

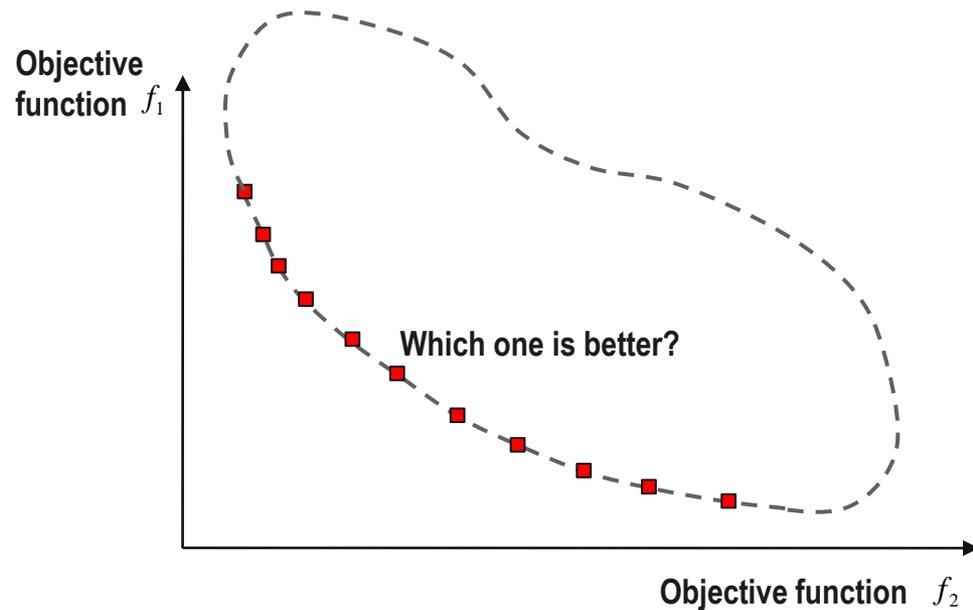
# Introductory Course: Using LS-OPT<sup>®</sup> on the TRACC Cluster

## 2.2b - User Defined Function - PERL or OCTAVE; Two Functions - Multiobjective Optimization

By: Cezary Bojanowski, PhD

# Multi-Objective Optimization (MOO)

- Most engineering problems deal with multiple objectives e.g. cost, weight, safety etc..
- They often conflict with each other e.g. increasing safety may involve increasing cost
- It is hard or even impossible to find single best solution



# Multi-Objective Optimization (MOO)

Objective functions

$$\min f_l(x); \quad l = 1, 2, \dots, n$$

Inequality constraints

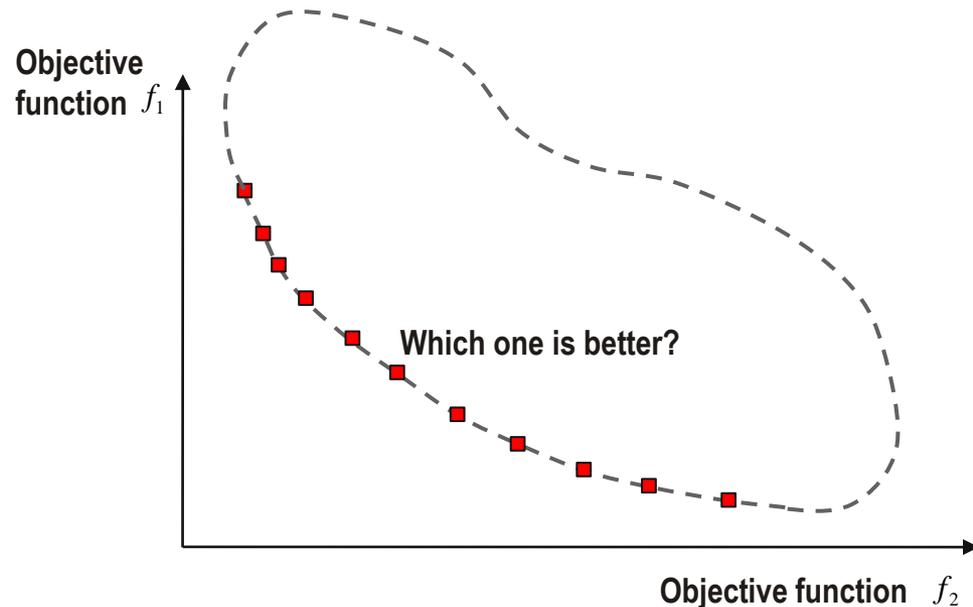
$$g_j(x) \leq 0; \quad j = 1, 2, \dots, m$$

Equality constraints

$$h_k(x) = 0; \quad k = 1, 2, \dots, l$$

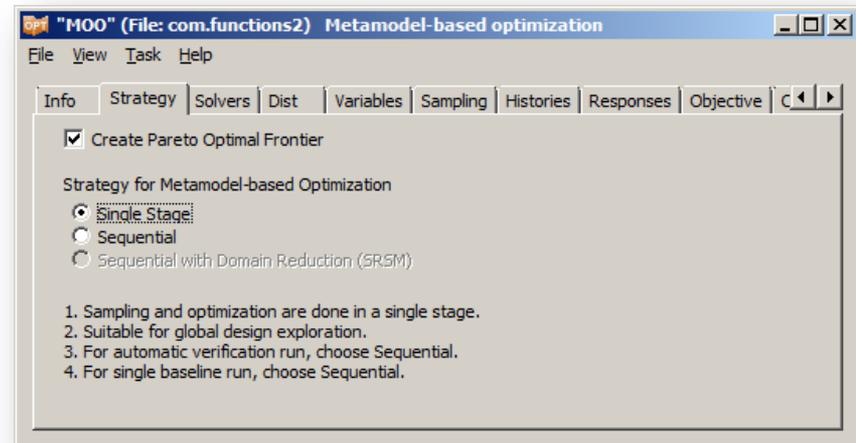
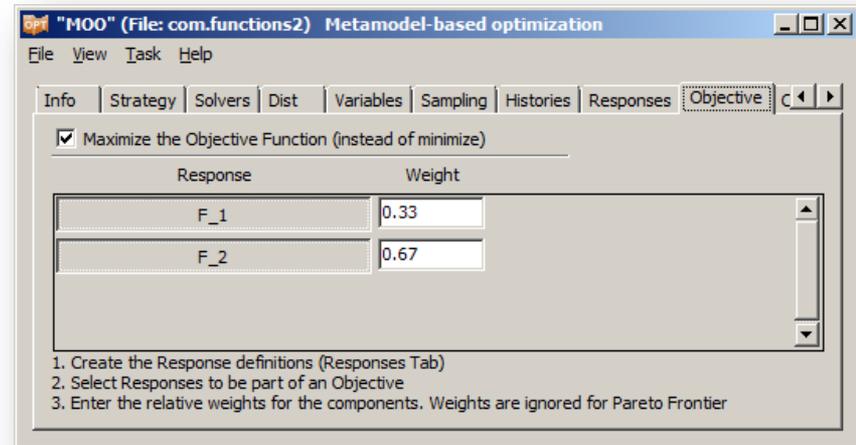
Bounds on variables (side constraints)

$$x_{i,L} \leq x_i \leq x_{i,U}; \quad i = 1, 2, \dots, p$$



# Methods of Solving Multiobjective Optimization Problems

- Weighed sum strategy
  - Convert multiple objectives into a single objective using weights
- $\varepsilon$  - constraint strategy
  - All but one objectives is treated as constraints and optimize for the left-out problem
- Multiobjective genetic algorithm for POF
  - All objectives are simultaneously optimized
  - Weights are ignored



# Multi-Objective Optimization (MOO)

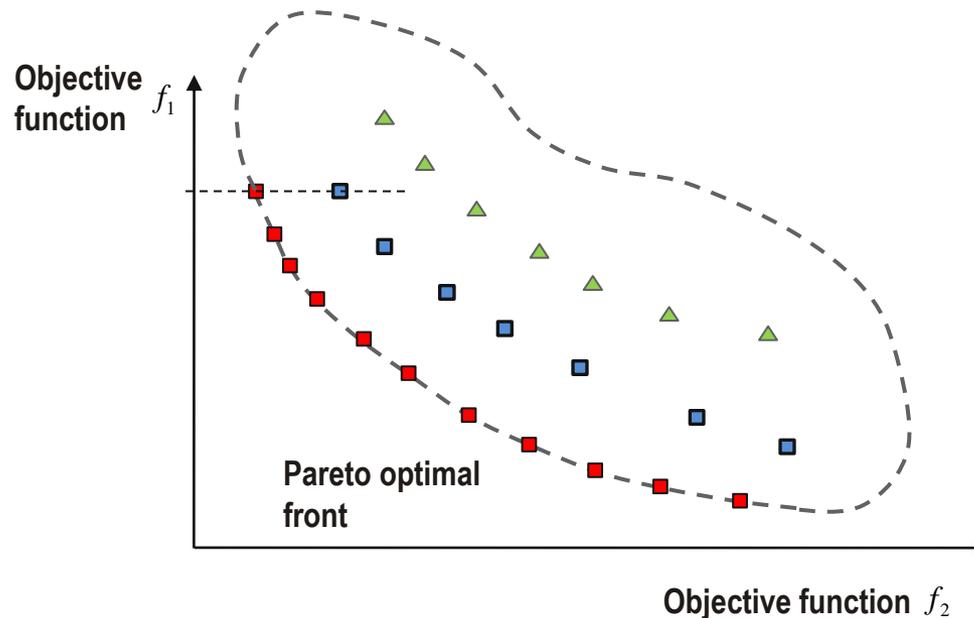
- MOO problems do not have a single optimal solution. Instead there is a set of solutions that reflects trade-off among objectives.
- Special considerations are required to compare different designs from MOO simulations.
- Non-dominant concept is used to compare different individuals
- A solution  $x^{(1)}$  dominates another solution  $x^{(2)}$  ( $x^{(1)} \succ x^{(2)}$ ), if either one of the following conditions is true:
  - $x^{(1)}$  is feasible and  $x^{(2)}$  is not
  - Both  $x^{(1)}$  and  $x^{(2)}$  are infeasible but  $x^{(2)}$  is more infeasible compared to  $x^{(1)}$

