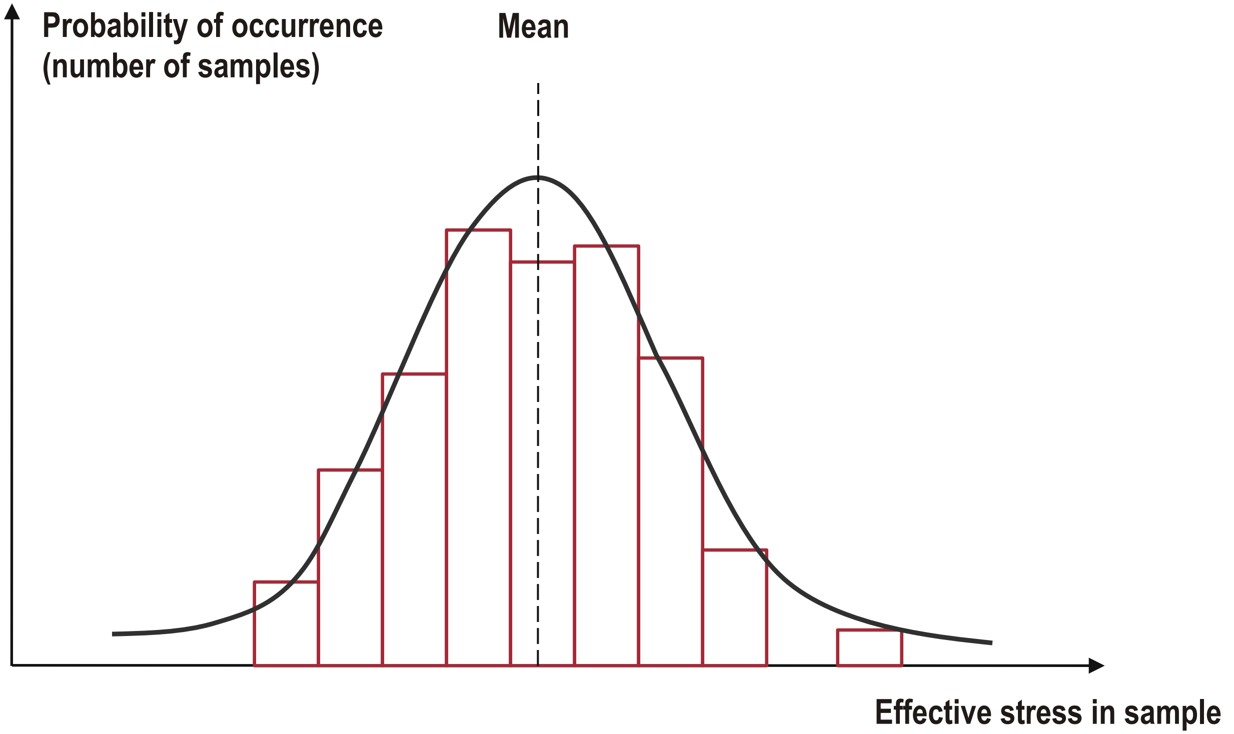
Sources of Variation

- Modeling variation
  - Differences in modeling will give different results as well as introduce noise to the results.
    - mesh density
    - choice between FEA, SPH, MMALE
    - resolution of output data
- Analysis variation
  - Variations in initial conditions can lead to noticeable differences in responses.
    - Physical: bifurcation events, sequence of impact (contact) due to changes in the design variables
    - Algorithmic: due to discretization a node can come into contact with an element or adjacent to it

Sources of Variation

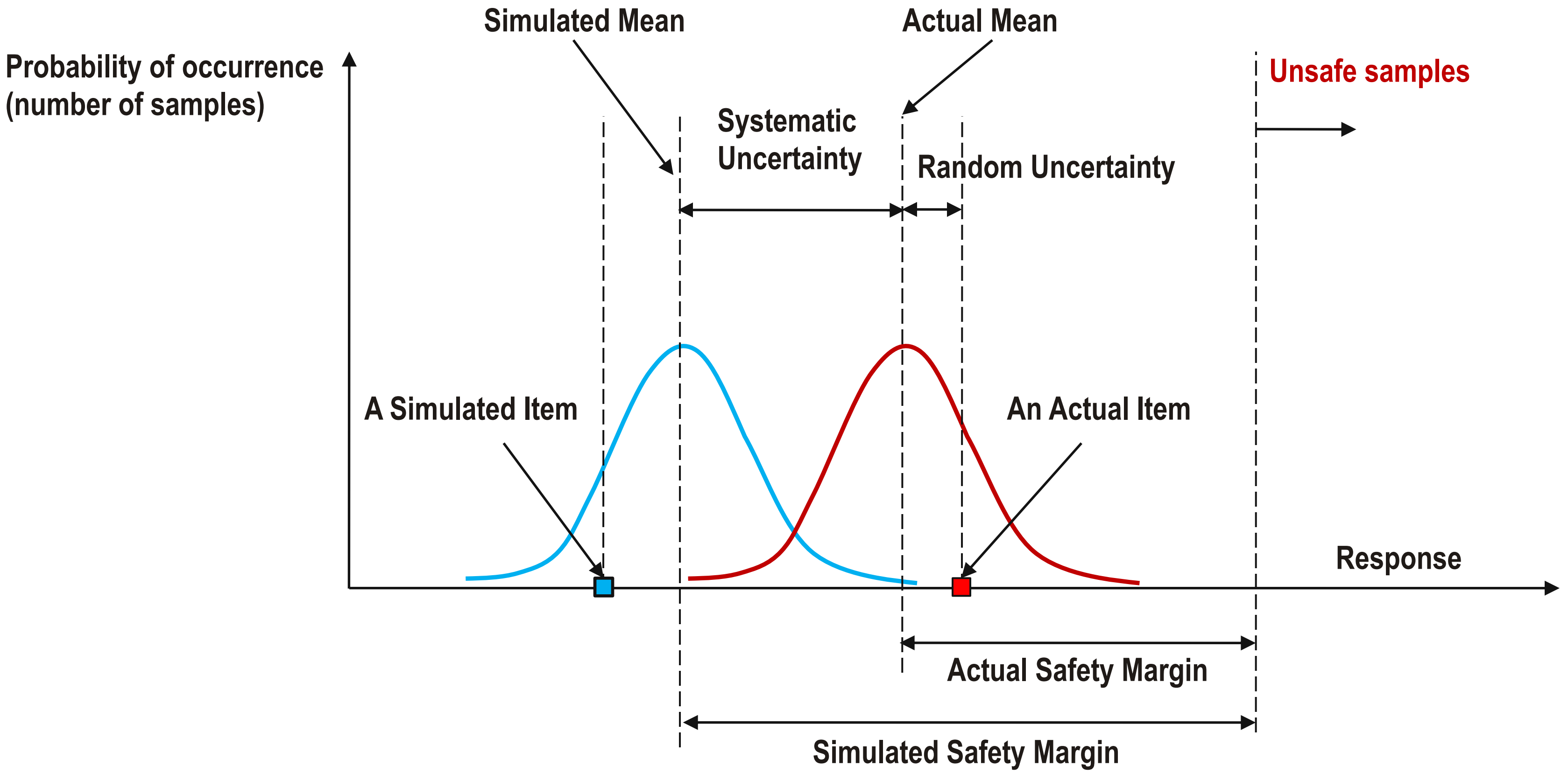
- Pure Variation
  - change in the results nonrelated to the input
    - different versions of solver
    - machine precision
    - round-off error

Response Variation



- What is the distribution of the response, given the distribution of parameters?
- What is the probability of structure failure?
- What are efficient redesign strategies?

Safety Margin

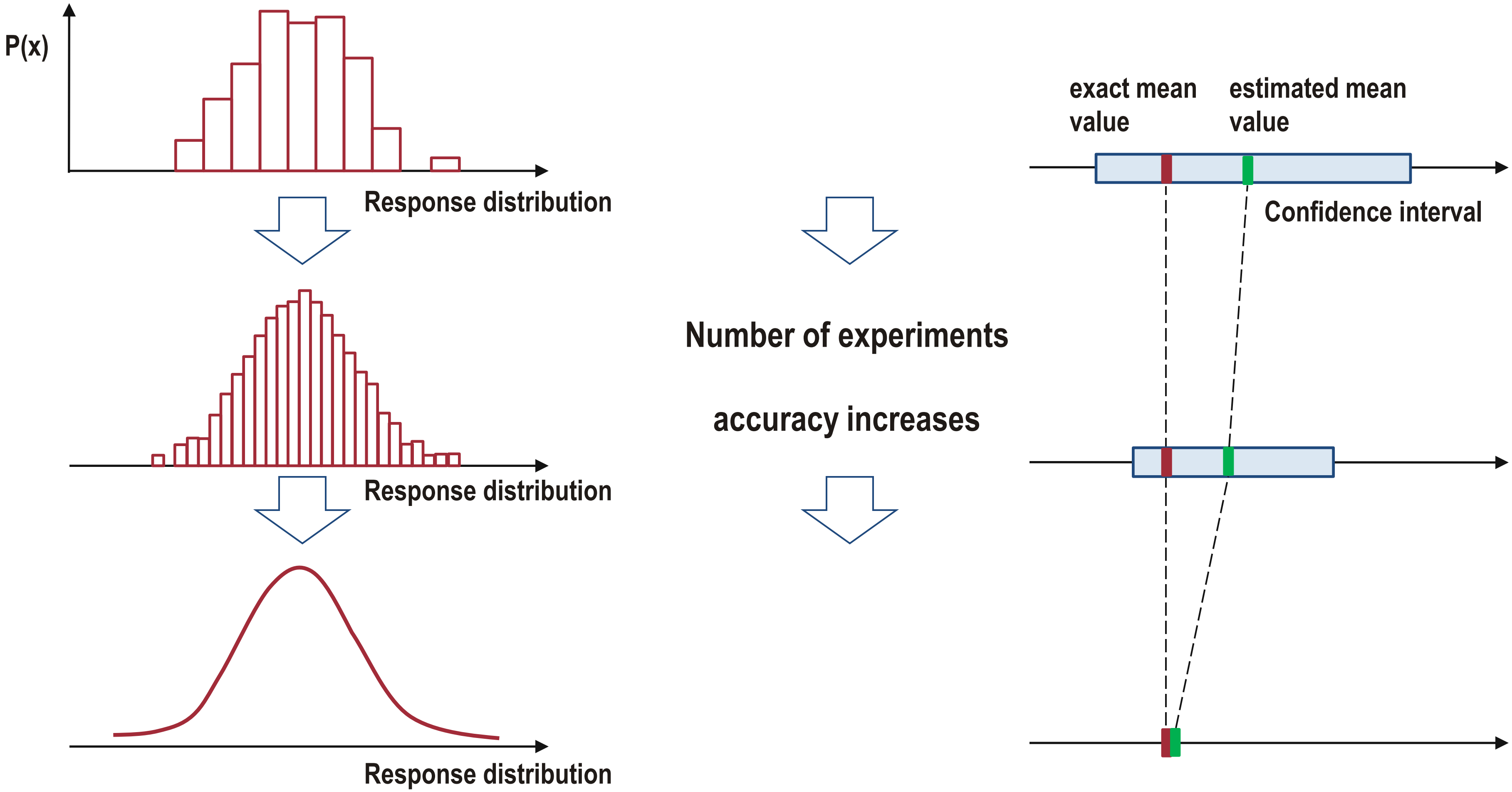


- What is the distribution of the response, given the distribution of parameters?
- What is the probability of structure failure?
- What are efficient redesign strategies?

Monte Carlo Analysis

- Monte Carlo simulation aims to compute results with the same scatter as what will occur in practice.
- Multiple analyses are conducted using values of the input variables selected considering their probability density function.
- The results from these analyses should have the scatter expected in practice.
- Requires large number of runs and random sampling.

Monte Carlo Analysis



Latin Hypercube sampling

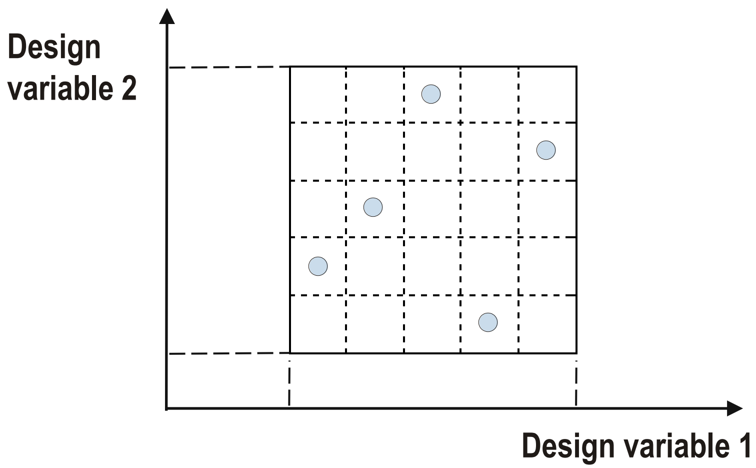
- The Latin Hypercube method is a random experimental design process. Was developed to generate a reasonable collection of parameter values from a multidimensional distribution.
- A square grid containing sample points is a Latin square iff there is only one sample in each row and each column.
- A Latin hypercube sampling is the generalization of this concept to an arbitrary number of dimensions.
- LH designs are independent of the mathematical model of the approximation and allow estimation of the main effects of all factors in the design in an unbiased manner

Latin Hypercube sampling

- When sampling a function of n variables, the range of each of them is divided into P equally probable intervals. P sample points are then placed to satisfy the LH requirements.

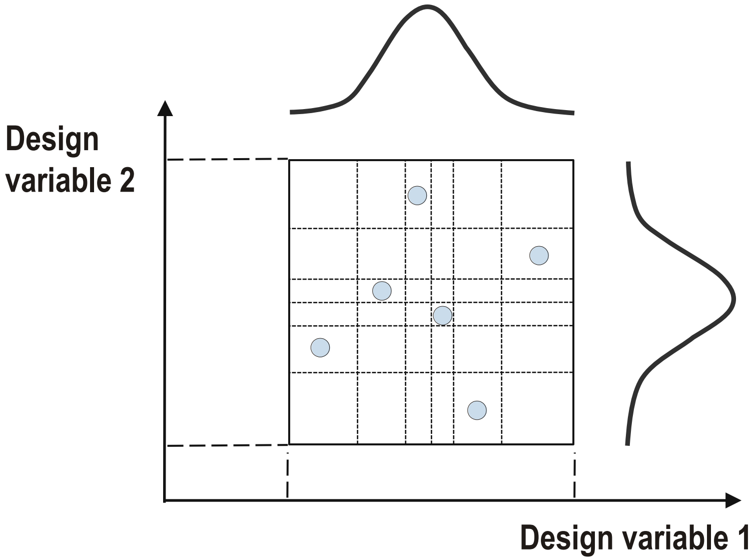
- This forces the number of divisions to be equal for each variable.

