Introductory Course: Using LS-OPT® on the TRACC Cluster

2.2b - User Defined Function - PERL or OCTAVE; Two Functions - Multiobjective Optimization

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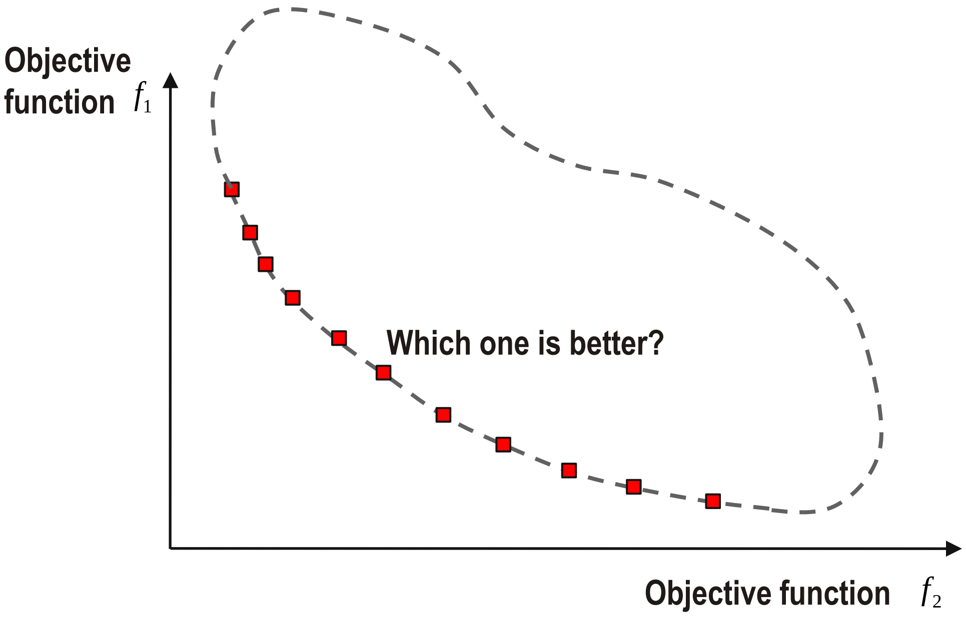
Multi-Objective Optimization (MOO)

- Most engineering problems deal with multiple objectives e.g. cost, weight, safety etc..
- They often conflict with each other e.g. increasing safety may involve increasing cost
- It is hard or even impossible to find single best solution



Multi-Objective Optimization (MOO)

Objective functions

$$\min \; f_l(x); \quad l = 1,2,\ldots,n$$

Inequality constraints

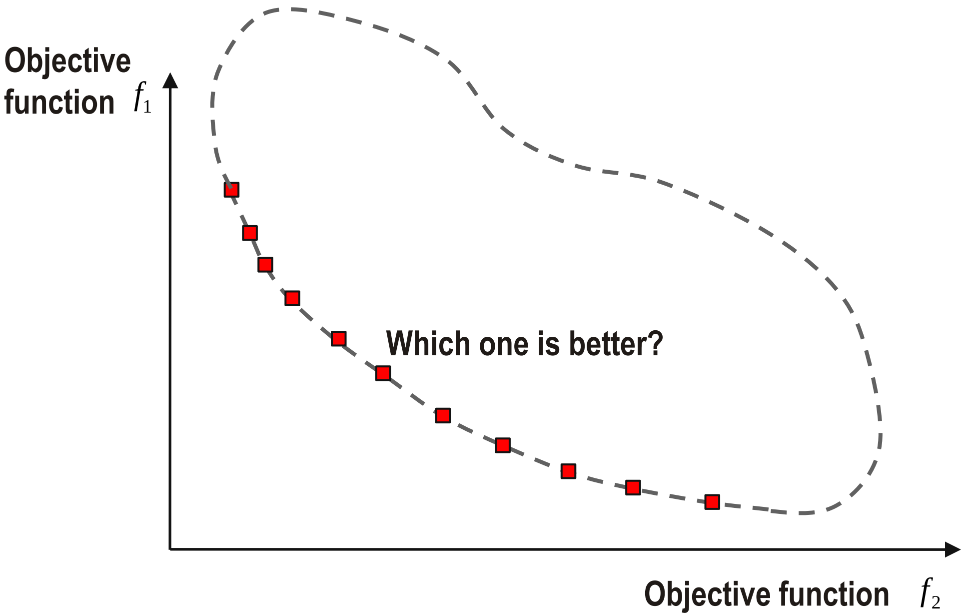
$$g_j(x) \le 0; \quad j = 1,2,\ldots,m$$

Equality constraints

$$h_k(x) = 0; \quad k = 1,2,\ldots,l$$

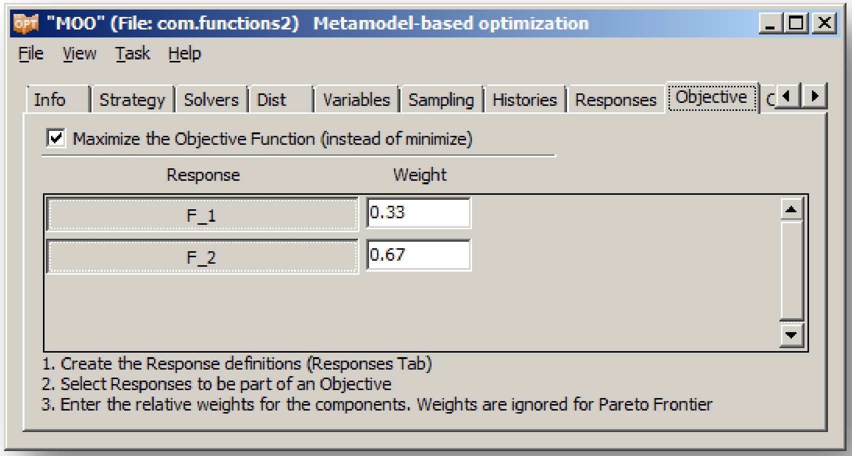
Bounds on variables (side constraints)