# Multi-Objective Optimization (MOO)

- When both  $x^{(1)}$  and  $x^{(2)}$  are feasible,  $x^{(1)}$  dominates  $x^{(2)}$  ( $x^{(1)} \succ x^{(2)}$ ) if following two conditions are satisfied:
  - $x^{(1)}$  is no worse than  $x^{(2)}$  in all objectives i.e.  $f_j(x^{(1)}) \not> f_j(x^{(2)})$ ,  $j \in [1, 2, \dots, M]$
  - $x^{(1)}$  is strictly better than  $x^{(2)}$  in at least one objective  $f_j(x^{(1)}) < f_j(x^{(2)})$
- If neither of the two solutions dominates the other, both solutions are non-dominated with respect to each other
- Any non-dominated solution in the entire domain is a Pareto optimal solution
- Function space representation of the Pareto optimal set is Pareto optimal front

# Multi-Objective Optimization (MOO)

- Blue solutions are dominated by red solutions but not by green ones
- Red solutions are not dominated by any other solutions
- Collection of red solutions creates Pareto optimal front

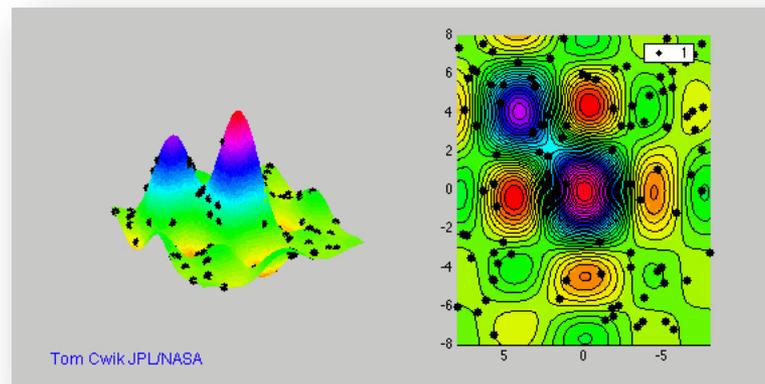


# Genetic Algorithms - “Survival of the Fittest”

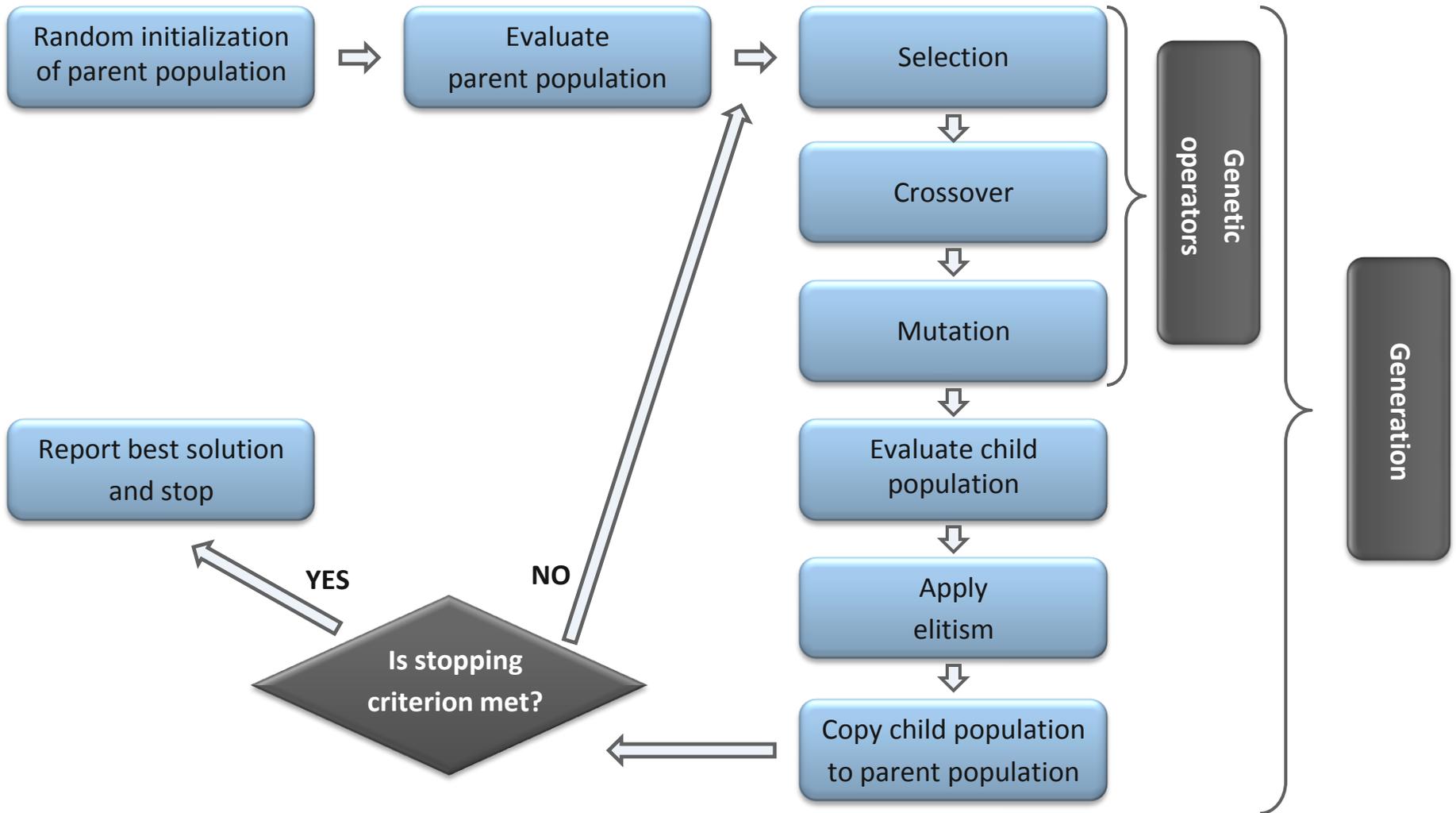
- Based on the evolutionist theory and Darwin’s principle “Survival of the Fittest”
- In nature , weak and unfit species within their environment are faced extinction by natural selection
  - GA is a population based approach (uses multiple points at a time)
  - GA is well suited for Pareto Front search
  - GA does not require derivative information to drive the search of optimal points
  - GA is a global optimizer, whereas conventional methods may get stuck in local optima
  - Requires large number of function evaluations

# Genetic Algorithms - Terminology

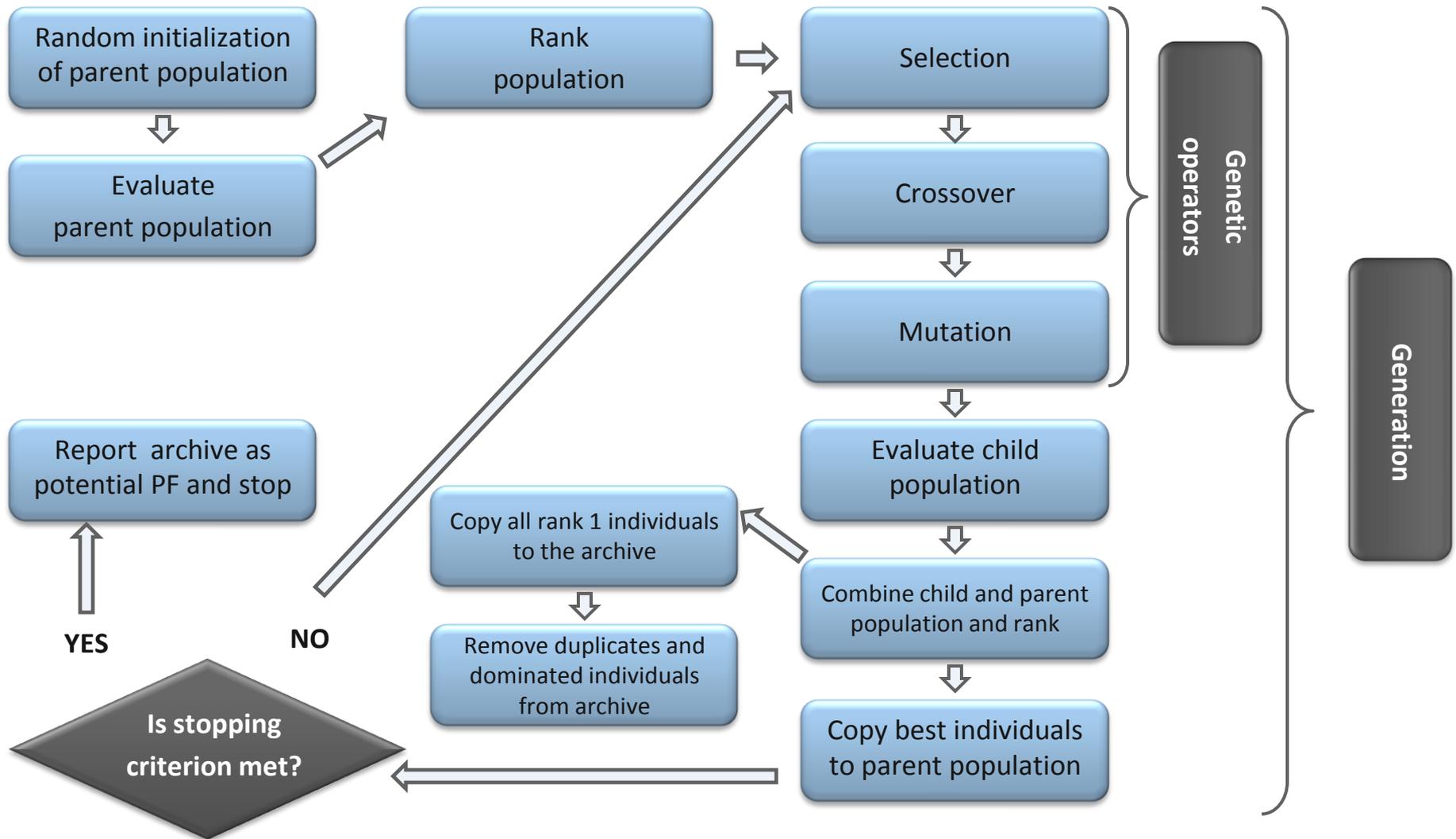
- Gene
  - each design variable  $x$
- Chromosome or individual
  - Group of design variables (vector of design variables)
- Population
  - Group of individuals
- Fitness
  - How good is the individual (analogous to the objective function)
- Genetic operators
  - Selection
  - Crossover
  - Mutation
- Generation
  - A generation comprises of application of genetic operations to create a child population that will become parent population in the next iteration