- LH sampling does not require more samples for more dimensions (variables)



Latin Hypercube sampling

- If the variable has assigned distribution, the partitions for LH sampling will be subdivided as to have equal probability

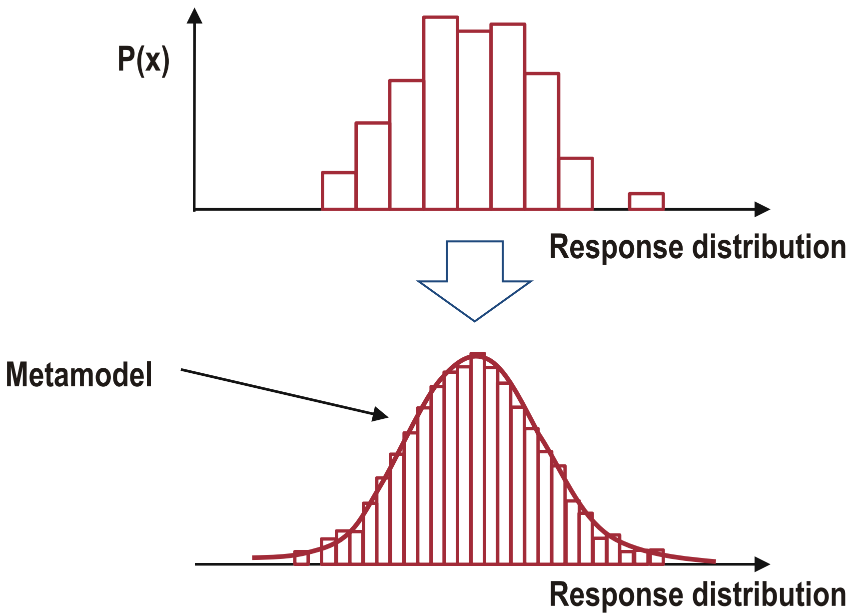


- LH sampling may be used further for space filling designs

Metamodel-based Monte Carlo

- Problem: Monte Carlo analysis requires large number of runs. May be prohibitive for large models (100+ runs).
- Solution: Use Monte Carlo analysis with reduced number of runs and construct metamodel (10-30+ runs). Very large number of function evaluations are possible when using metamodels (10^6).
- The results are exact for linear or quadratic responses approximated with linear or quadratic response surfaces.
- For random shapes of response use Neural Networks (NN) or Radial Basis Function (RBF) Networks

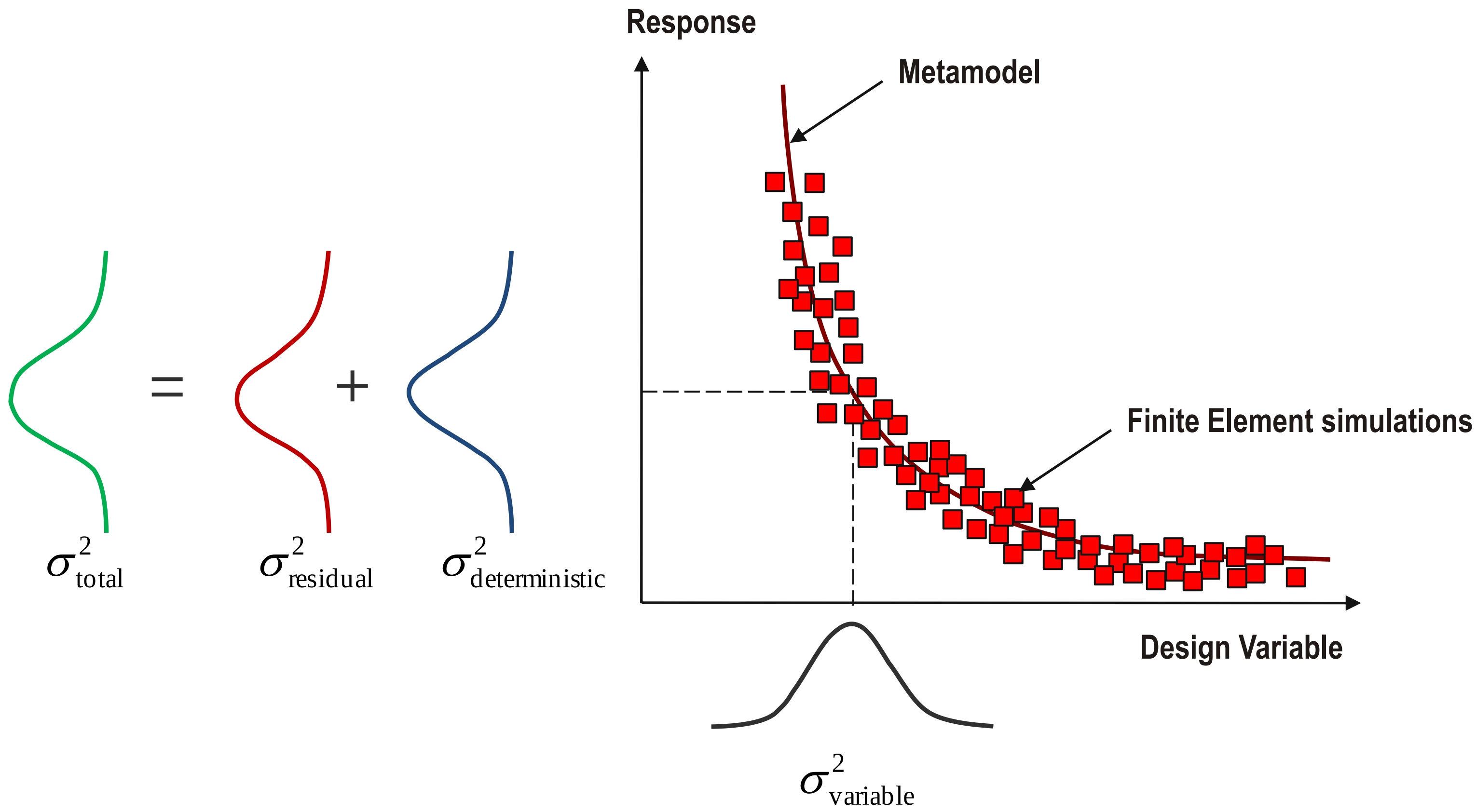
Metamodel Based Monte Carlo Analysis



Build a metamodel on medium number of simulation points

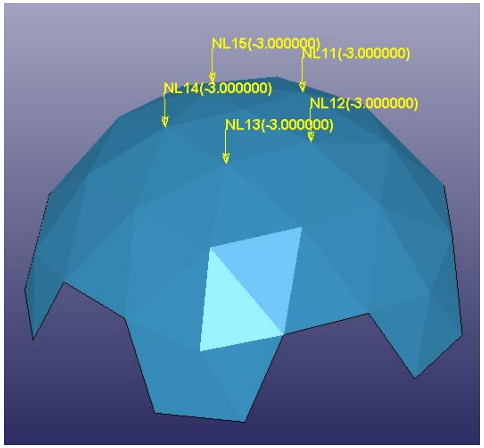
Perform hundreds of function evaluations on cheap metamodel

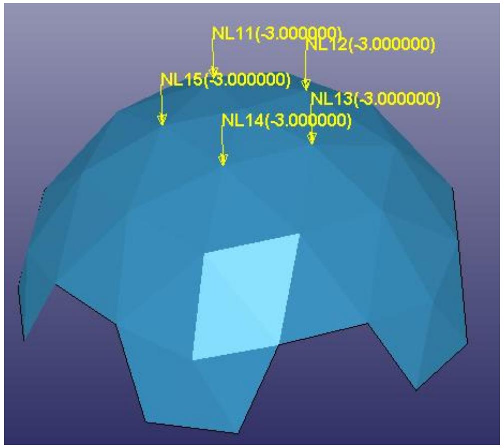
Stochastic Contribution in Metamodel based Monte Carlo Simulations

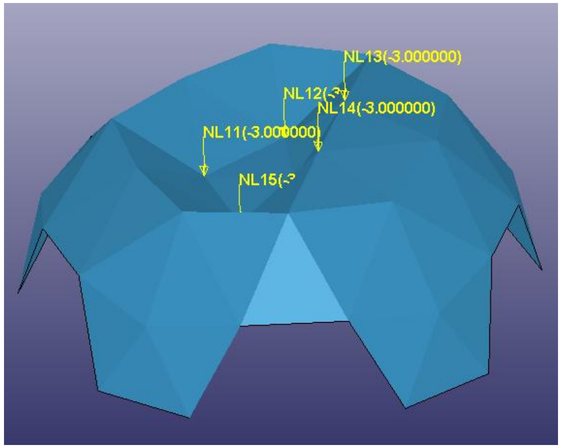


Monte Carlo Analysis with LS-OPT Direct Simulations

Example: Deflection of the dome structure.



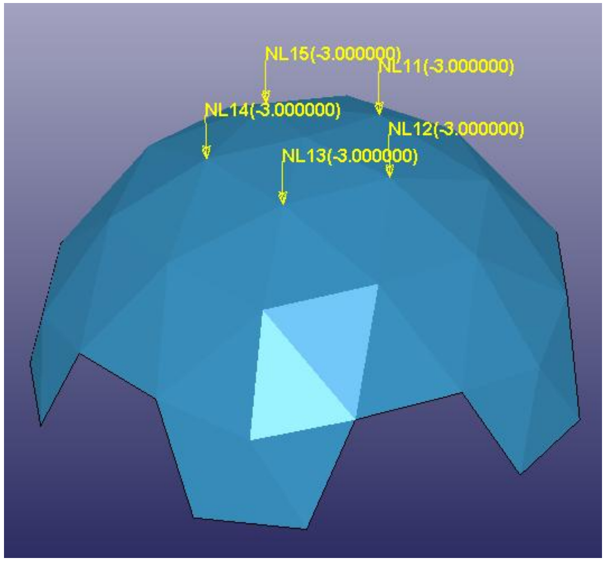




Problem description

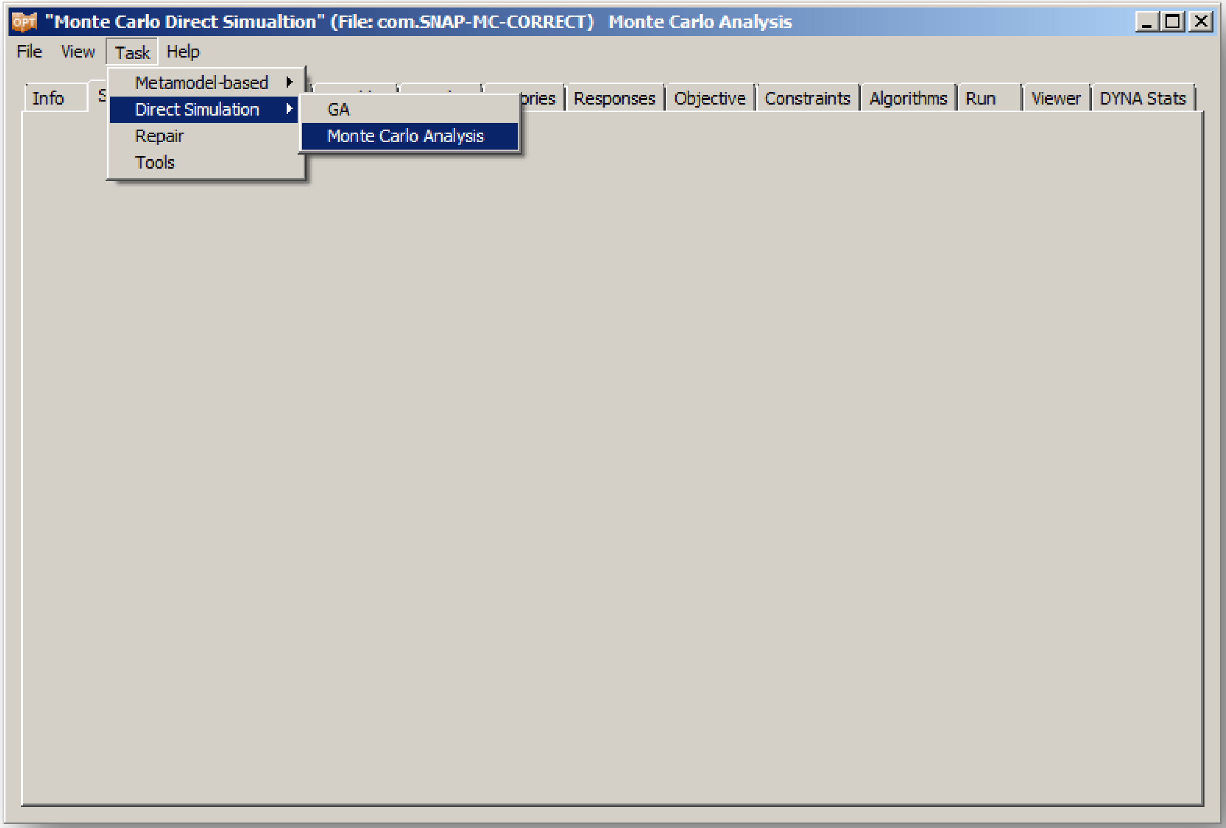
- This example is a Monte Carlo analysis of a dome structure being loaded from the top. The effect of variations in:
  - material properties,
  - thickness,

  are investigated.
- The geometry is shown in Figure below.
- The x, y and z – displacements are restrained at the bottom of the structure.
- It is loaded from the top in five nodes.
- Maximum z – displacement of the central node is used as the response variable