# Genetic Algorithm - Generation Flowchart



# Genetic Algorithm for Multiobjective Optimization



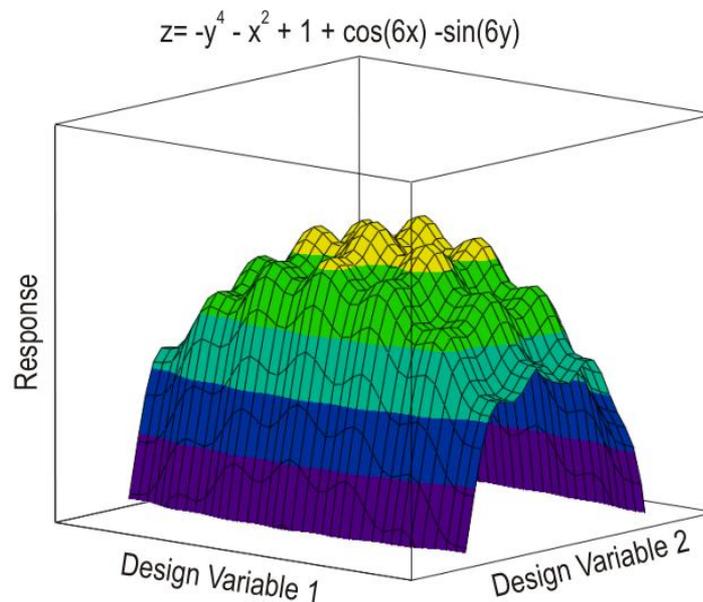
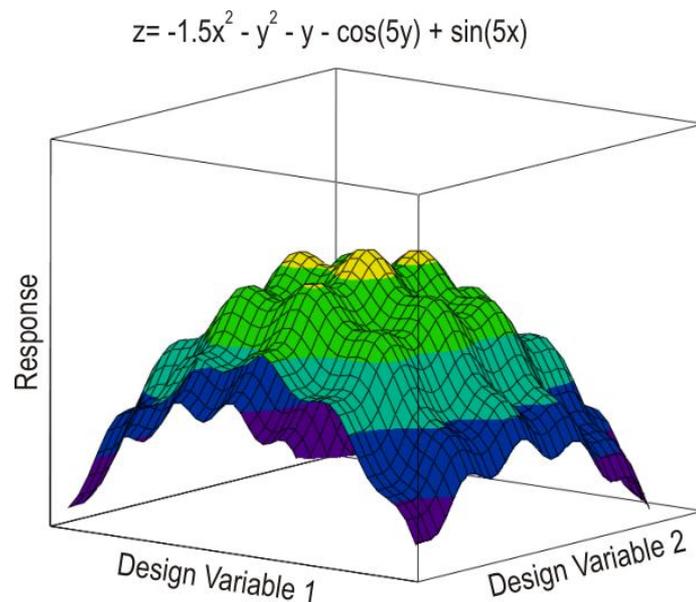
# Problem Definition

- Objective: find  $x$  and  $y$  that would maximize expression  $0.33 z_1 + 0.67 z_2$  where:

$$z_1 = -1.5x^2 - y^2 - y - \cos(5y) + \sin(5x)$$

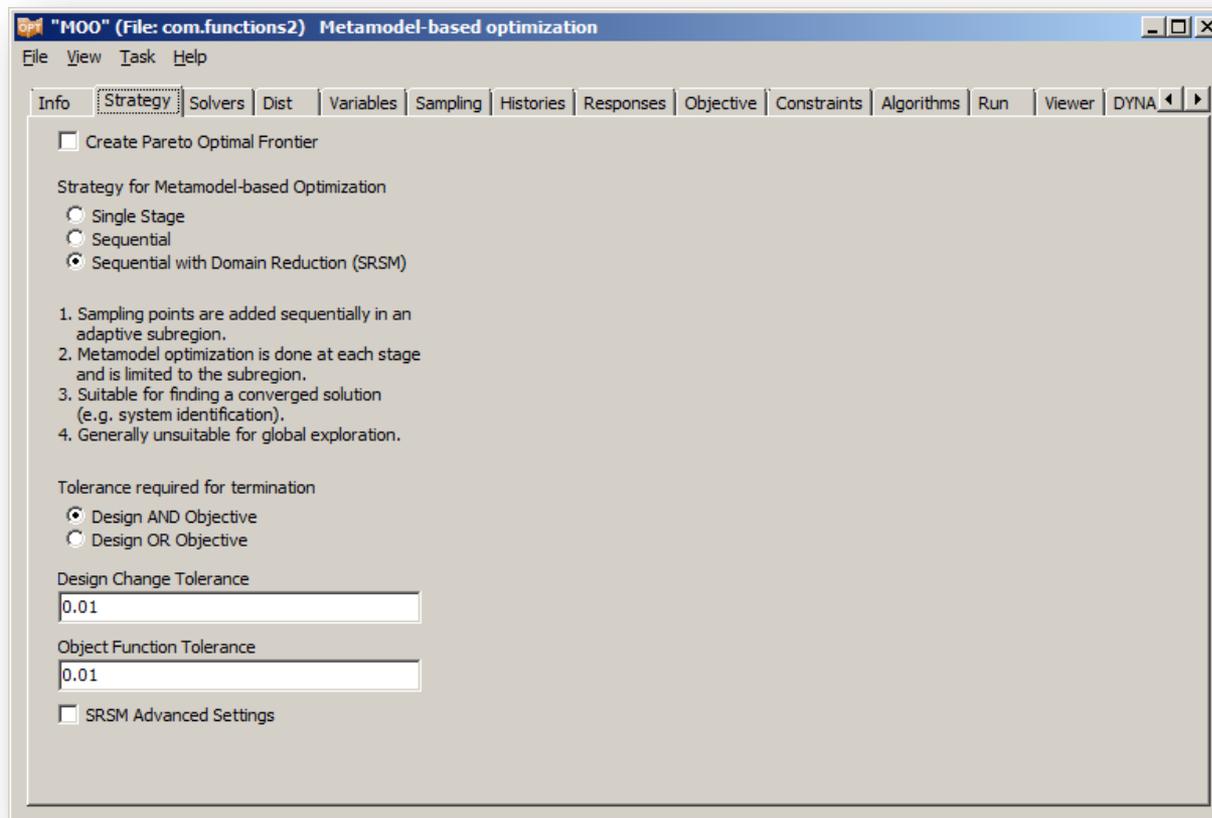
$$z_2 = -y^4 - x^2 + 1 + \cos(6x) - \sin(6y)$$

- Two design variables:  $x$  and  $y$  in the range  $(-3, 3)$



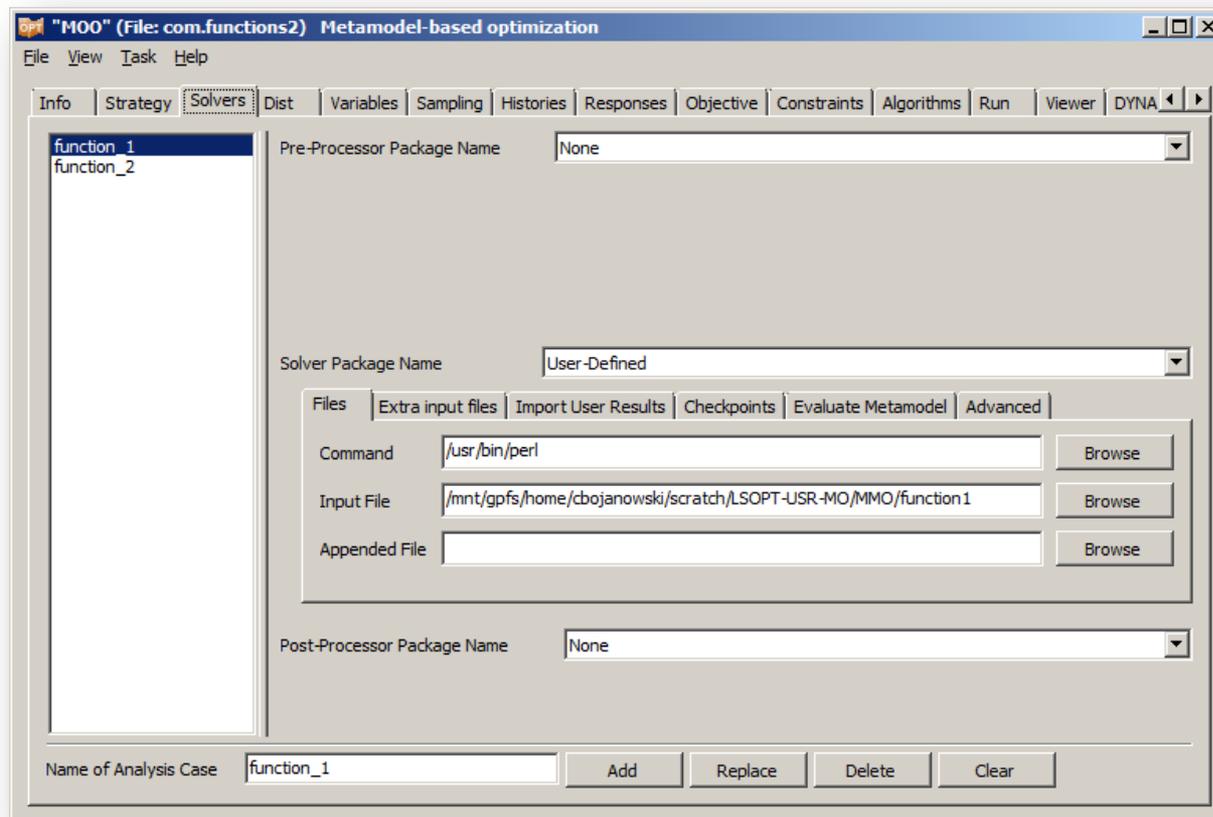
# Strategy Panel

- Copy the command file and scripts **function1** and **function2** to new working directory
- Go to Strategy panel and select Sequential with Domain Reduction
- Leave defaults for convergence check criteria



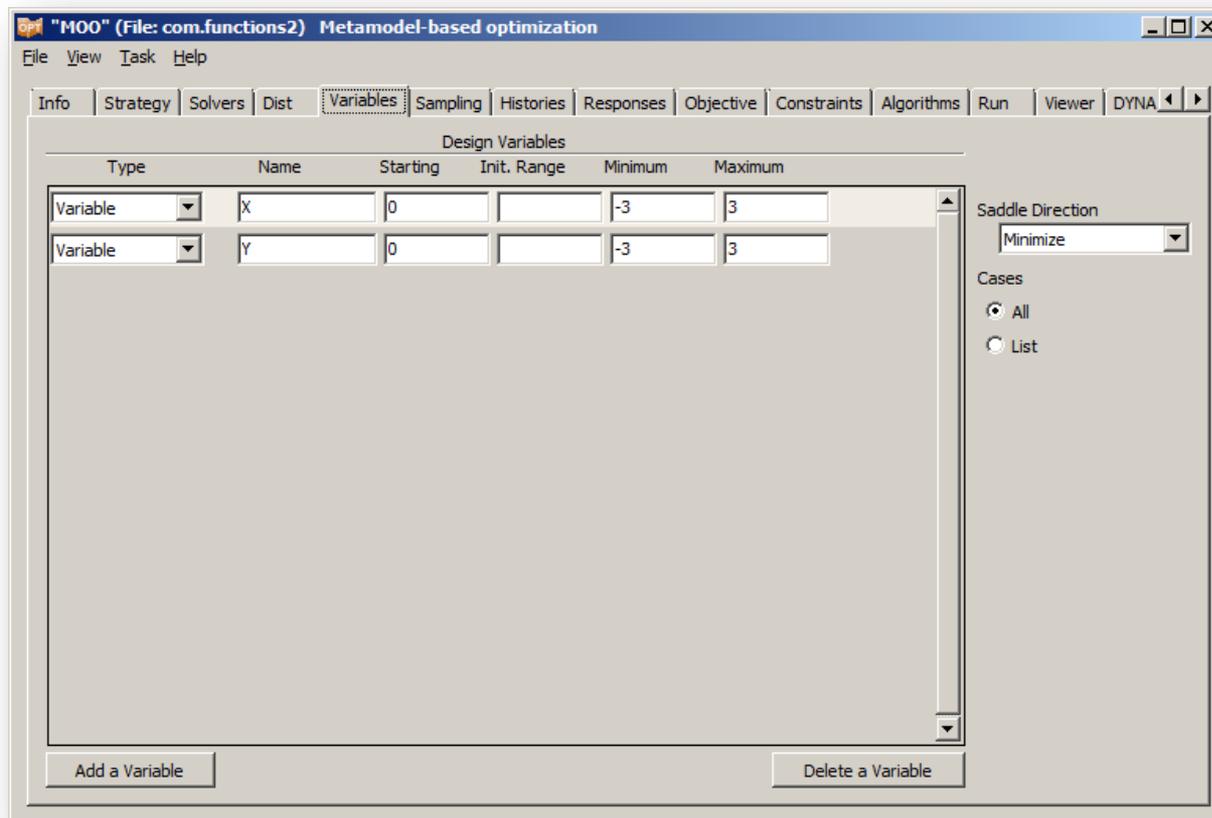
# Solvers Panel

- Go to Solvers panel, Analysis Case **function1** should exist already
- Modify the path to the copied input file (**function1**) and press Replace
- Similarly create Analysis Case **function\_2** with path to the **function2** input file



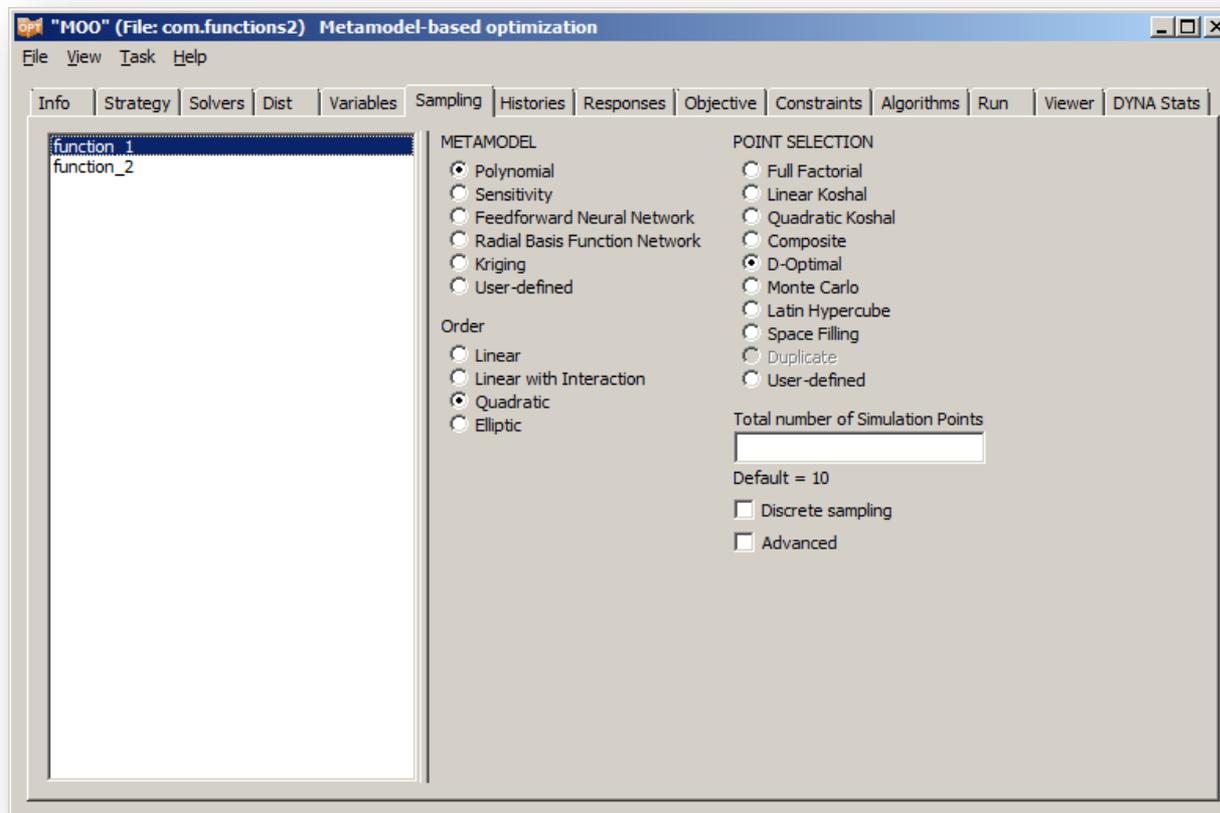
# Variables Panel

- Go to Variables panel
- The variables **X** and **Y** should exist
- Leave the min and max bounds as they are



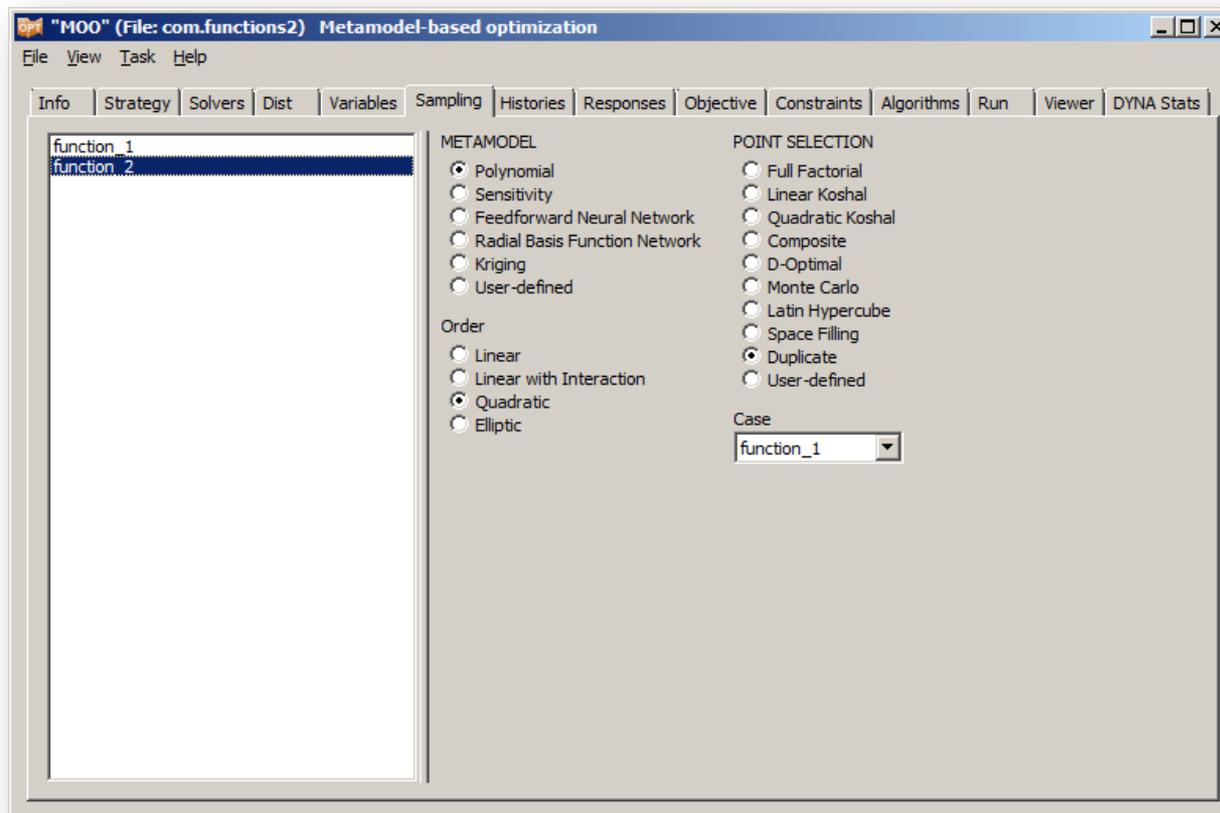
# Sampling Panel

- Go to Sampling panel
- For **function\_1** choose Quadratic Polynomial Metamodel with D-Optimal Point Selection method