Approach 1 – Monte Carlo Direct Analysis

- Go to Task tab
- Select Monte Carlo Analysis from Direct Simulations group

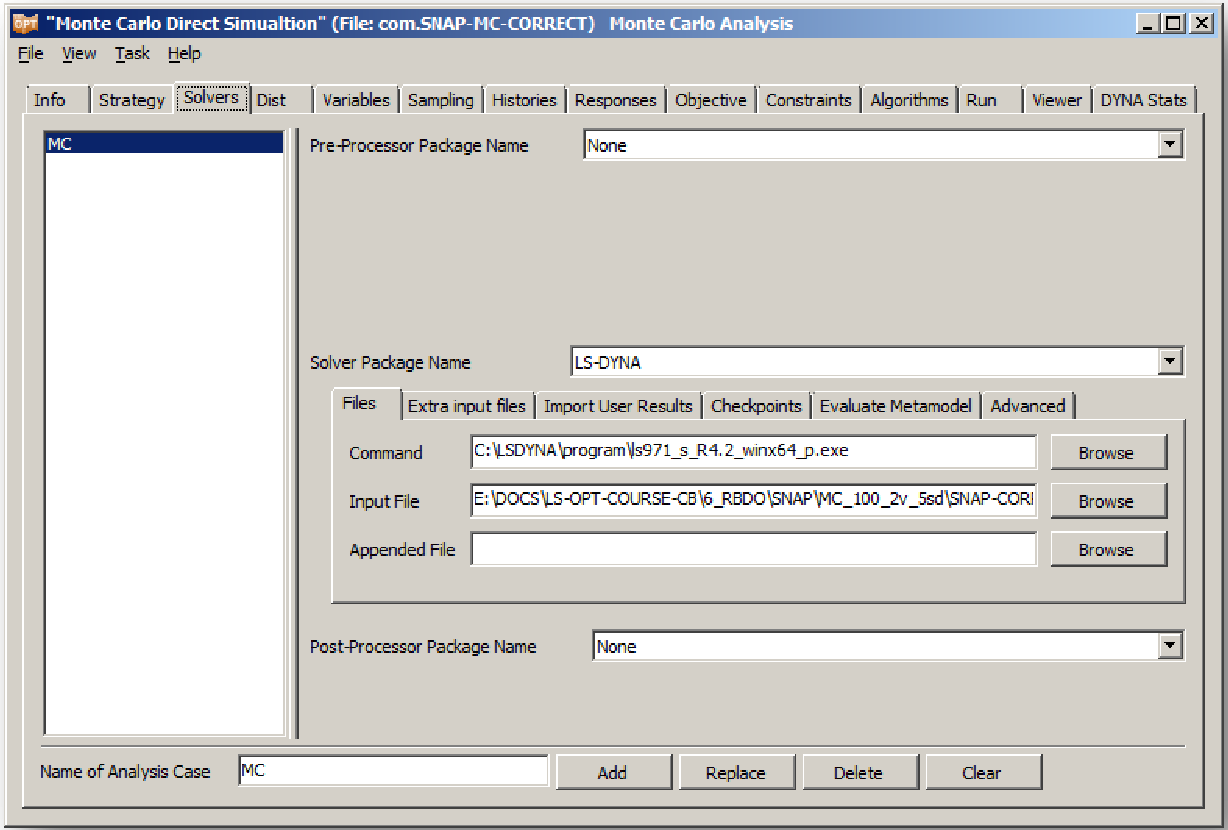


Info Panel

- Describe the problem
- Give author's name

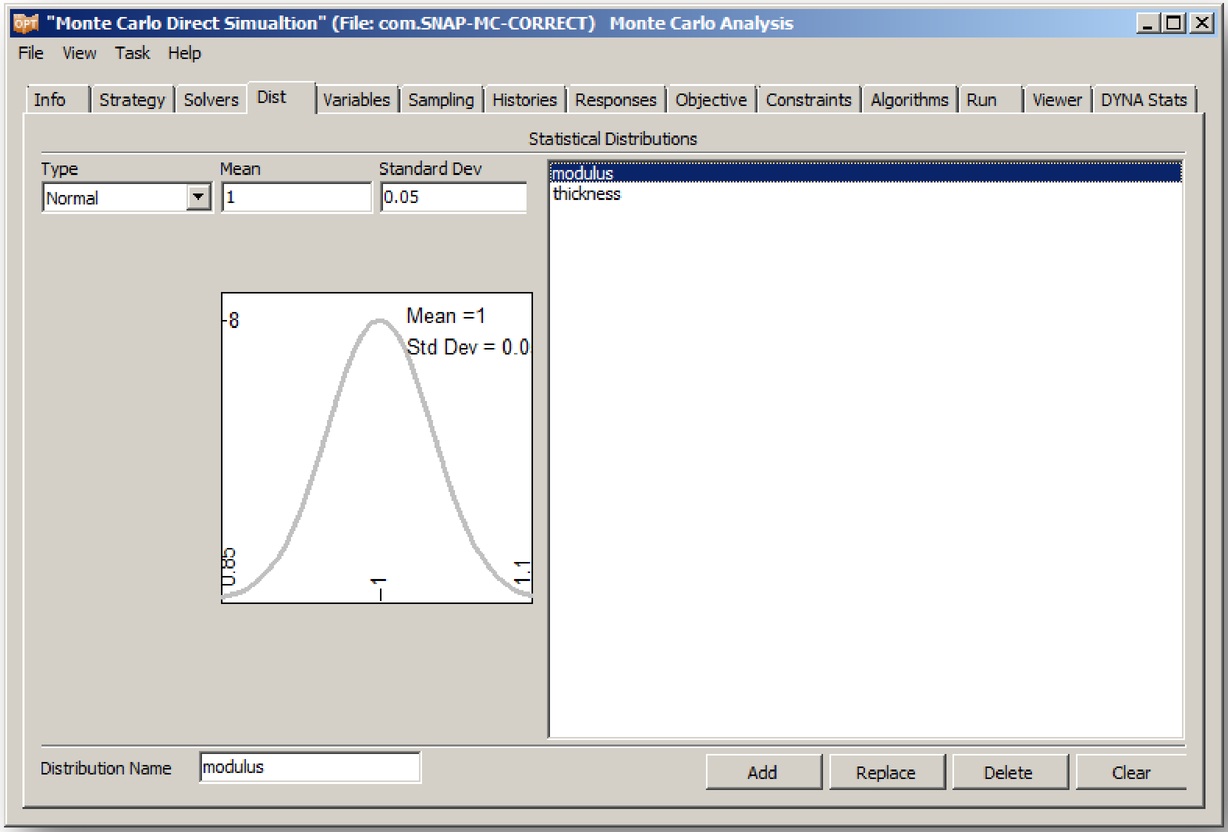
Solvers Panel

- On TRACC cluster specify the command as full path to `lsoptscript`
- Navigate to the input file
- Name the analysis `MC` and hit Add



Distributions Panel

- In distributions panel select Type Normal, type 1 for Mean and 0.05 for Std. dev.
- Name the distribution – modulus and hit Add button
- Repeat the same procedure for thickness distribution



LS-DYNA k-file

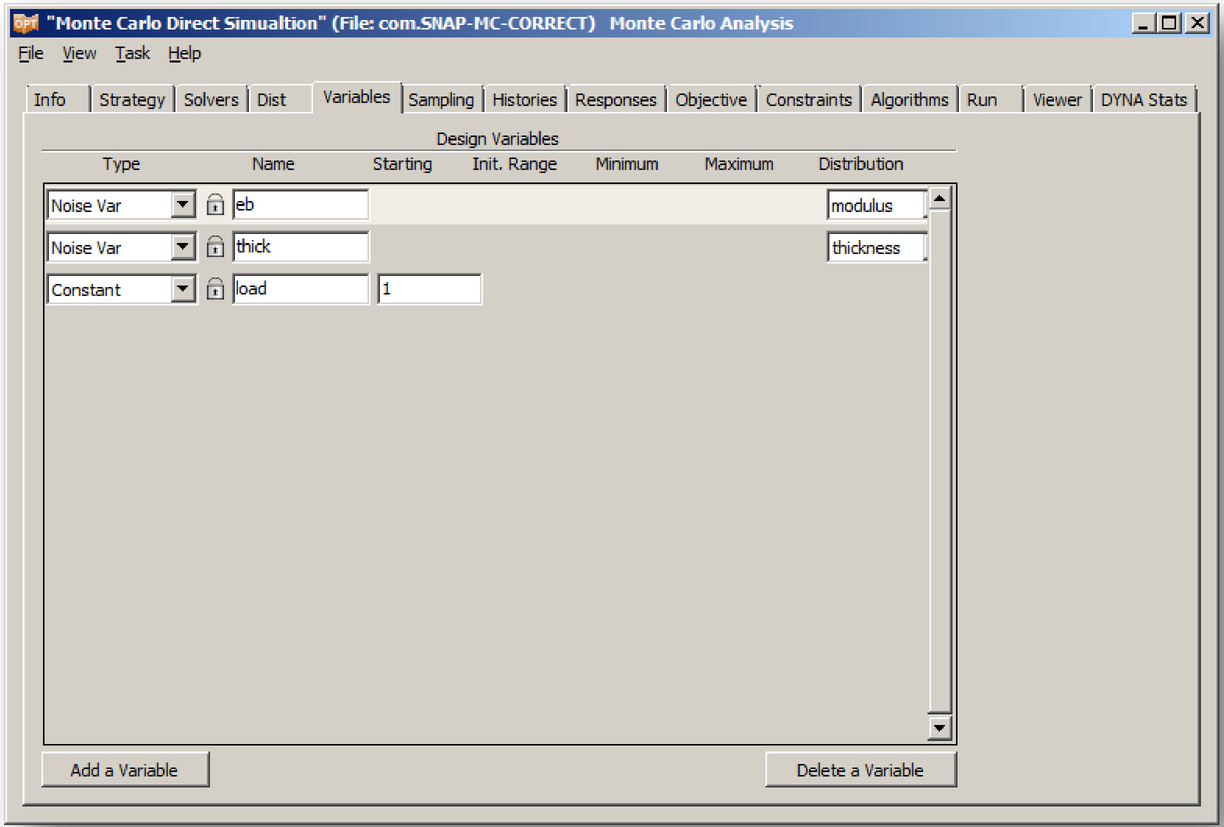
- Defined noise variables will be the multipliers of the design parameters – thickness and elastic modulus

```
*SECTION_SHELL
$#   secid    elform      shrf       nip     propt   qr/irid     icomp     setyp
         2         4     0.000         3         0         0         0         0
$#      t1        t2        t3        t4      nloc     marea      idof    edgset
<<65*thick>>,<<65*thick>>,<<65*thick>>,<<65*thick>>
```

```
*MAT_ORTHOTROPIC_ELASTIC
$#     mid        ro        ea        eb        ec      prba      prca      prcb
         2 2.1500E-6 10.000000 <<0.3*eb>> 1.0000E-5  0.050000 1.0000E-5 1.0000E-5
$#     gab       gbc       gca      aopt         g      sigf
  0.500000  0.001000  0.001000     0.000     0.000     0.000
$#      xp        yp        zp        a1        a2        a3      macf
     0.000     0.000     0.000     0.000     0.000     0.000         1
$#      v1        v2        v3        d1        d2        d3      beta       ref
     0.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
```

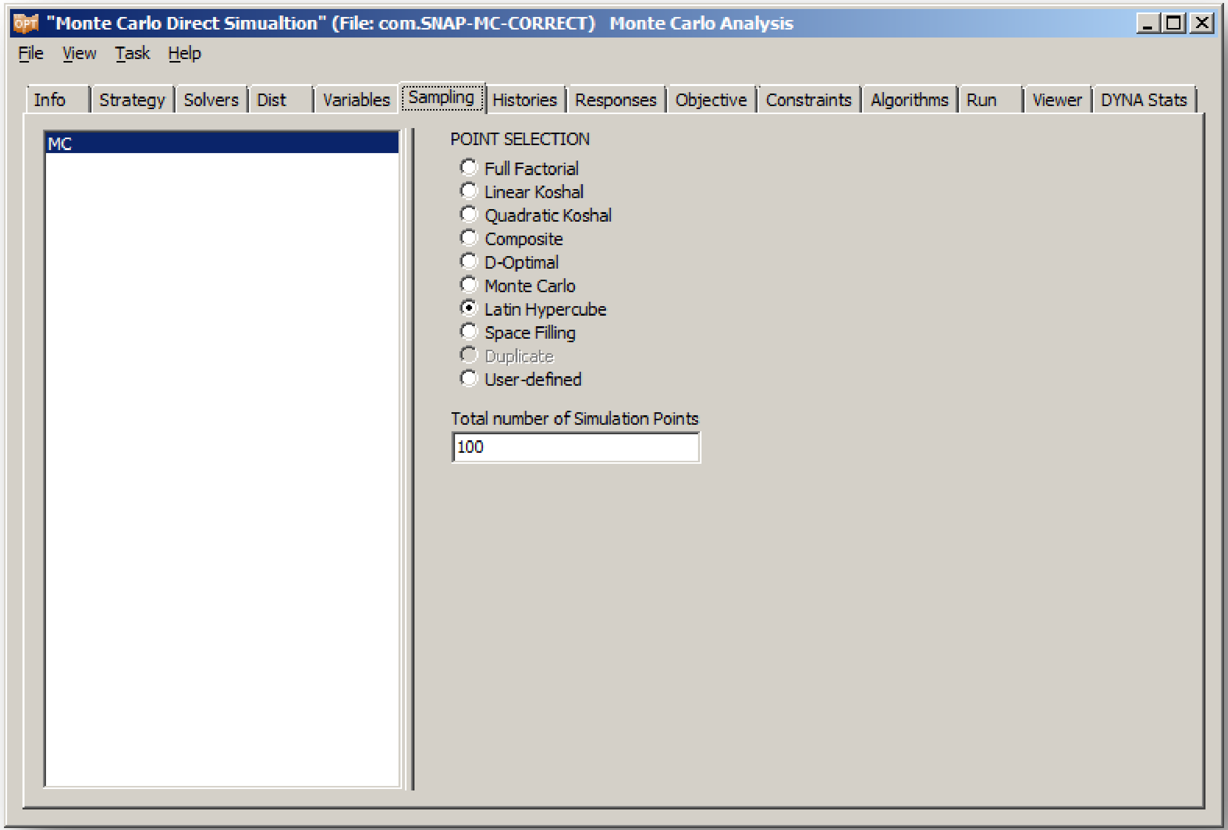
Variables Panel

- After reading the k-file, variables should be defined already.
- Change type for variable `eb` to Noise variable
- Select distribution type `modulus`
- For `thick` variable change type to Noise variable and select `thickness` distribution
- For Constant load type `1` as starting value



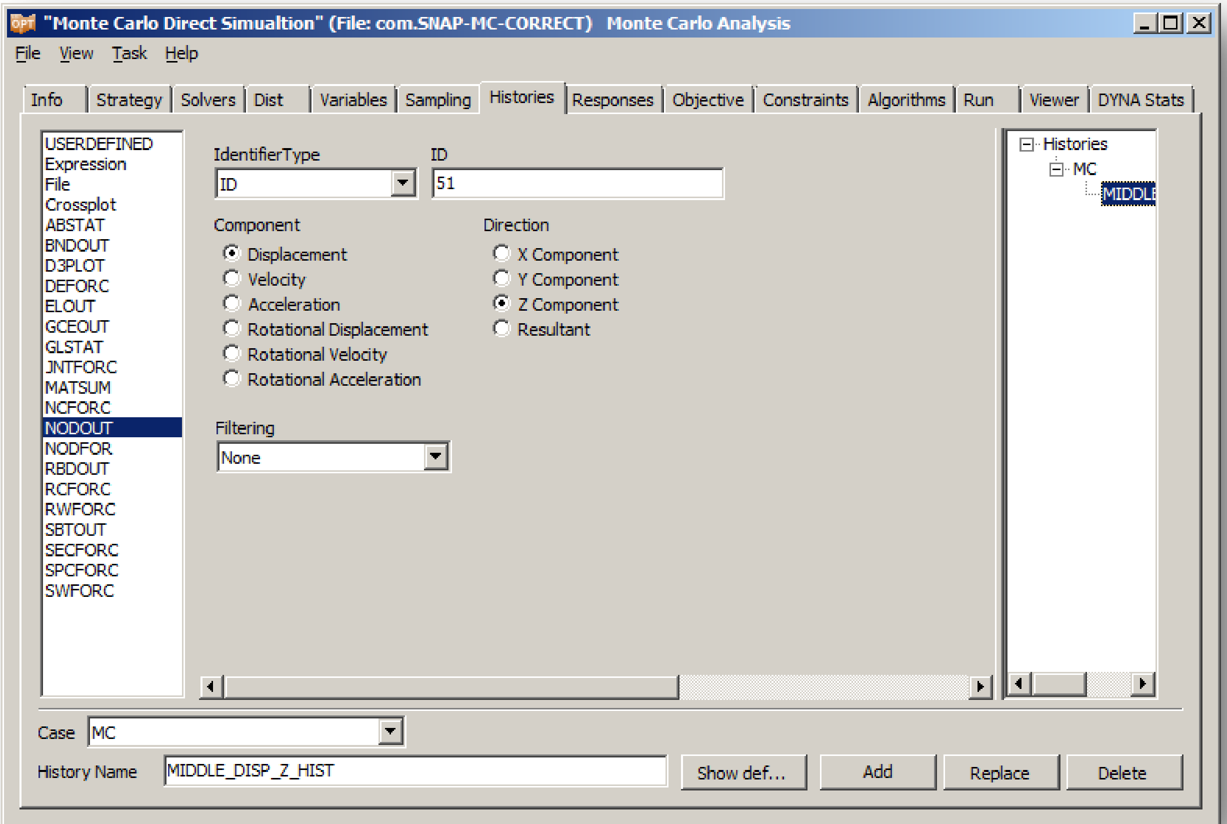
Sampling Panel

- In Sampling Panel pick Latin Hypercube as Point selection method
- Type 100 for Total number of Simulation Points