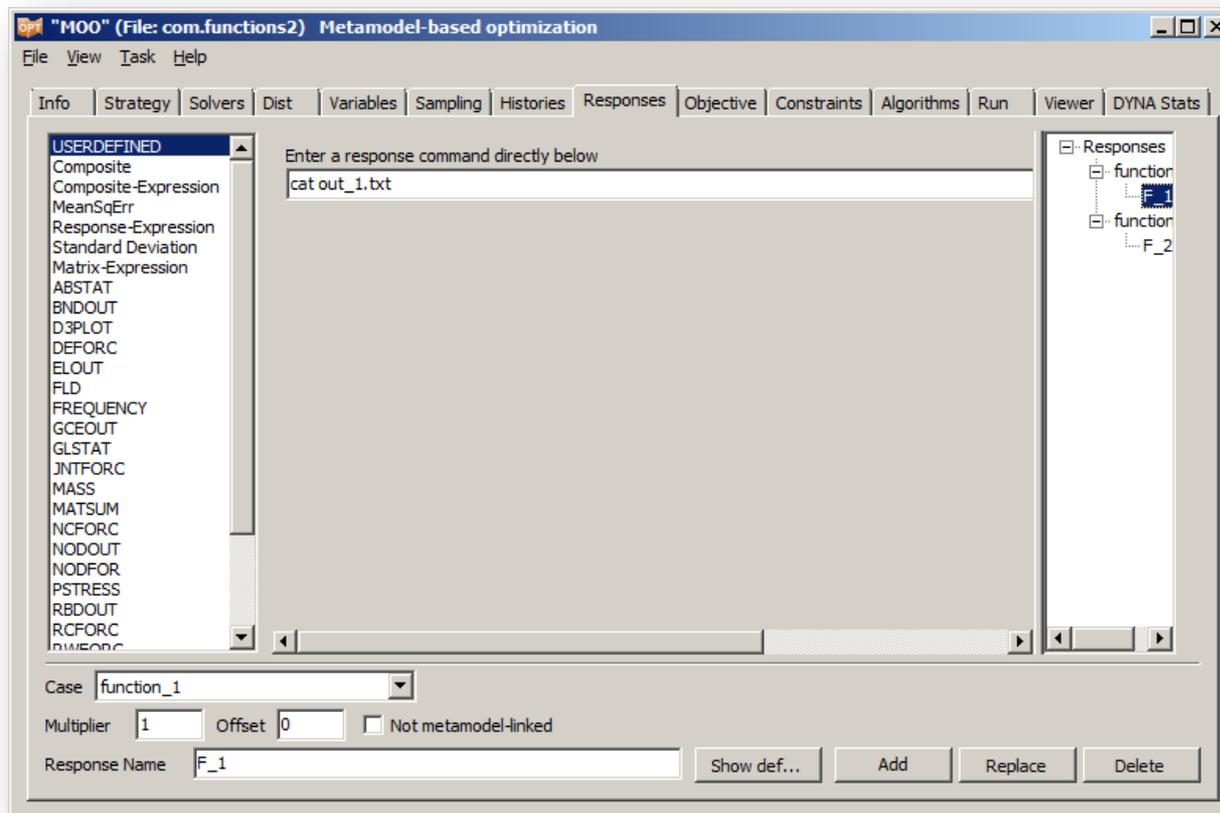
# Sampling Panel

- For **function\_2** Analysis Case also choose Polynomial Quadratic Metamodel
- For Point selection choose Duplicate and from drop down menu select Case **function\_1**



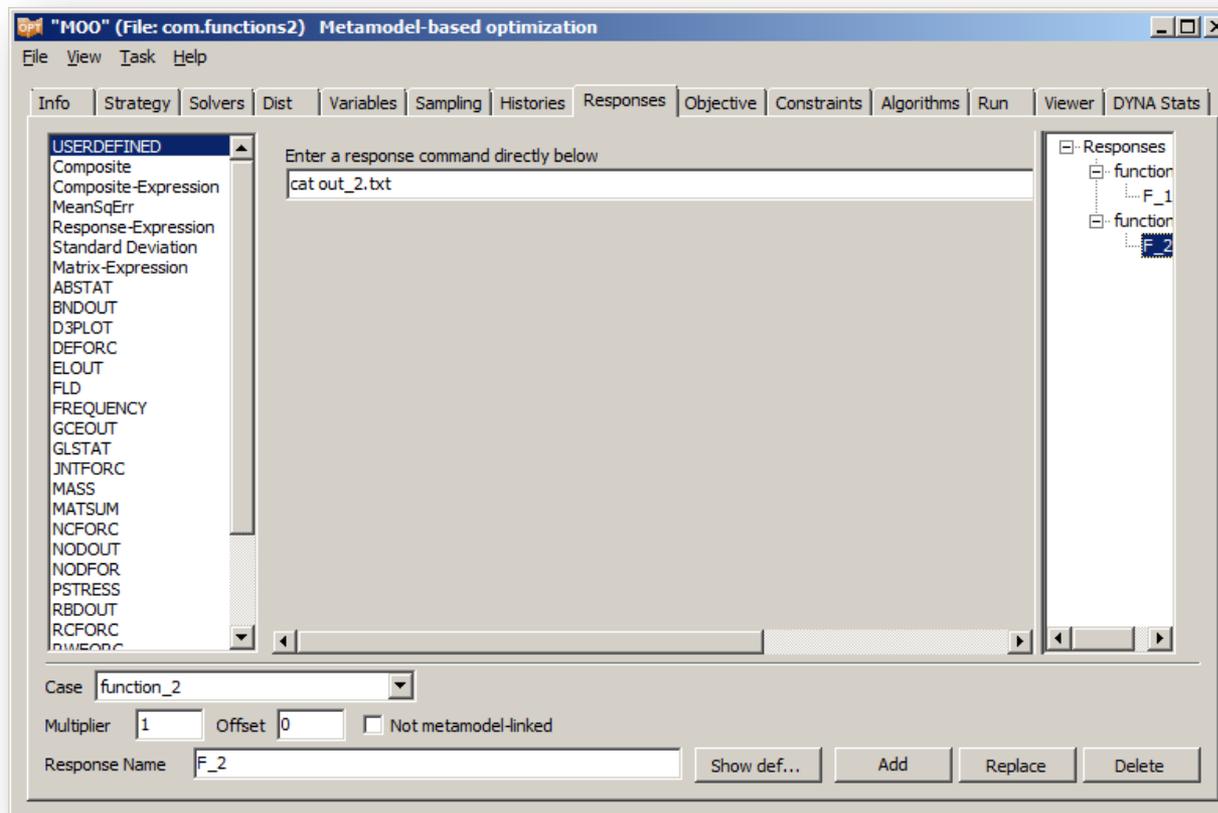
# Responses Panel

- Go to Responses panel
- Response **F\_1** for case **function\_1** should exist



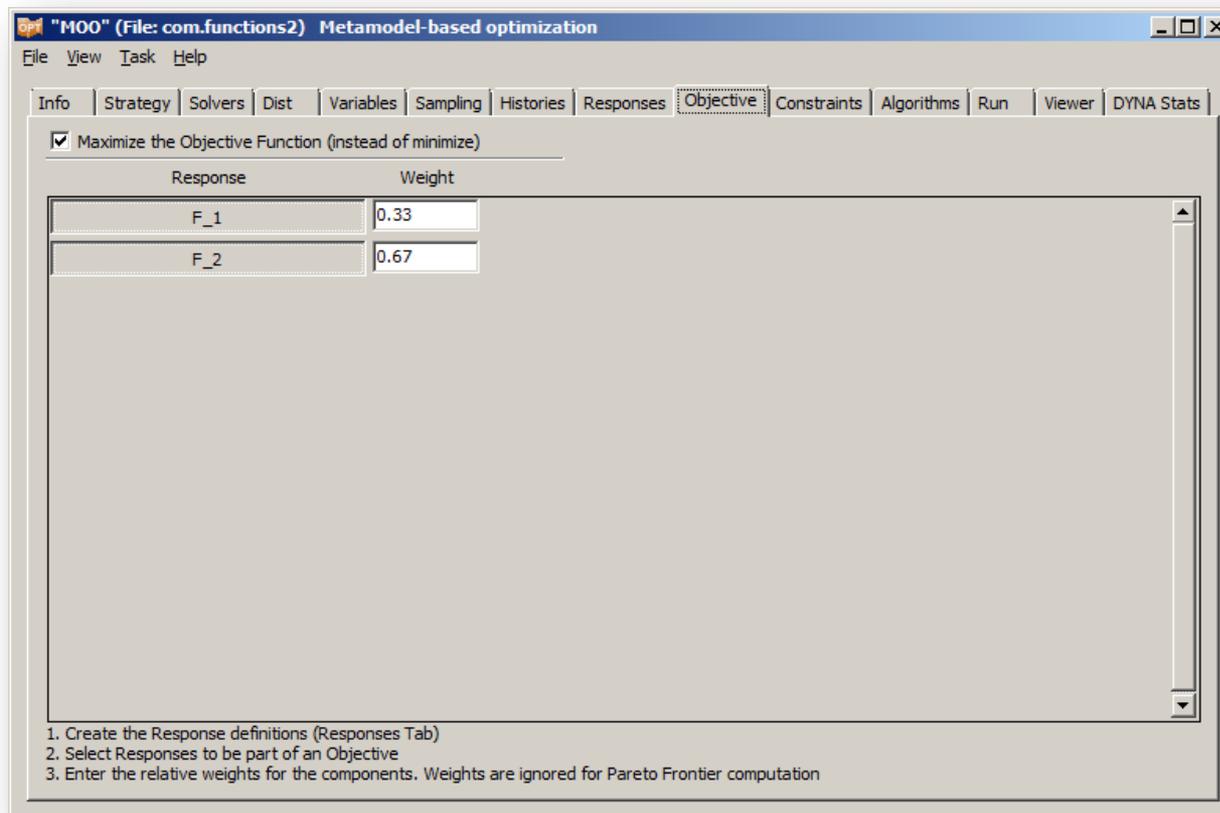
# Responses Panel

- Select Case `function_2`
- Enter a response command “`cat out_2.txt`”
- Enter `F_2` in Response Name field and press Add