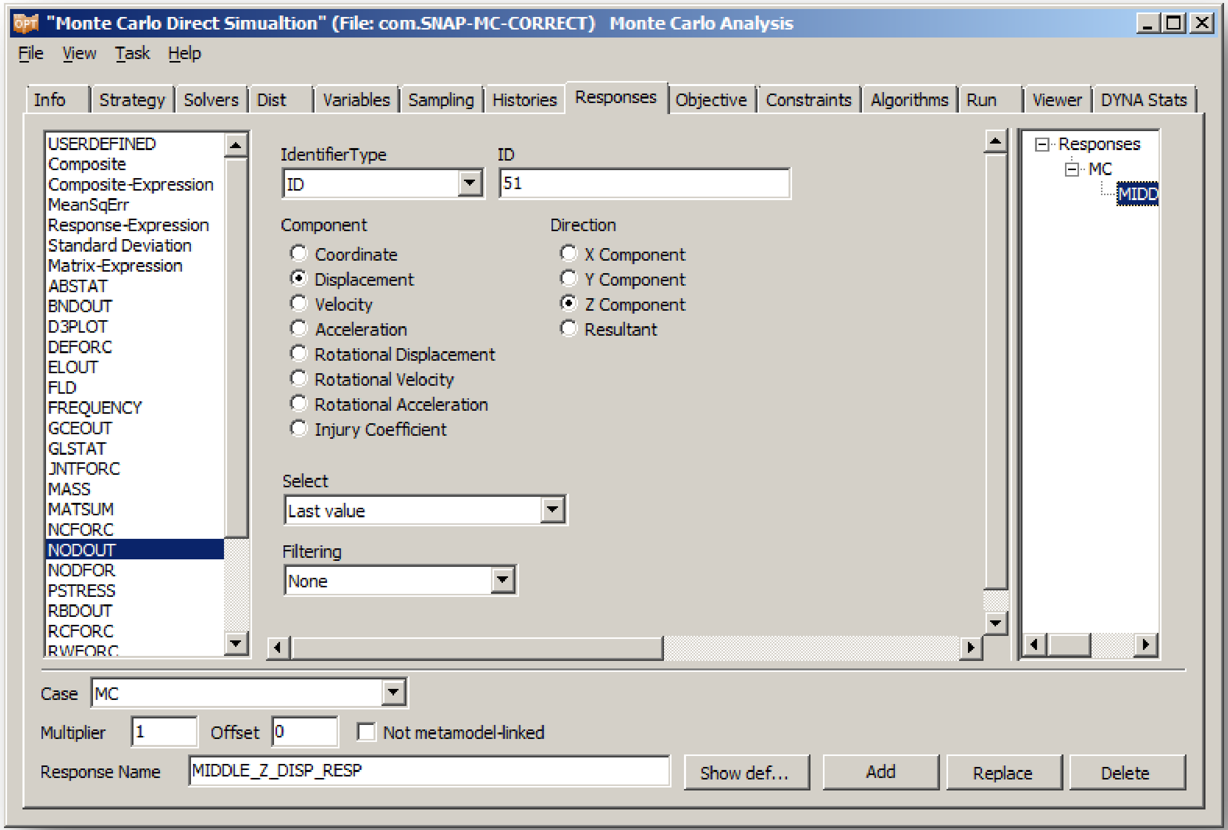
Histories Panel

- In Histories Panel pick NODOUT from left window
- Choose ID for Identifier Type
- Type 51 for ID and select Z Component as Direction of Displacement
- Give name to the history MIDDLE_DISP_Z_HIST and press Add



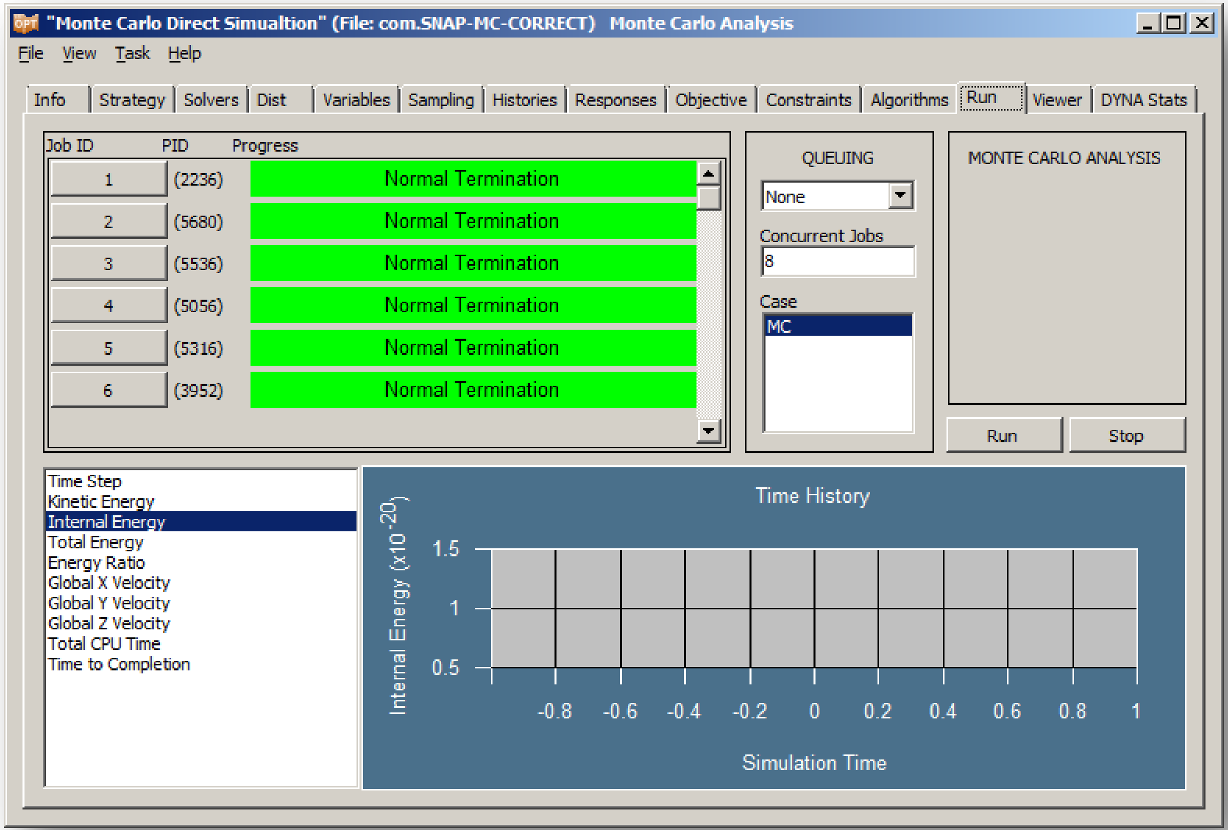
Responses Panel

- In Histories Panel pick NODOUT from left window
- Choose ID for Identifier Type
- Type 51 for ID and select Z Component as Direction of Displacement
- Give name to the history MIDDLE_DISP_Z_RESP and press Add



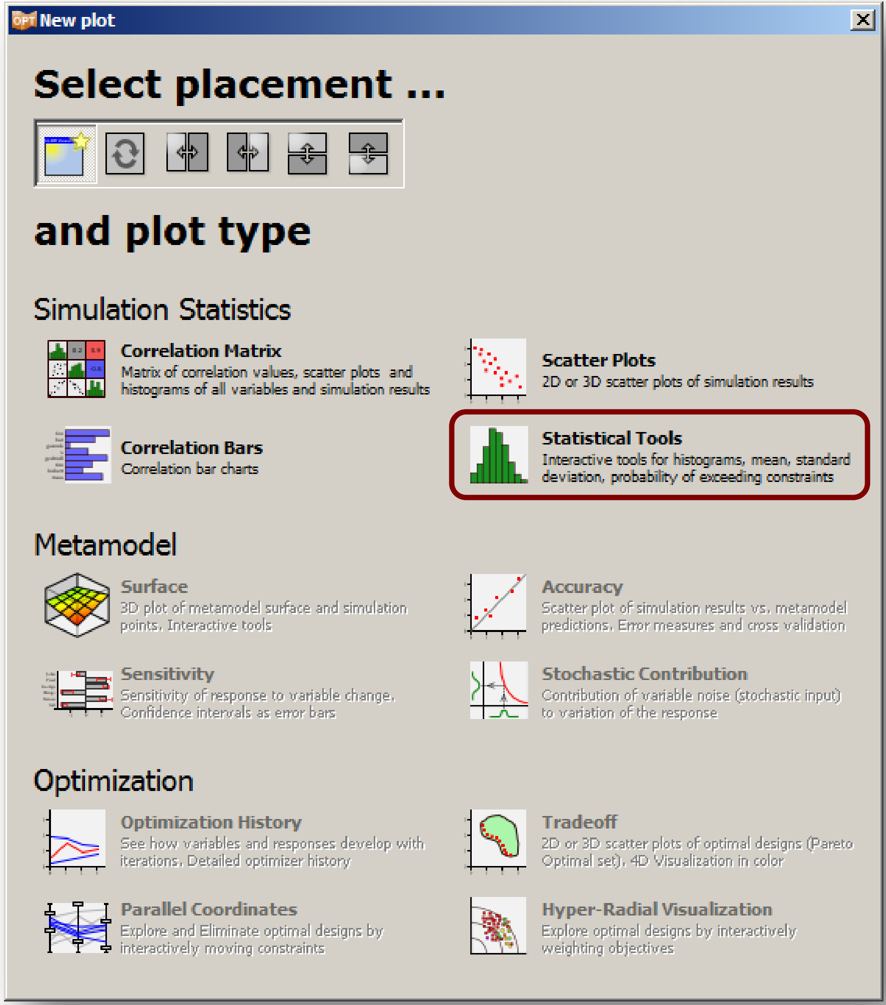
Run Panel

- In Run Panel select PBS for Queuing system if run on TRACC cluster
- Leave none if run locally on Windows machine
- Type 8 for concurrent jobs and press Run button (one core is enough for this job)



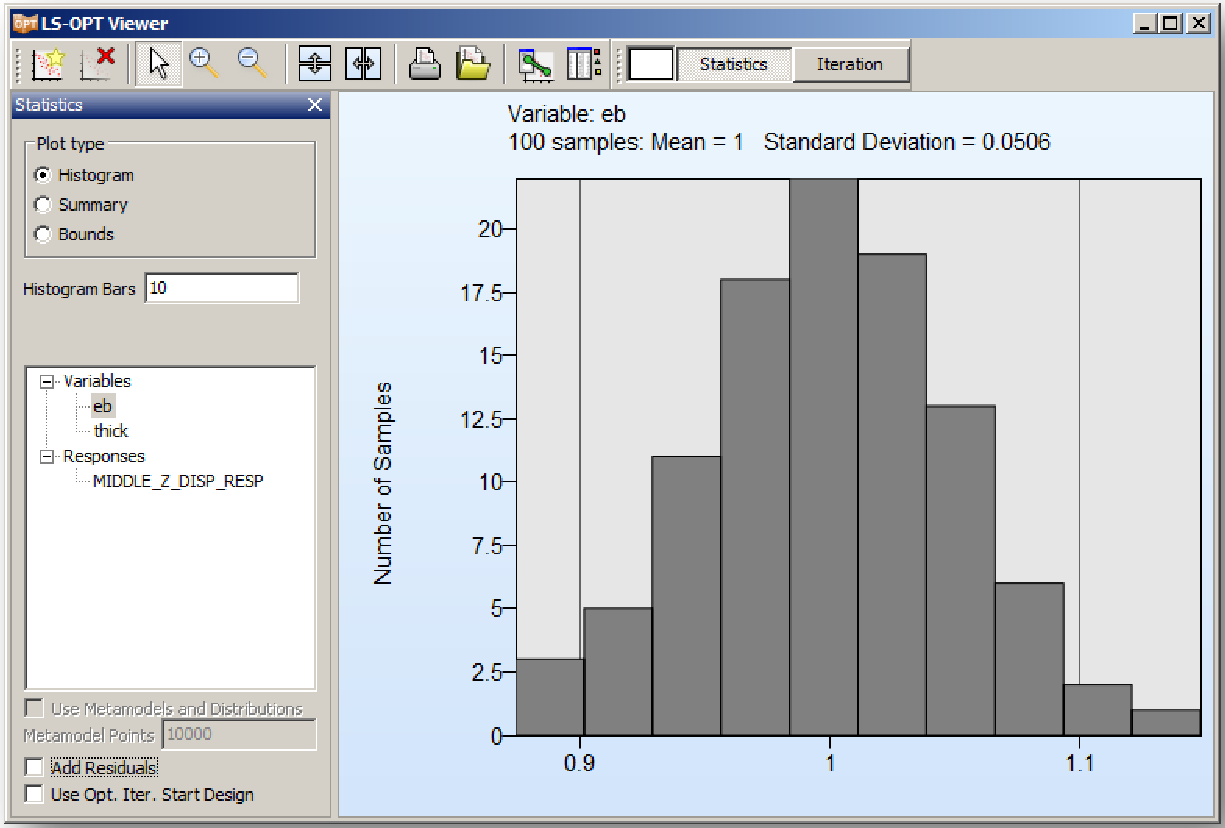
Viewer

- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select "Statistical Tools"



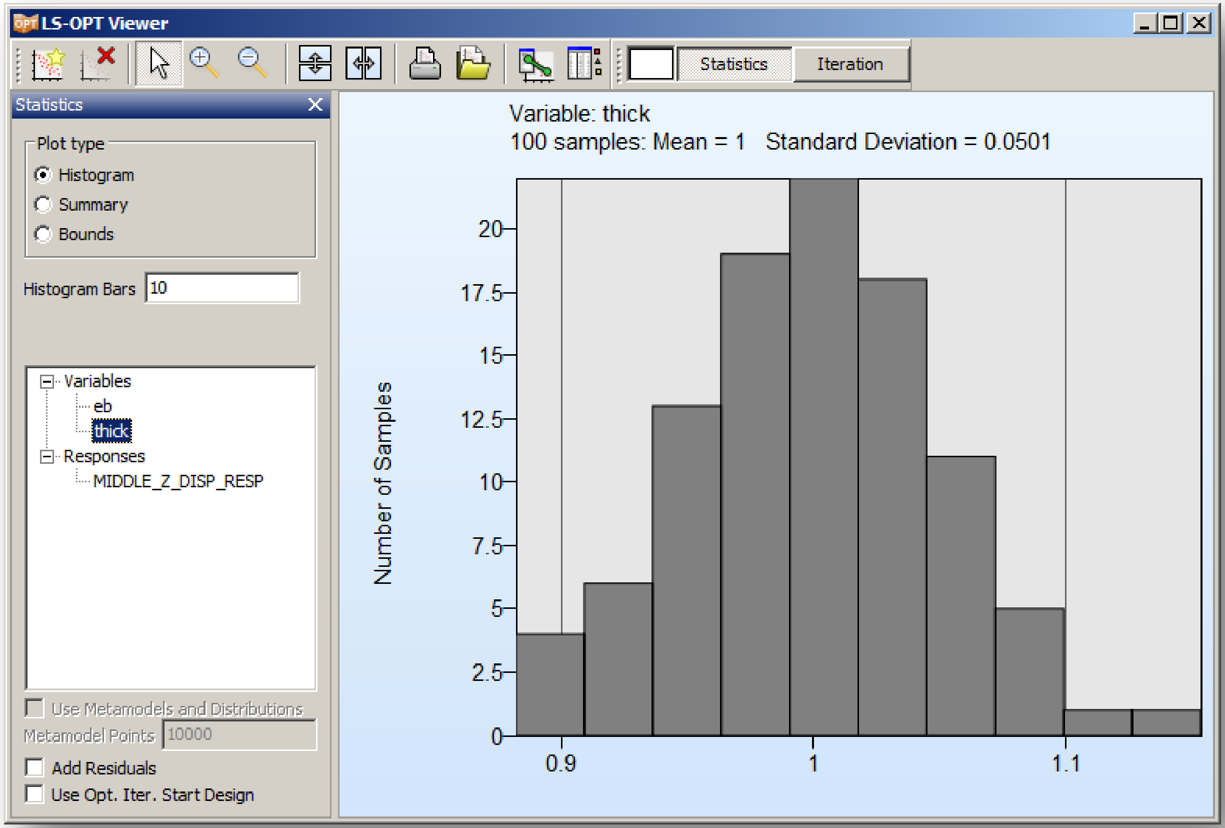
Statistical Tools

- From Histogram Plot type select eb Variable statistics to display
- As requested mean is 1 and the std. deviation is ~0.05



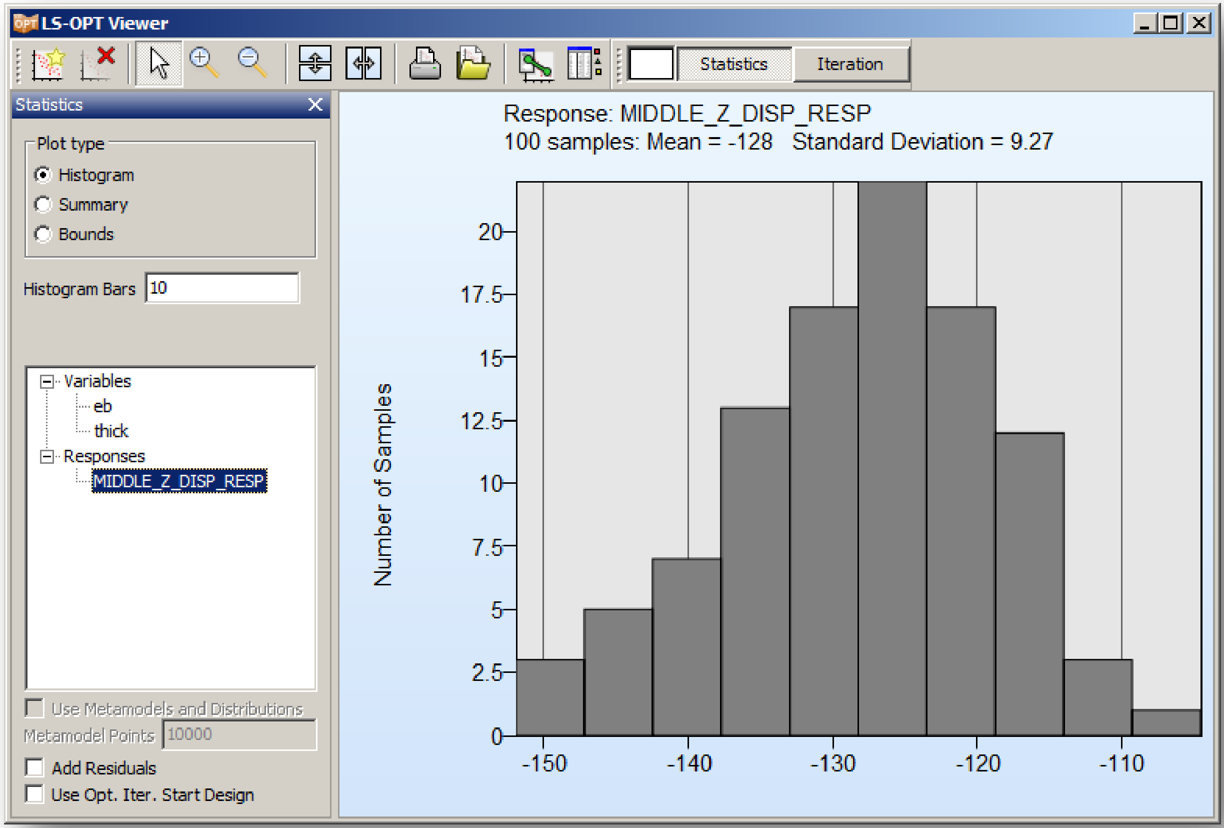
Statistical Tools

- From Histogram Plot type select thick Variable statistics to display
- As requested mean is 1 and the std. deviation is ~0.05



Statistical Tools

- From Histogram Plot type select MIDDLE_Z_DISP_RESP Response statistics to display
- The mean of the responses is -128 with standard deviation of 9.27



Correlation of Responses

- The mean of a set of responses is:

$$\mu = \frac{1}{n}\sum_{i=1}^{n} y_i$$

- Its variance is defined as:

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n} (y_i - \mu)^2$$

- The standard deviation is:

$$\sigma = \sqrt{\sigma^2}$$

- The covariance of two variables indicates whether a change in the one is associated with a change in the other:

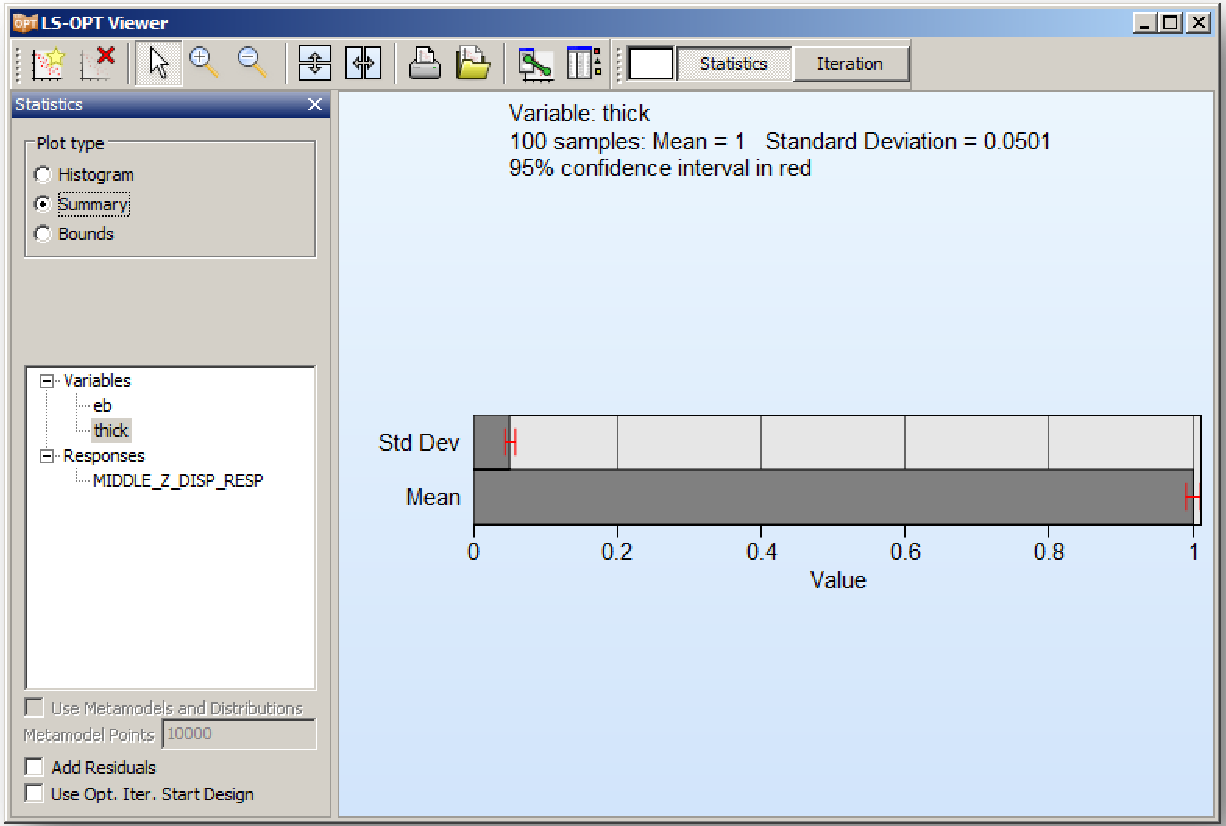
$$Cov(y_1, y_2) = E[(y_1 - \mu_1)(y_2 - \mu_2)]$$

- Coefficient of correlation is used instead as scaled quantity (always between -1 and 1)

$$\rho = \frac{Cov(y_1, y_2)}{\sigma_1 \sigma_2}$$

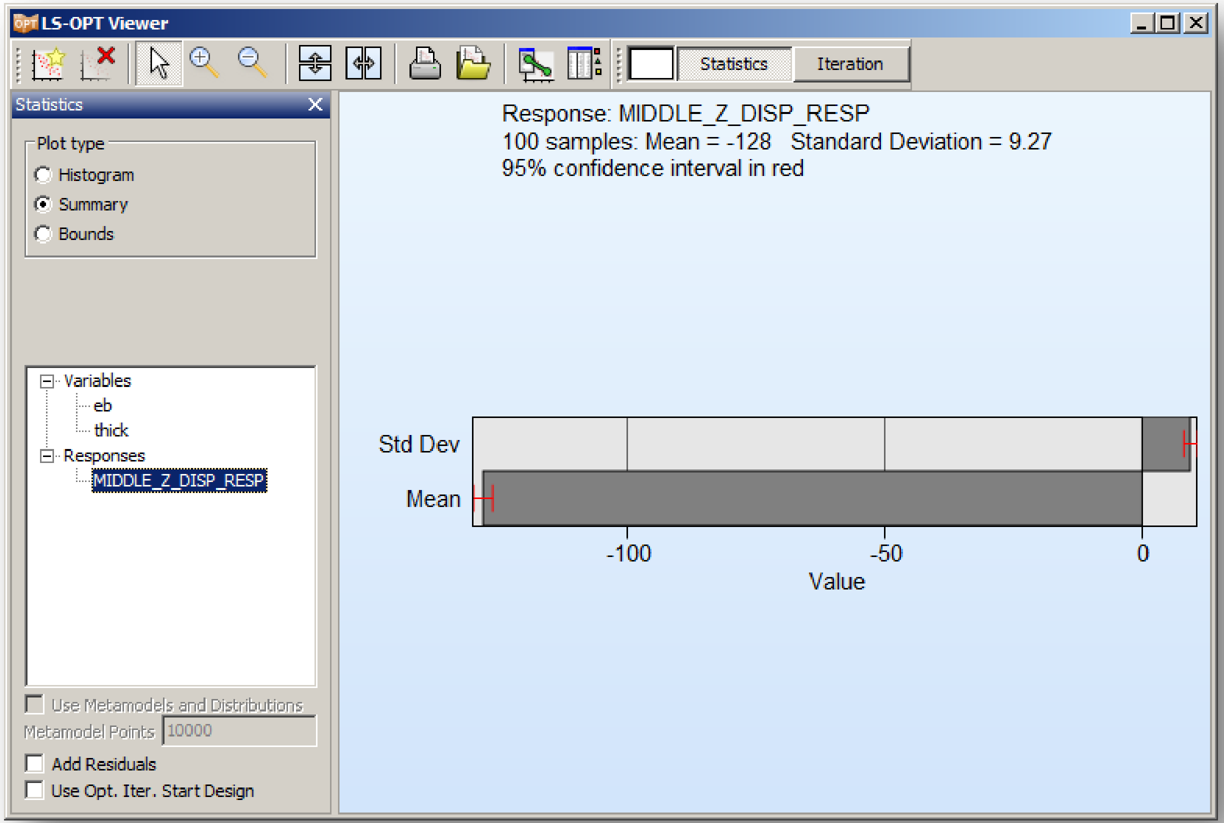
Statistical Tools

- From Summary Plot type select thick Variable



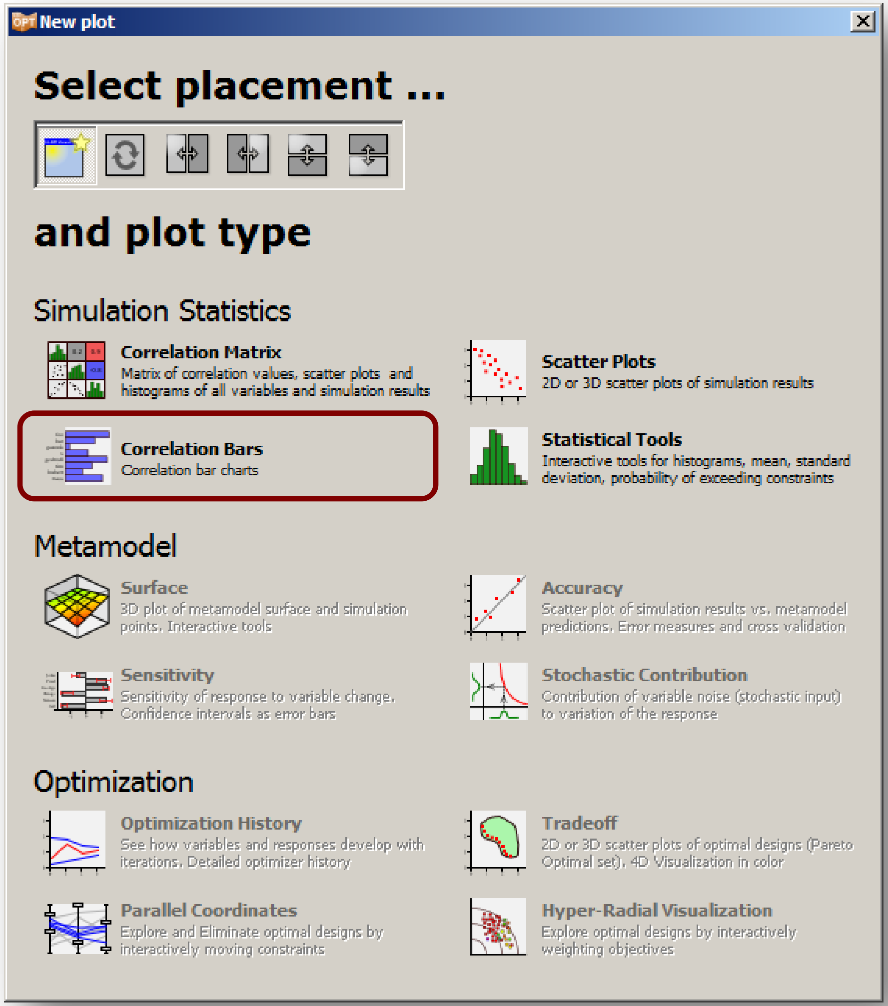
Statistical Tools

- From Summary Plot type select MIDDLE_Z_DISP_RESP Response to see as bars mean and standard deviation of the item