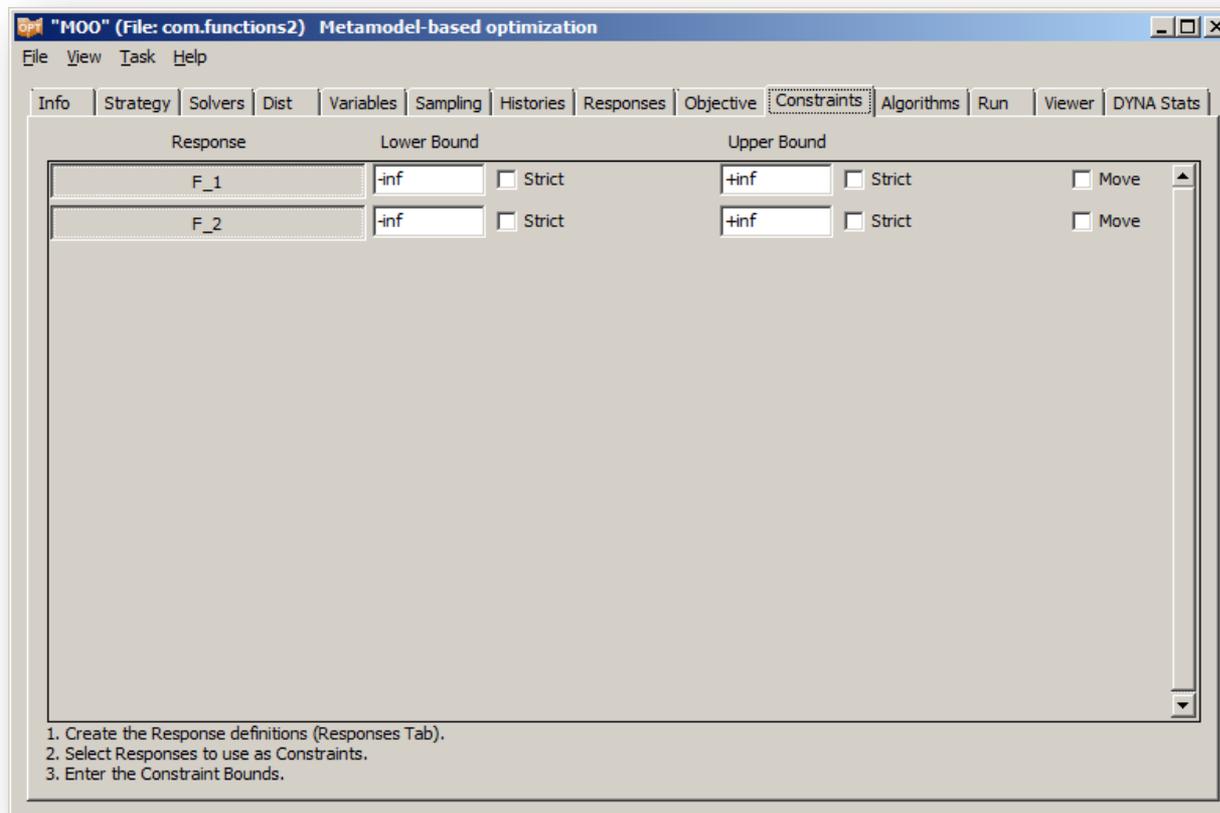
# Objective Panel

- Go to Objective Panel
- Select **F\_1** and **F\_2** responses for objective
- Assign weight **0.33** to **F\_1** and **0.67** to **F\_2** for the multi-objective case
- Note: weights are ignored for Pareto Front computation



# Constraints Panel

- There is no constraints on the functions
- Leave the default infinite bounds for the Responses



# Run Panel

- Go to Run Panel
- Type **10** for Iteration Number
- With None as a Queuing software and one Concurrent Job press Run
- 182 calculations should be performed

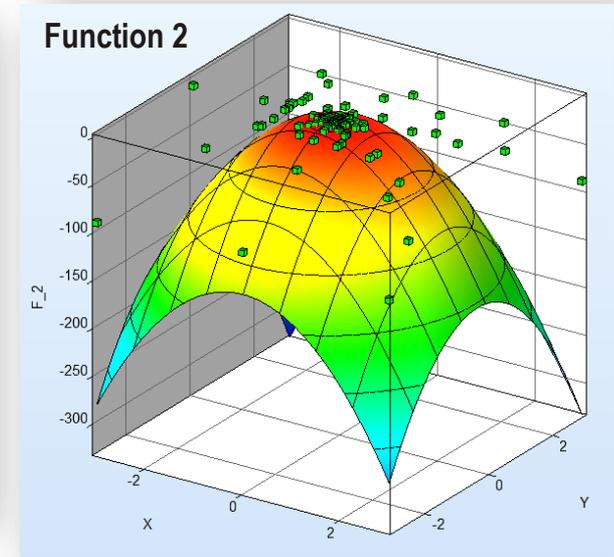
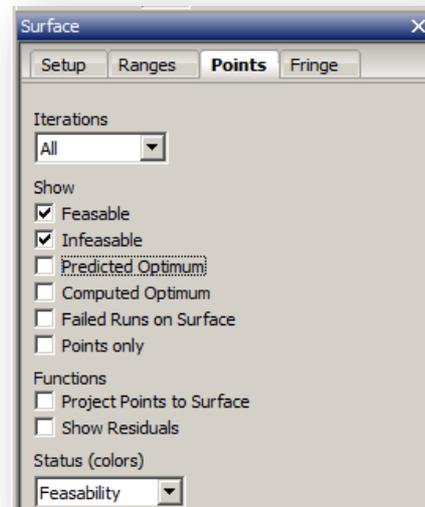
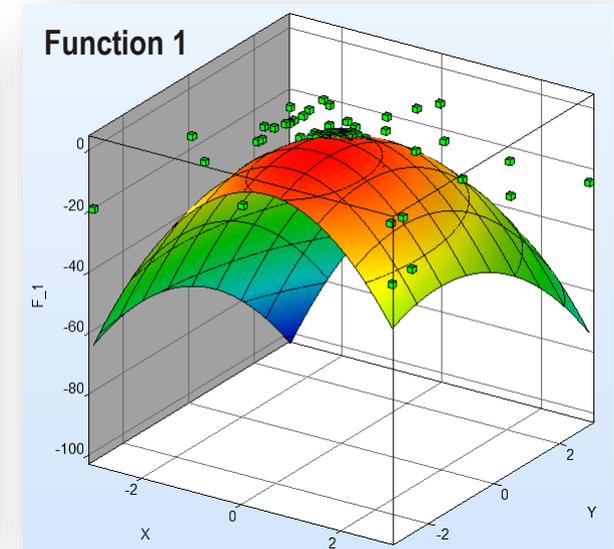
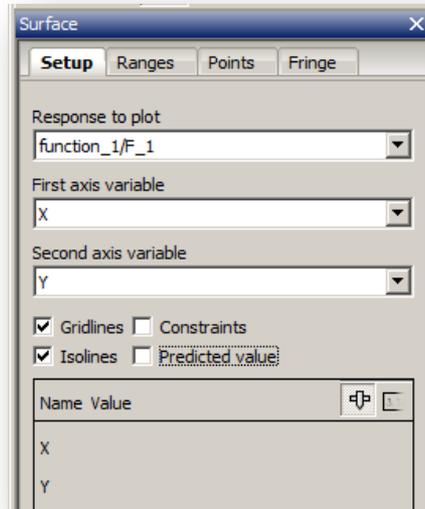
The screenshot shows the MOO (Metamodel-based optimization) software interface. The title bar reads "MOO" (File: com.functions2) Metamodel-based optimization. The menu bar includes File, View, Task, and Help. The toolbar contains buttons for Info, Strategy, Solvers, Dist, Variables, Sampling, Histories, Responses, Objective, Constraints, Algorithms, Run, Viewer, and DYNA Stats.

The main panel is divided into several sections:

- Job ID Table:** A table with columns for Job ID, PID, and Progress. The progress column shows "Normal Termination" for all jobs, highlighted in green.
- QUEUING:** A section with a dropdown menu set to "None", a "Concurrent Jobs" input field set to "1", and a "Case" dropdown menu with options "function\_1" and "function\_2".
- SEQUENTIAL OPTIMIZATION:** A section with a "Number of iterations" input field set to "10", and two checkboxes: "Omit last verification run" and "Clean Start from Iteration".
- Run and Stop buttons:** Two buttons labeled "Run" and "Stop".
- Time Step List:** A list of time steps including Kinetic Energy, Internal Energy, Total Energy, Energy Ratio, Global X Velocity, Global Y Velocity, Global Z Velocity, Total CPU Time, and Time to Completion.
- Graph Area:** A graph area with the text "No Processes Selected" and a y-axis ranging from 0 to 1.

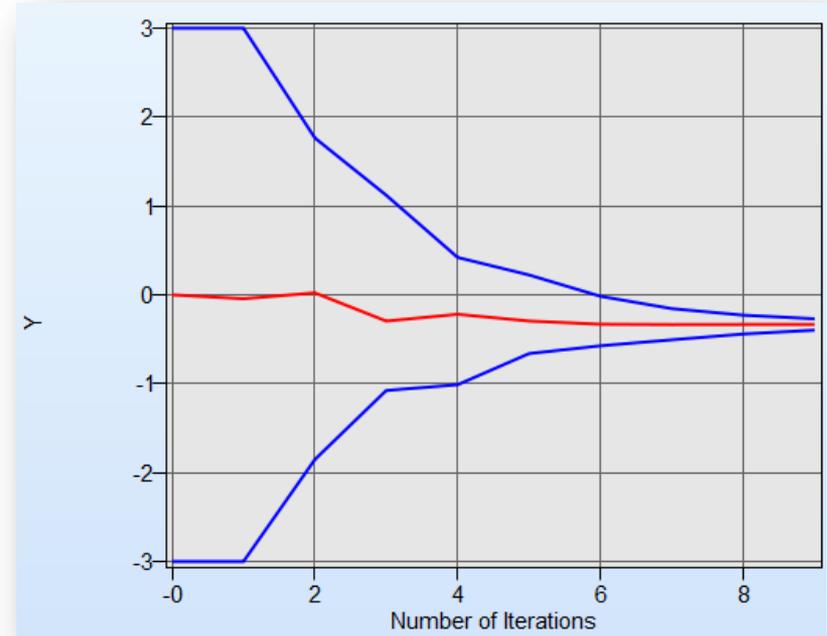
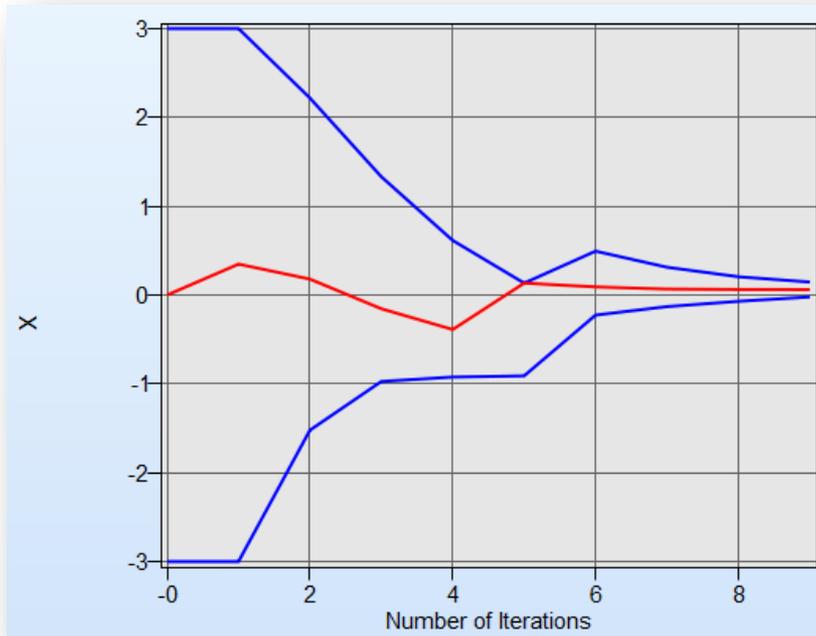
# Viewer - Metamodel