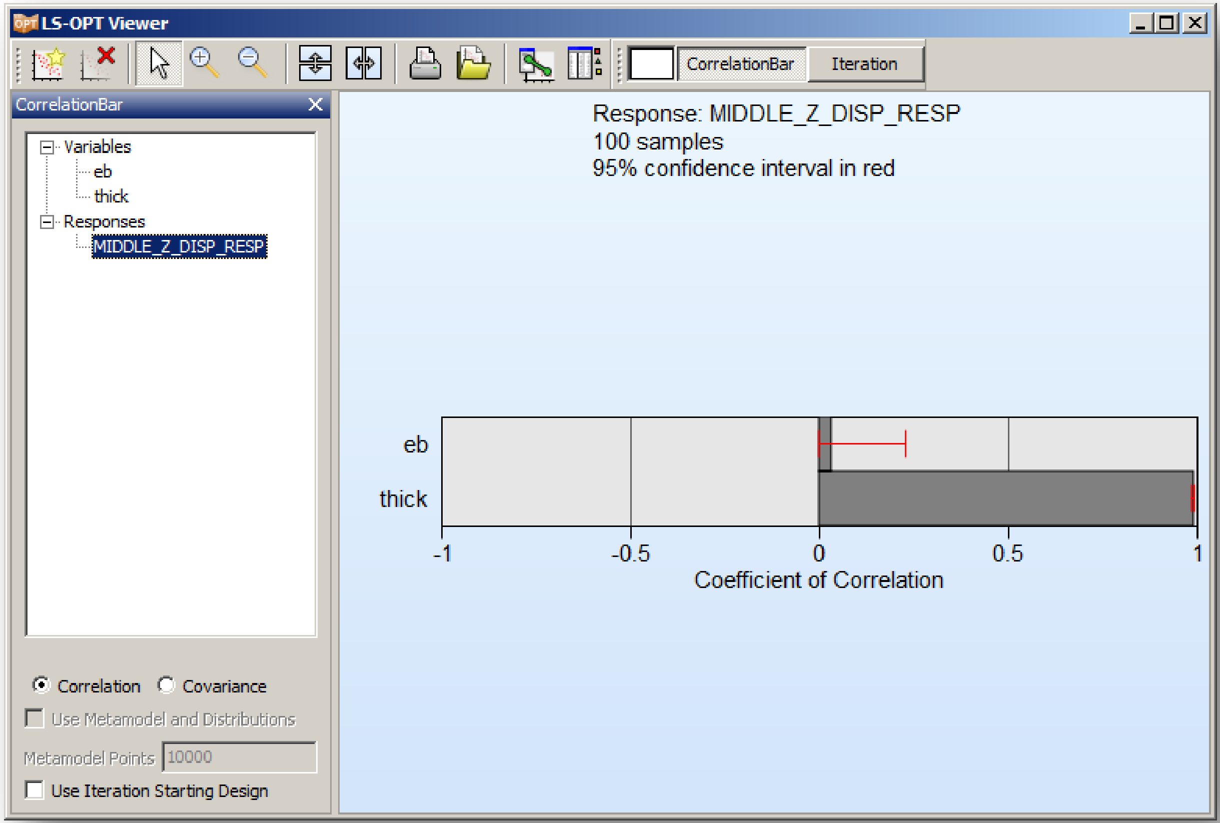
Viewer

- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select "Correlation Bars"



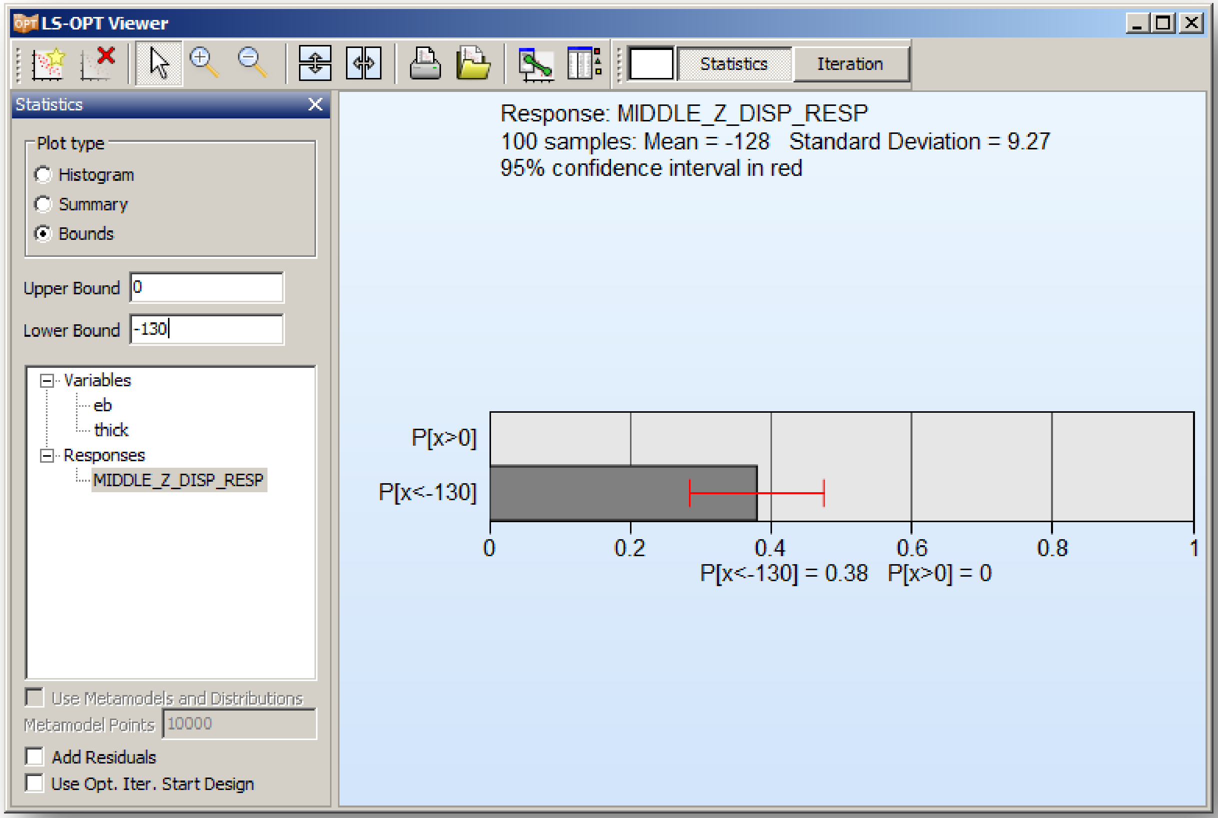
Correlation Bars

- From Correlation Bar menu select MIDDLE_Z_DISP_RESP Response to see correlation bars between the response and design variables



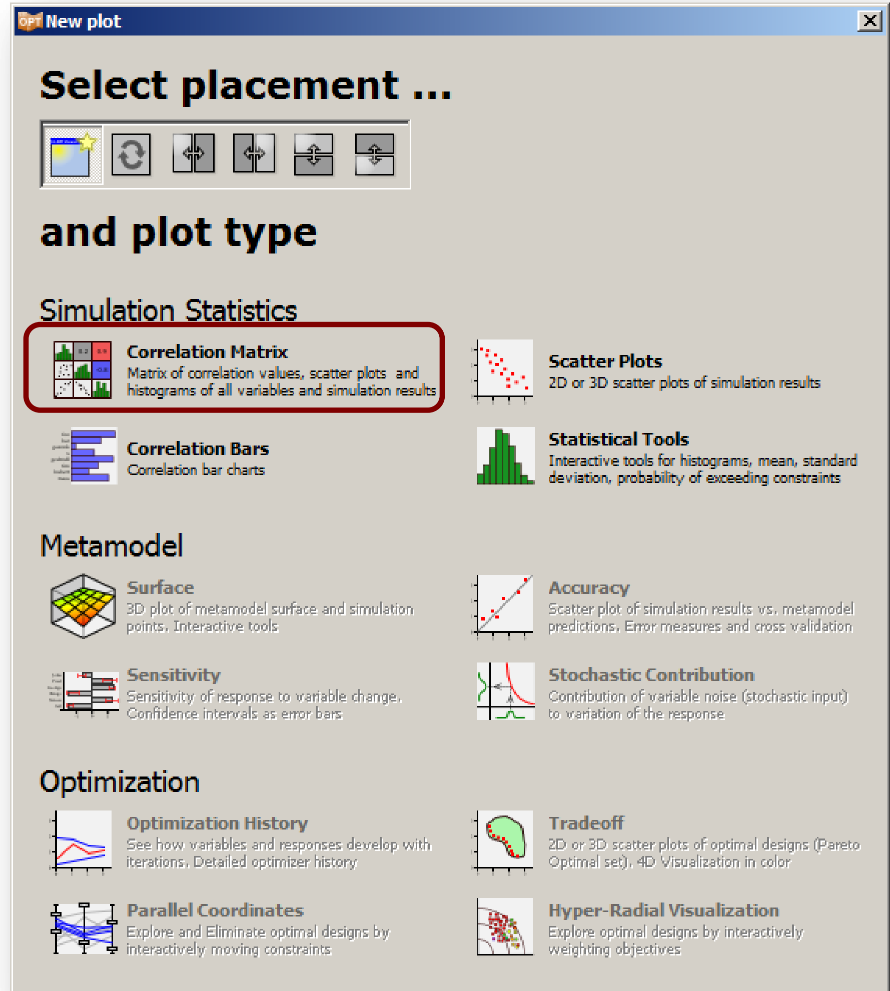
Statistical Tools

- Go back to Statistical Tools
- Pick Bounds and type `-130` as Lower bound for `MIDDLE_Z_DISP_RESP` Response
- Probability of z-displacement exceeding `-130` is 38%



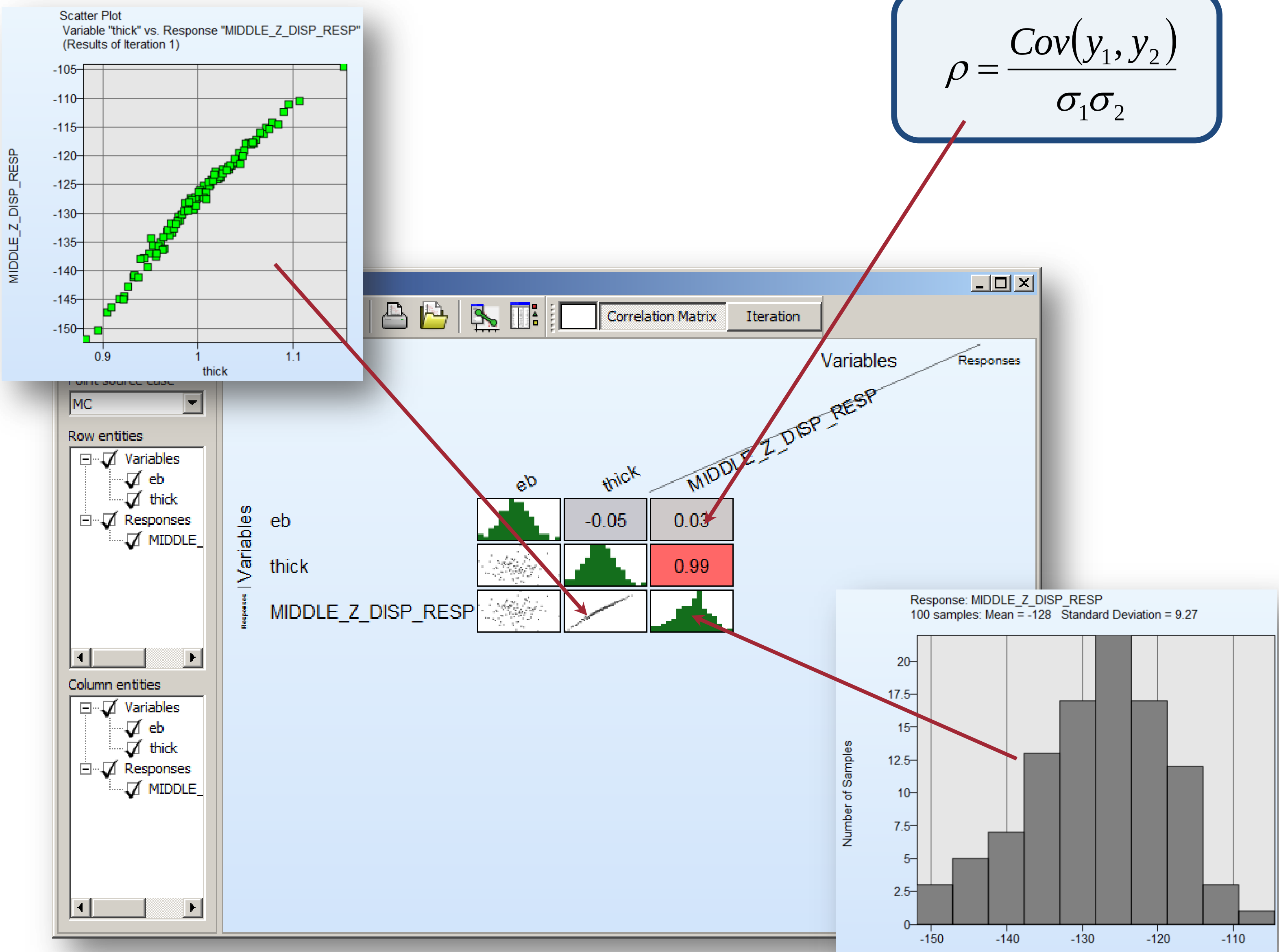
Viewer

- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select “Correlation Matrix”



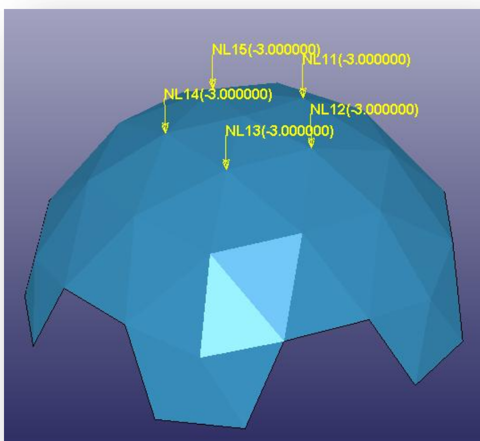
Correlation Matrix

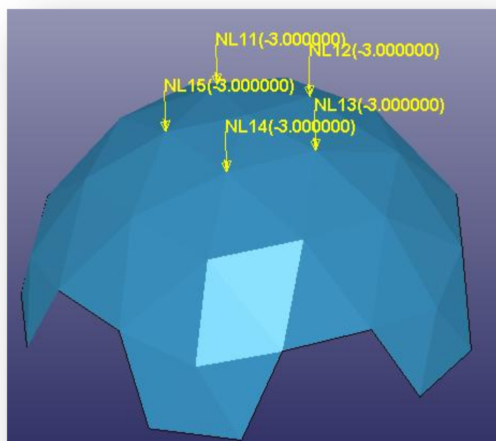
$$\rho = \frac{Cov(y_1, y_2)}{\sigma_1 \sigma_2}$$

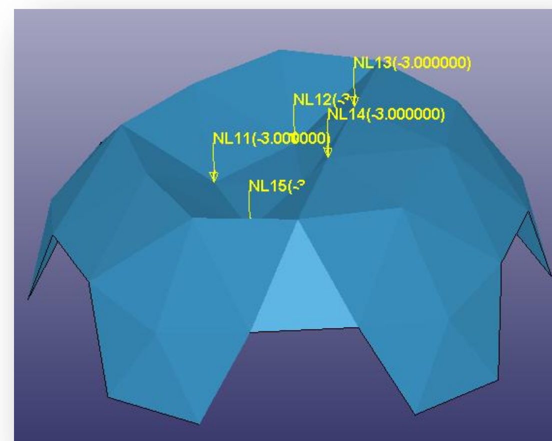


Monte Carlo Analysis with LS-OPT Metamodel Based Simulation

Example: Deflection of the dome structure.