- Go to Viewer panel and select Surface from Metamodel menu
- In the Points tab select All Iterations
- Check Feasible and Infeasible Points
- For colors choose Feasibility



# Viewer Panel

- Go to Viewer panel and press Restart Viewer
- Select Optimization History from the Optimization menu
- See the histories for design variables
- Select the last iteration with clicking close to the plot
- Point selection window should appear

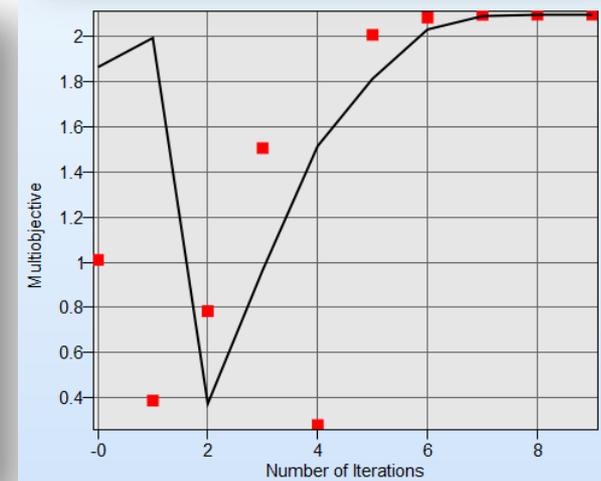
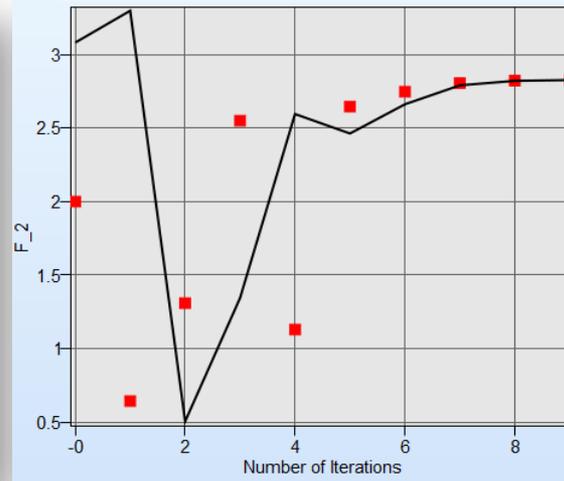
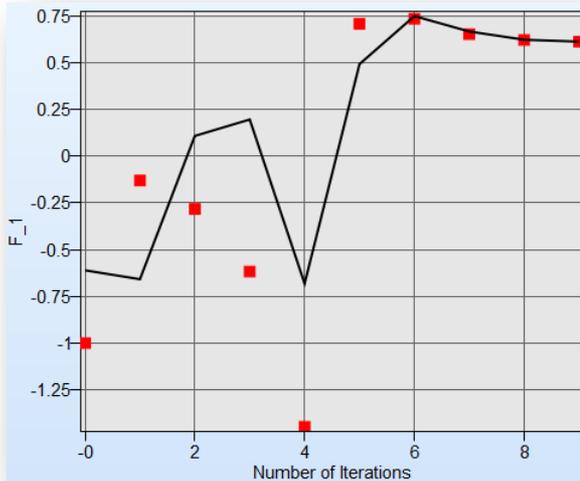


# Optimization History

- Check the values of variables for optimal point
- Are they different from the single case problem?
- In the Setup window select Objectives and see the history of **F\_1**, **F\_2** and Multiobjective function
- The Multiobjective is more influenced by second objective (bigger weight)

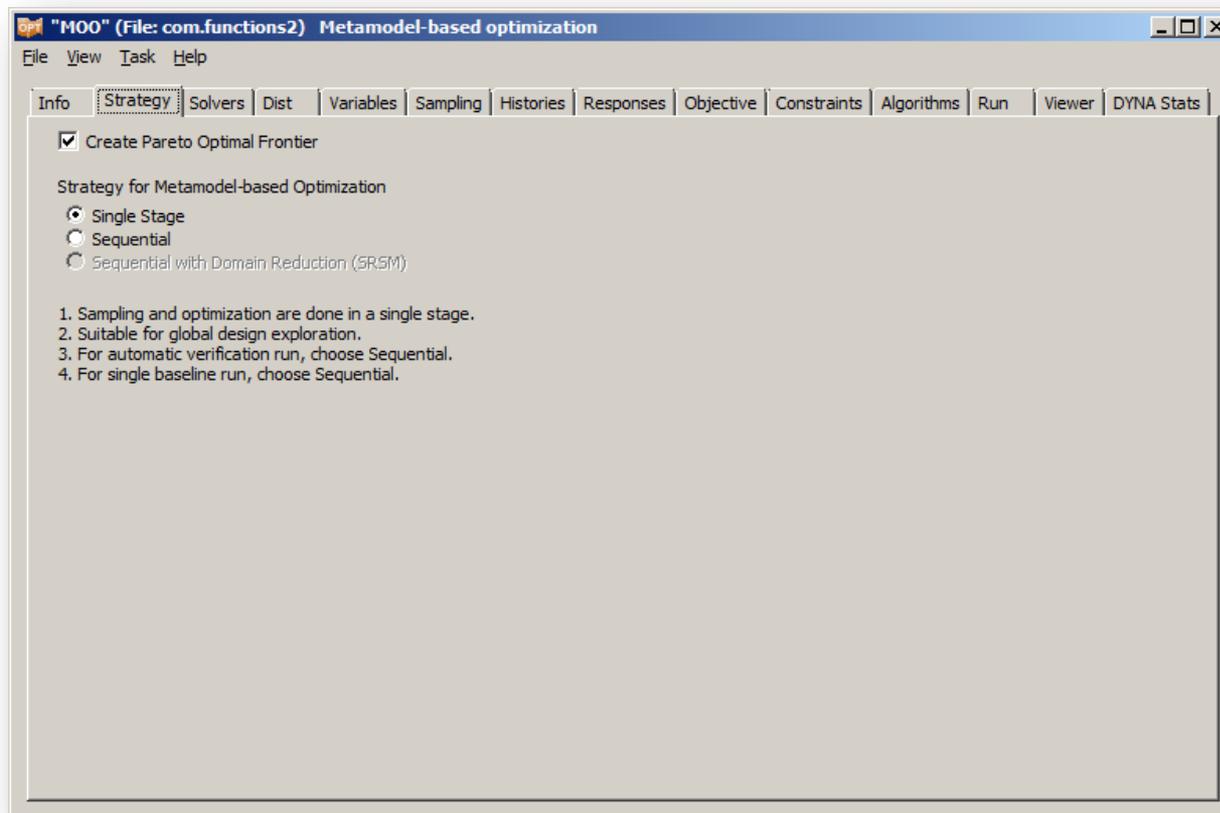
Point selection

Entity	Computed	Predicted
Point		
Variables		
X	0.0612921	0.0612921
Y	-0.332879	-0.332879
Responses		
F_1	0.611582	0.611607
F_2	2.82753	2.82751
Constraints		
F_1	0.611582	0.611607
F_2	2.82753	2.82751
Objectives		
F_1	0.611582	0.611607
F_2	2.82753	2.82751
Multiobjective	2.09627	2.09626
Max Constraint Violation	0	0



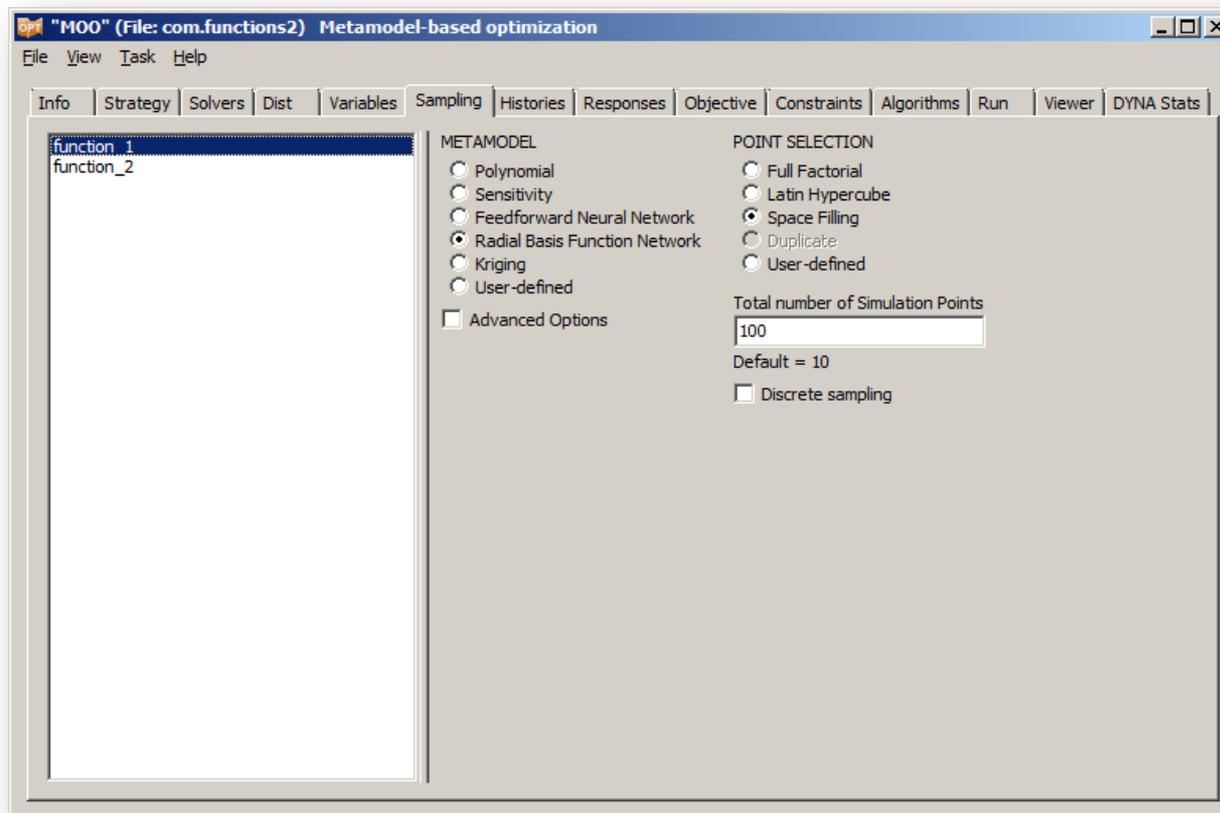
# Pareto Front

- Go back to Strategy panel
- Select Single Stage Strategy
- Check the option Create Pareto Optimal Frontier
- Make sure that In Algorithms Tab GA is selected now