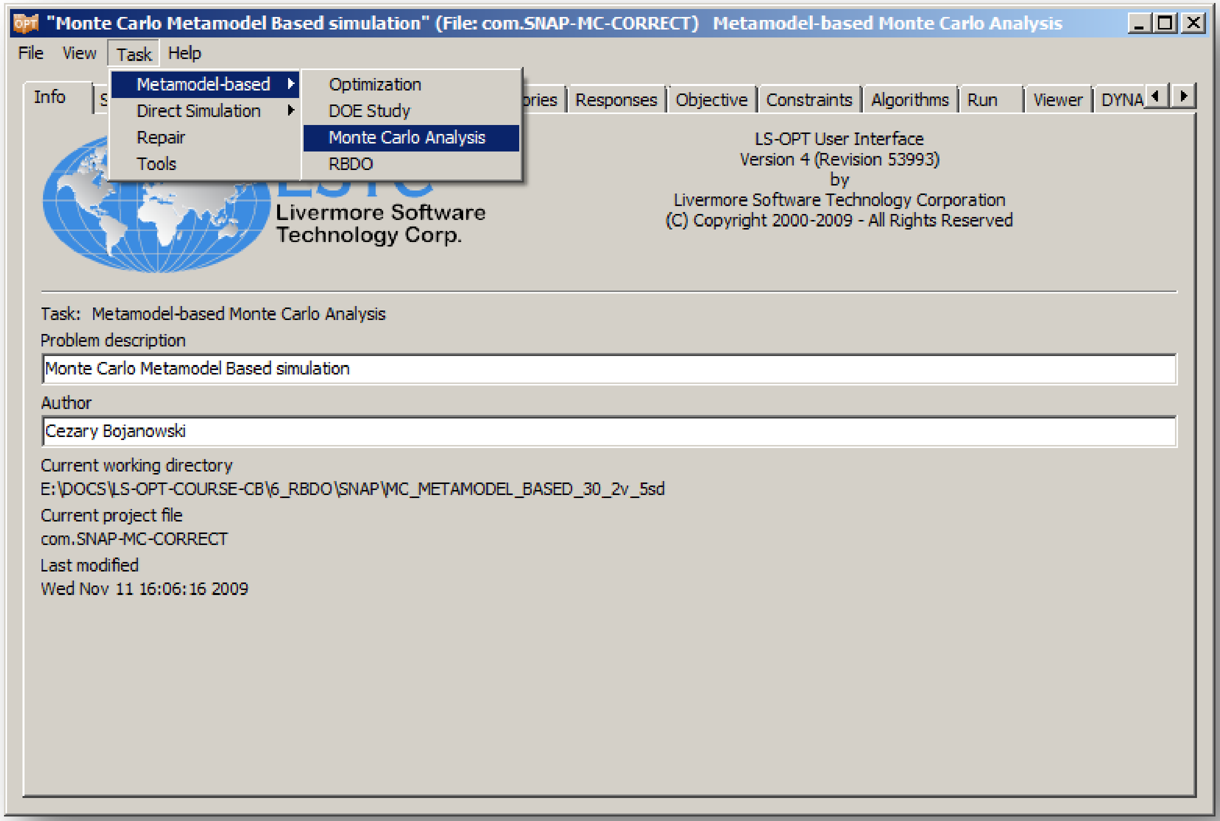






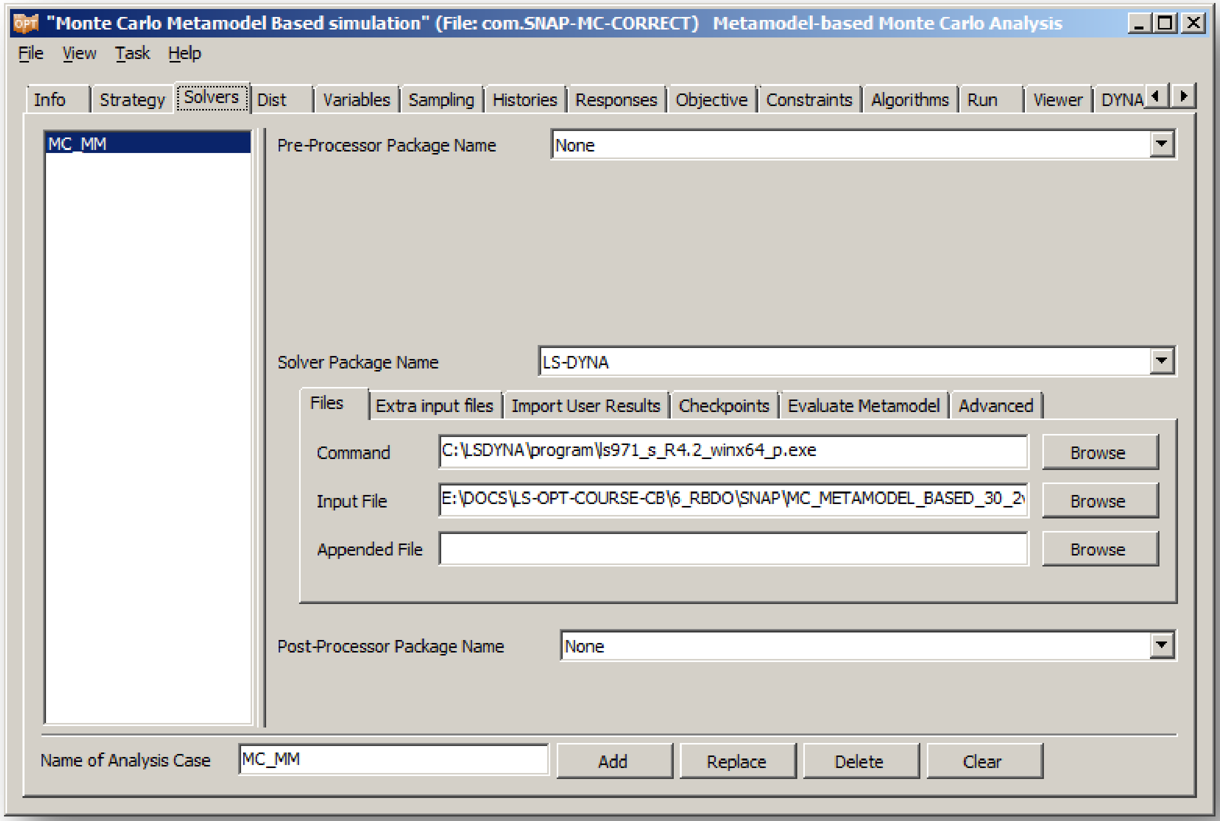
Approach 2 - Metamodel based Monte Carlo Analysis

- Go to Task tab
- Select Monte Carlo Analysis from Metamodel based group



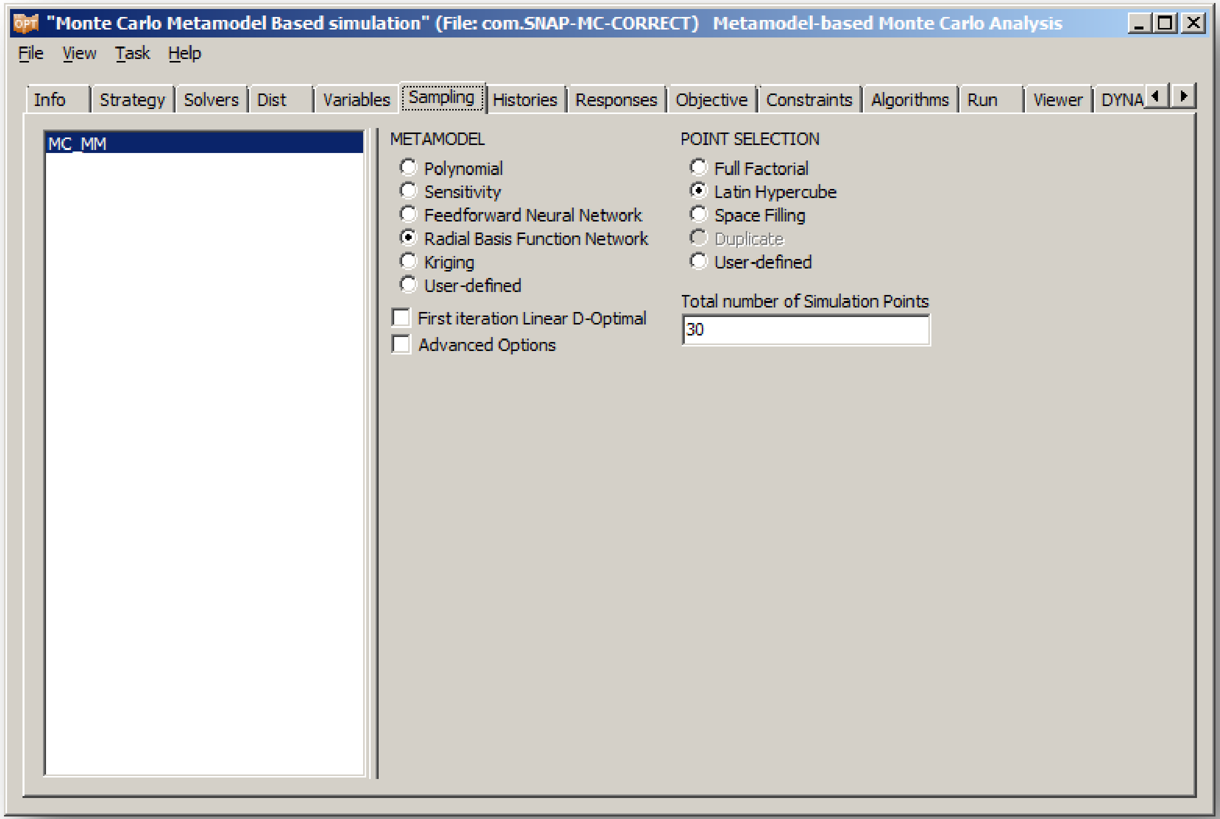
Solvers Tab

- Go to Solvers tab
- Navigate to the `lsoptscript` (or ls-dyna executable) in Command field
- Find the k-file in Input file tab
- For Name of Analysis Case type `MC_MM` and press Add



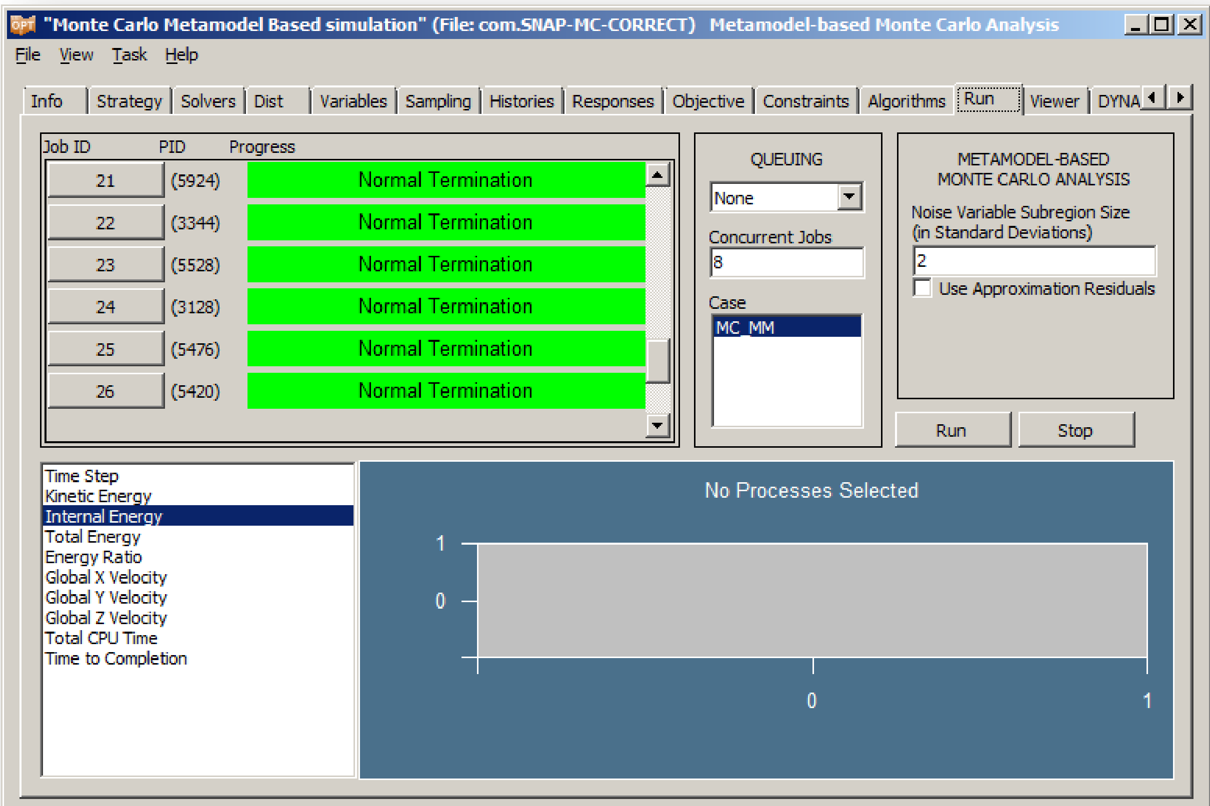
Sampling Tab

- Go to Sampling tab
- Select Radial basis Function Network for Metamodel type and
- Choose 30 for Total number of Simulation Points



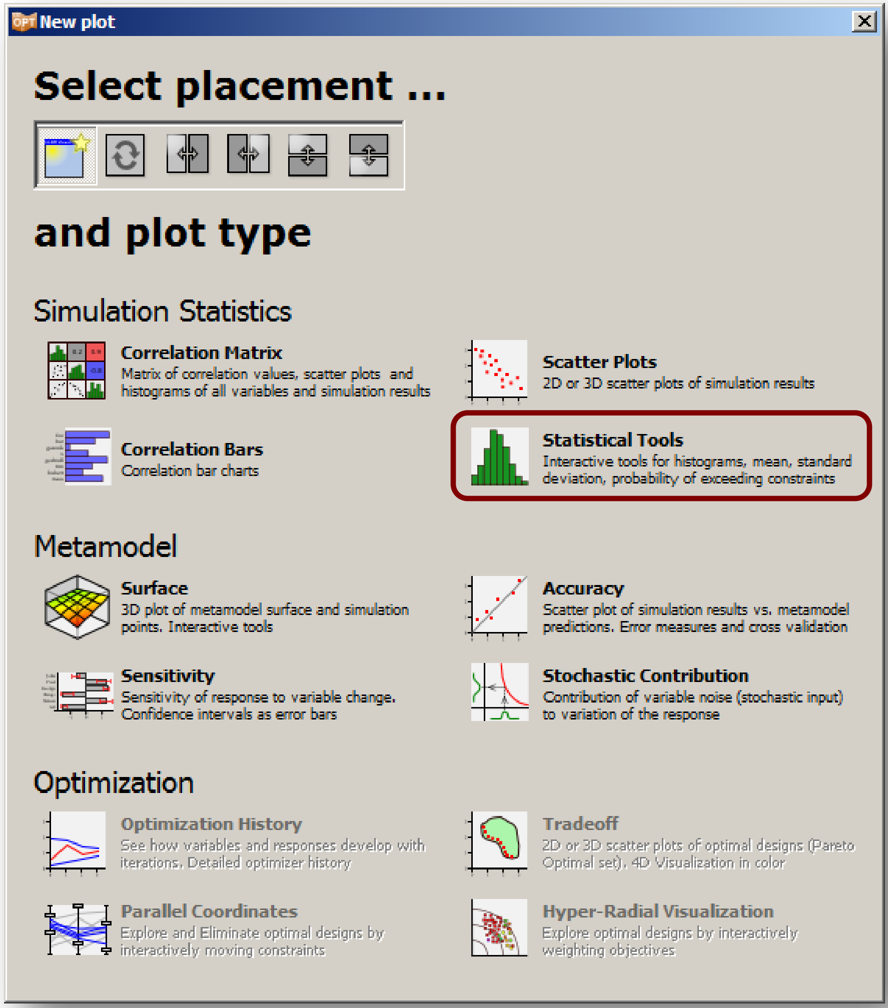
Run Tab

- Go to Run tab
- Select PBS for your Queuing system (if on TRACC cluster) or leave none
- Press Run button