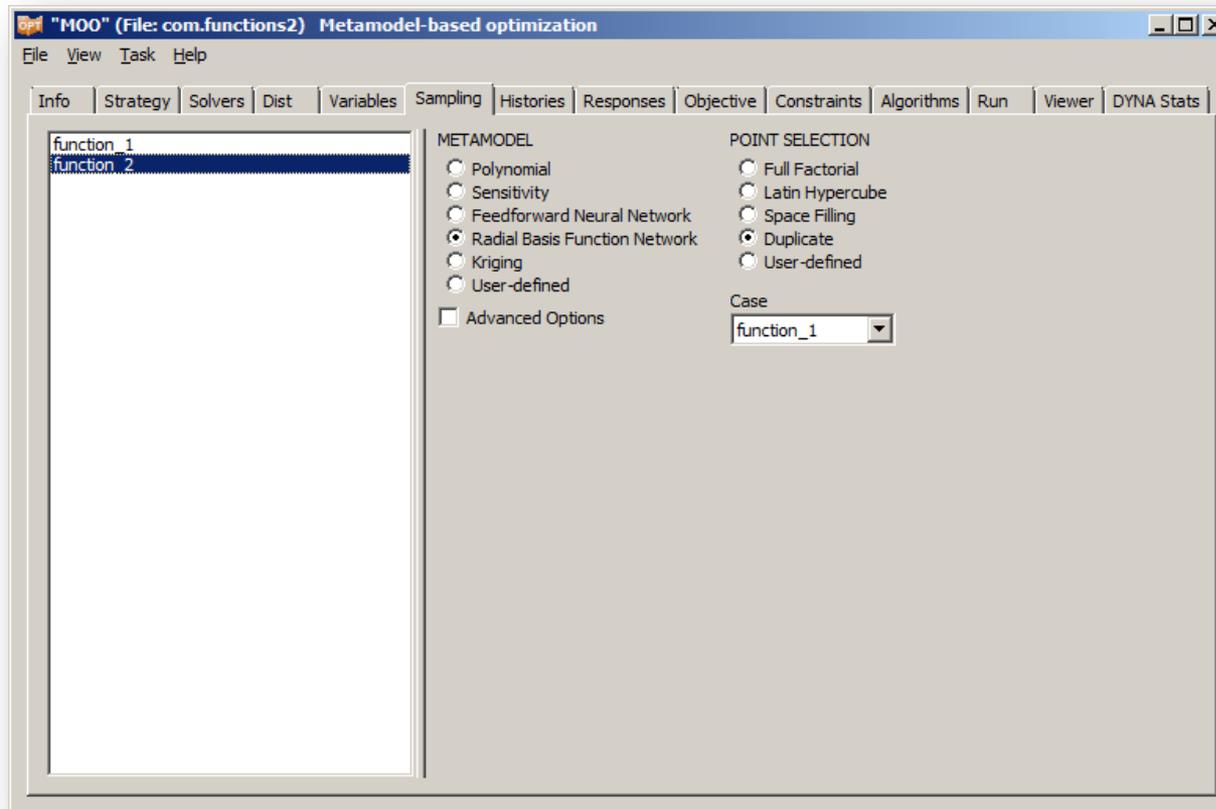
# Sampling Panel

- In the Sampling panel make sure that Metamodel has automatically changed to Radial Basis Function Network with Space filling Point Selection for **function\_1** case
- Type **100** for the Total number of Simulation Points



# Sampling Panel

- For `function_2` case make sure that the points are duplicated from case 1



# Run Panel

- Go to Run panel
- Hit Run and wait for **200** jobs to be finished

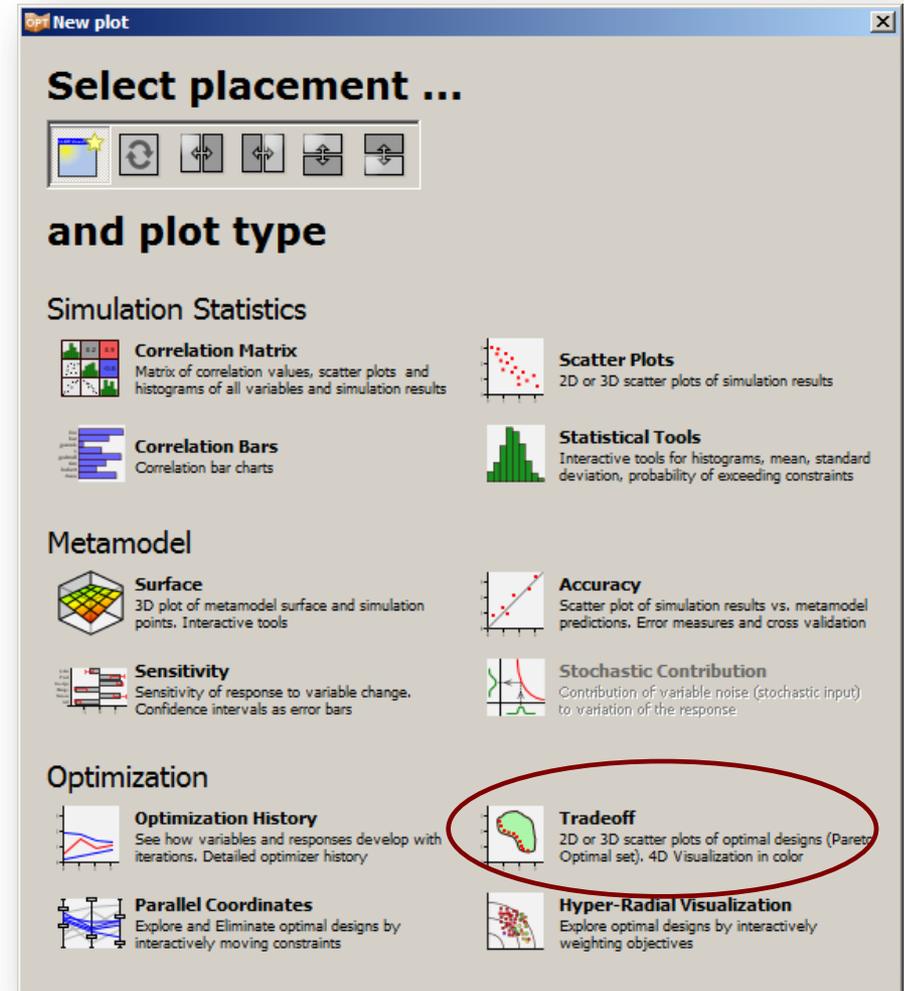
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The main area is divided into several sections:

- Job Progress Table:** A table with columns Job ID, PID, and Progress. It shows 6 jobs (195-200) all with "Normal Termination" status, indicated by green bars.
- QUEUING:** A panel with a dropdown menu set to "None", a "Concurrent Jobs" input field set to "1", and a "Case" list containing "function\_1" and "function\_2".
- SINGLE STAGE OPTIMIZATION:** A panel with a "Clean Start" checkbox.
- Run/Stop Buttons:** Two buttons labeled "Run" and "Stop".
- Plot Area:** A large blue area with the text "No Processes Selected" and a coordinate system with axes labeled 0 and 1.
- Left Panel:** A list of variables for monitoring, including Time Step, Kinetic Energy, Internal Energy, Total Energy, Energy Ratio, Global X Velocity, Global Y Velocity, Global Z Velocity, Total CPU Time, and Time to Completion.

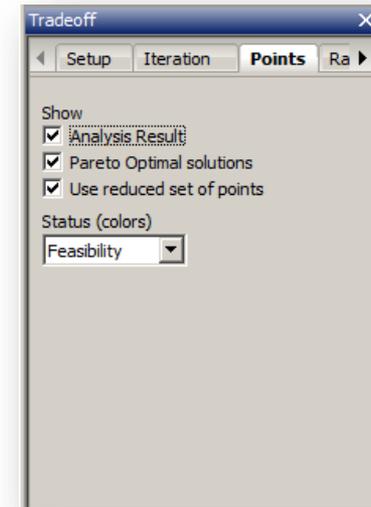
# Viewer

- Go to Viewer panel
- From Optimization select Tradeoff



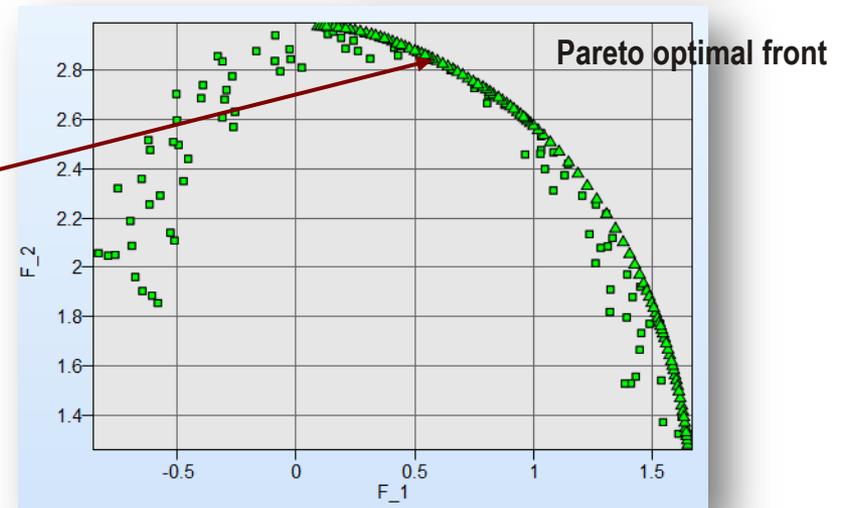
# Tradeoff study

- In the Tradeoff tab go to Points
- Check Analysis Results, Pareto Optimal solution
- For Color leave Feasibility
- Locate the “optimal solution” from Sequential Analysis in the Pareto Front
- Note: weights are ignored here



The Point selection dialog box displays a table with columns for Entity, Computed, and Predicted values. The 'Responses' section is circled in red, highlighting the values for F\_1 and F\_2.

Entity	Computed	Predicted
Point		
Variables		
X	0.0612921	0.0612921
Y	-0.332879	-0.332879
Responses		
F_1	0.611582	0.611607
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F_1	0.611582	0.611607
F_2	2.82753	2.82751
Multiobjective	2.09627	2.09626
Max Constraint Violation	0	0



# Tradeoff study - Pareto Front

- In the Tradeoff tab go to Setup
- Select 3D from the menu
- For X-axis select **X** variable and for Y-axis select **Y** variable
- For Z-Axis select Responses – **F\_1**