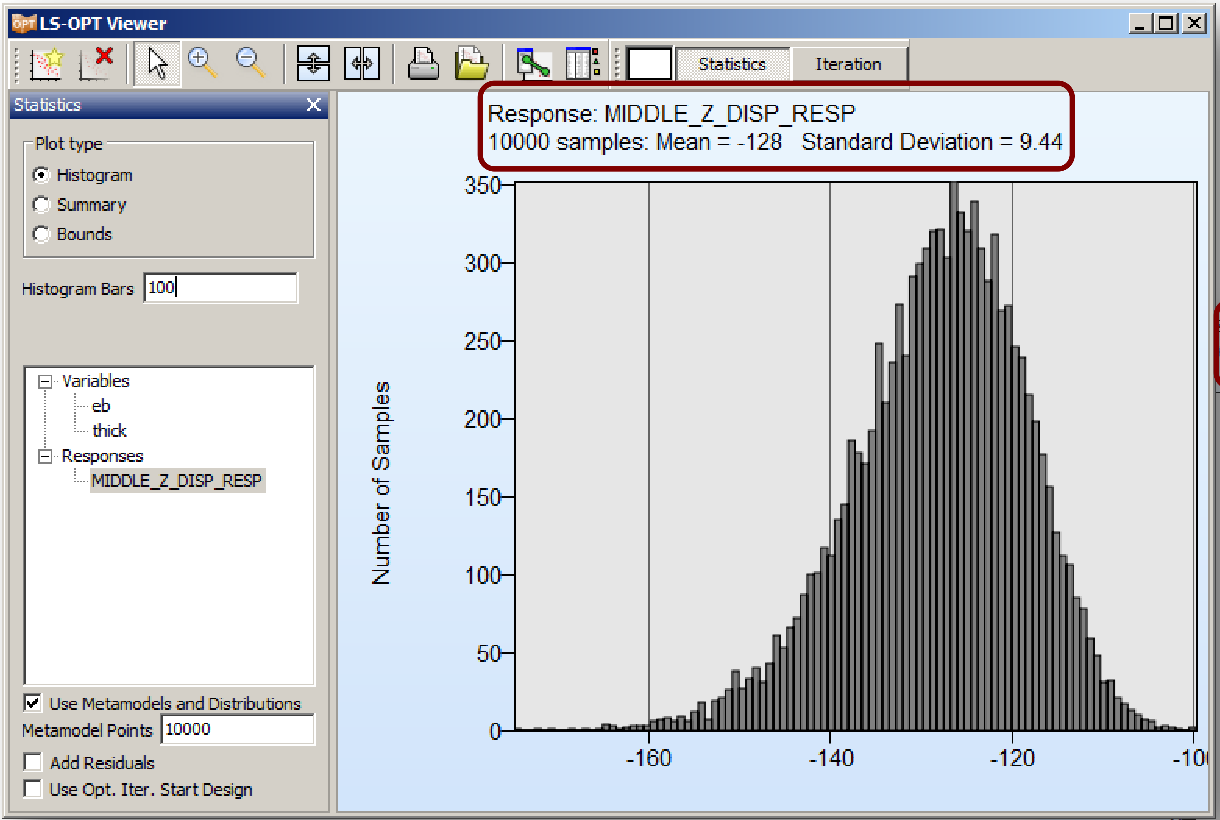
Viewer

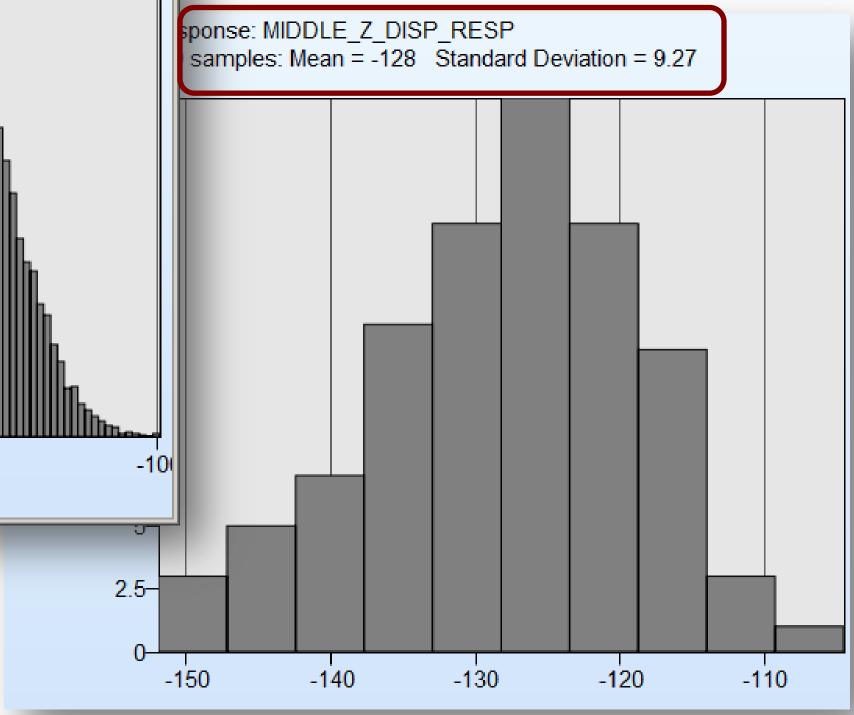
- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select "Statistical Tools"



Statistical Tools

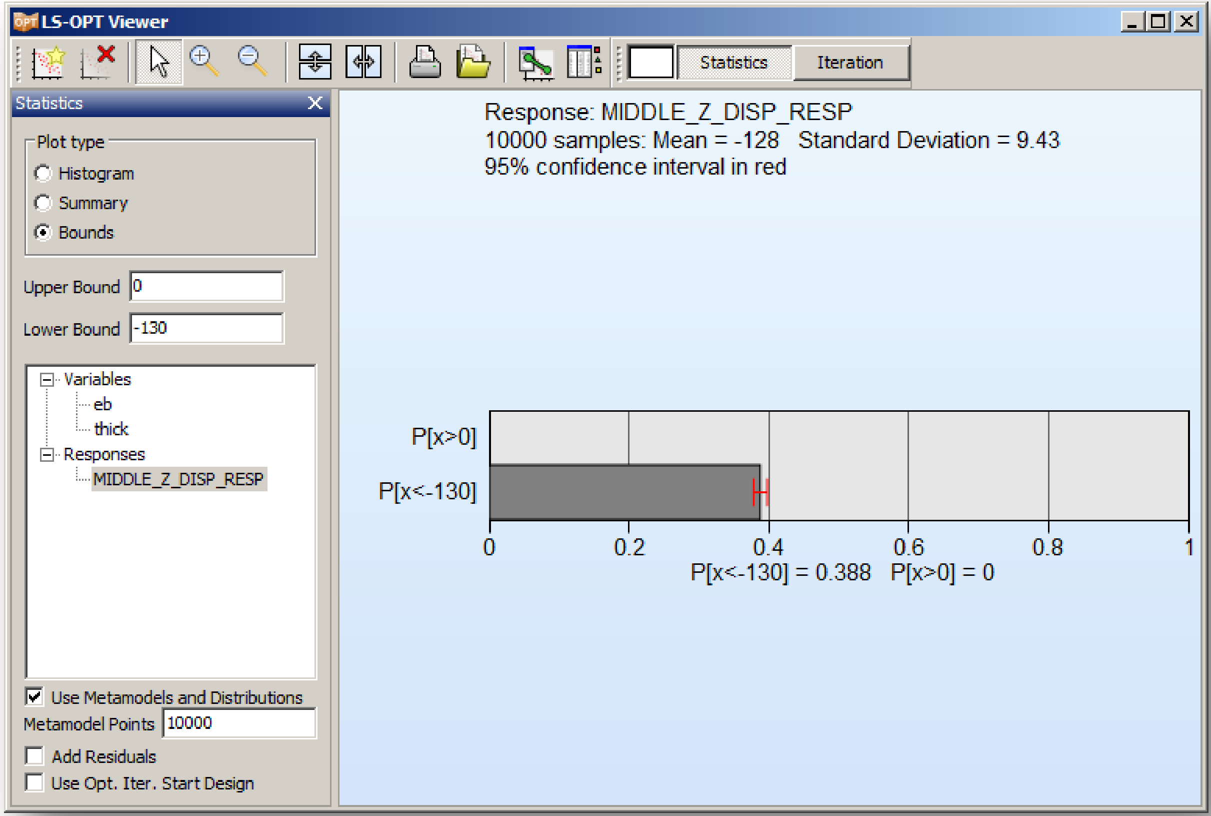
- Plot histograms for Variables and for Response `MIDDLE_Z_DISP_RESP`
- In Histogram Bars field type `100`





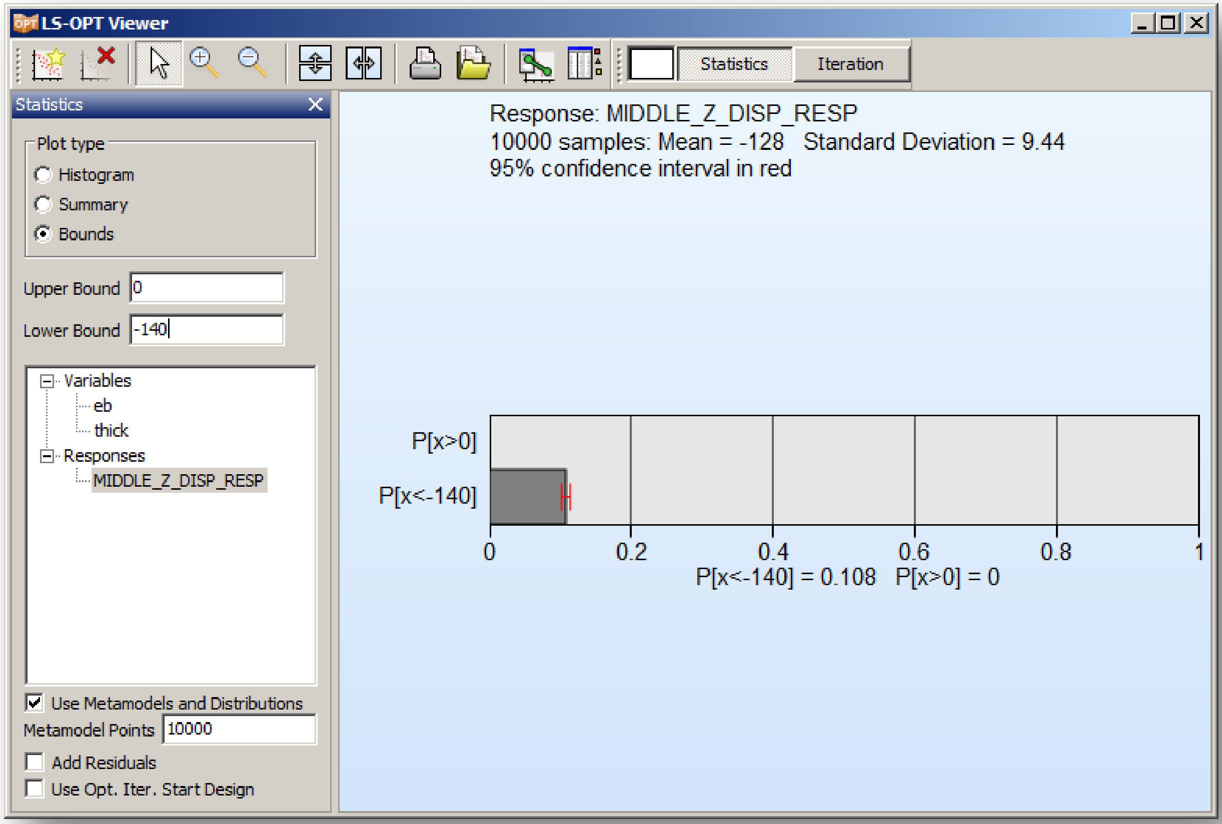
Statistical Tools

- Go back to Statistical Tools
- Pick Bounds and type `-130` as Lower bound for `MIDDLE_Z_DISP_RESP` Response
- Probability of z-displacement exceeding `-130` is `38.8%` (previously `38.0%`)



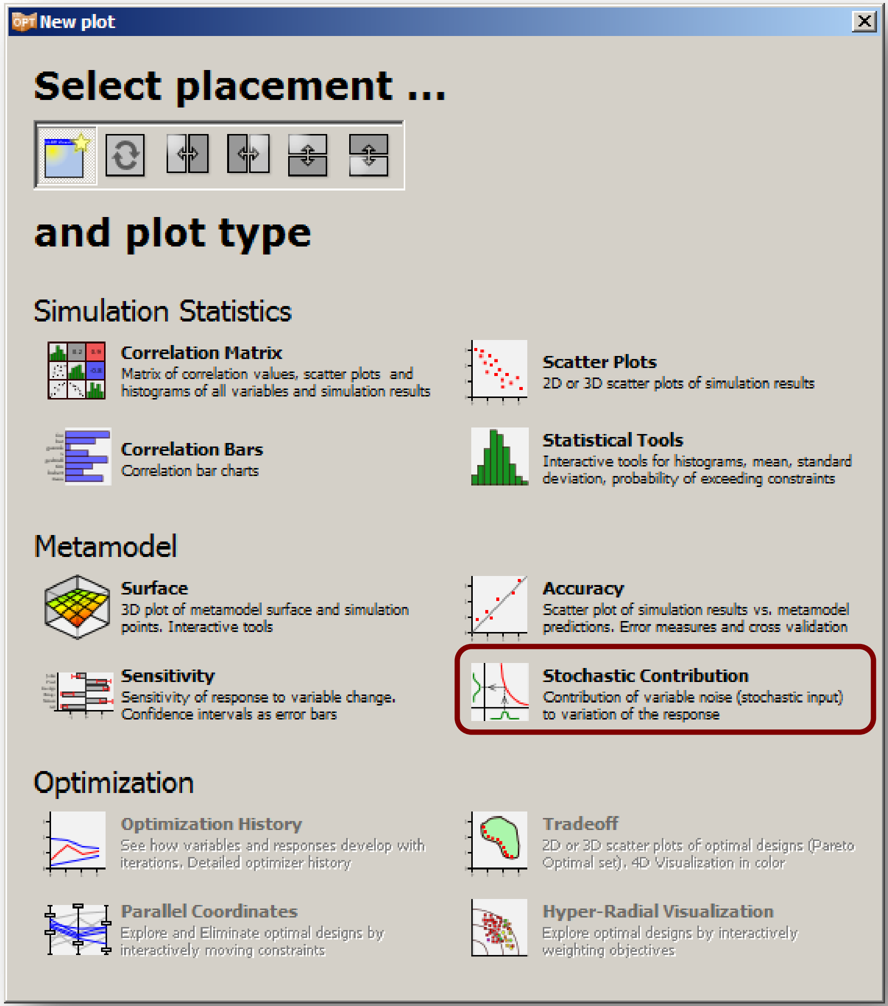
Statistical Tools

- Go back to Statistical Tools
- Pick Bounds and type `-140` as Lower bound for `MIDDLE_Z_DISP_RESP` Response
- Probability of z-displacement exceeding `-140` is `10.8%`



Viewer

- Go to Viewer tab in LS-OPTui
- Press Restart viewer button
- From New plot panel select "Stochastic Contribution"



Stochastic Contribution

- Standard deviation in response due to distributions of variables should be shown