$$x_{i,L} \le x_i \le x_{i,U}; \quad i = 1,2,\ldots,p$$

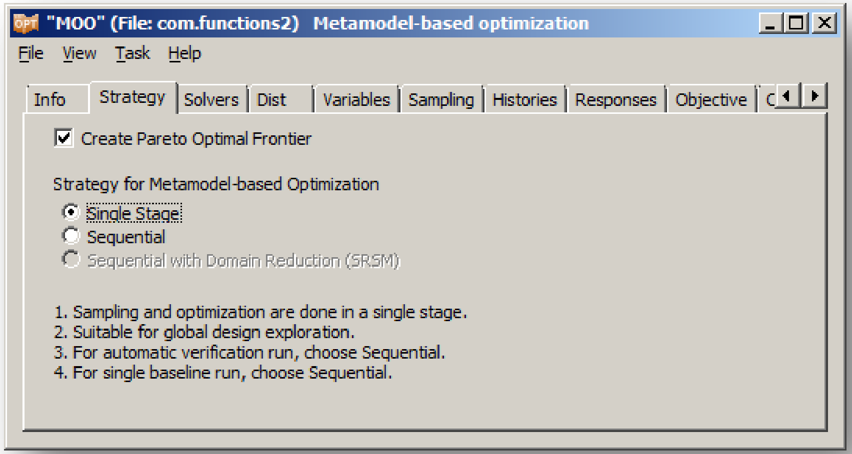


Methods of Solving Multiobjective Optimization Problems

- Weighed sum strategy
  - Convert multiple objectives into a single objective using weights



- ε - constraint strategy
  - All but one objectives is treated as constraints and optimize for the left-out problem



- Multiobjective genetic algorithm for POF
  - All objectives are simultaneously optimized
  - Weights are ignored

Multi-Objective Optimization (MOO)

- MOO problems do not have a single optimal solution. Instead there is a set of solutions that reflects trade-off among objectives.

- Special considerations are required to compare different designs from MOO simulations.

- Non-dominant concept is used to compare different individuals

- A solution $x^{(1)}$ dominates another solution $x^{(2)}$ $\left(x^{(1)} \succ x^{(2)}\right)$, if either one of the following conditions is true:
  - $x^{(1)}$ is feasible and $x^{(2)}$ is not
  - Both $x^{(1)}$ and $x^{(2)}$ are infeasible but $x^{(2)}$ is more infeasible compared to $x^{(1)}$

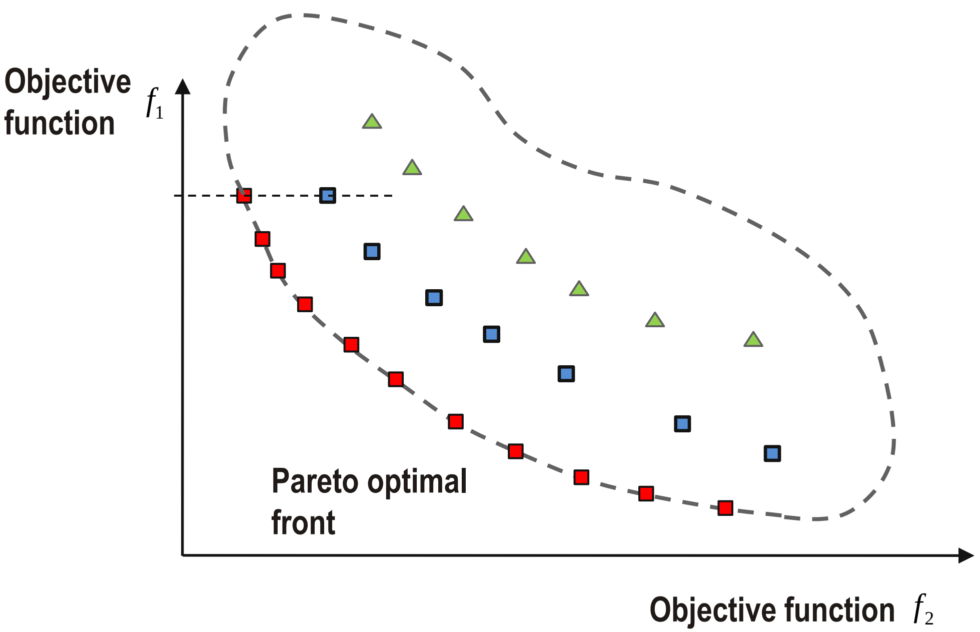
Multi-Objective Optimization (MOO)

- When both $x^{(1)}$ and $x^{(2)}$ are feasible, $x^{(1)}$ dominates $x^{(2)}$ $\left(x^{(1)} \succ x^{(2)}\right)$ if following two conditions are satisfied:
  - $x^{(1)}$ is no worse than $x^{(2)}$ in all objectives i.e. $f_j\left(x^{(1)}\right) \not> f_j\left(x^{(2)}\right),\ j \in [1,2,\ldots,M]$
  - $x^{(1)}$ is strictly better than $x^{(2)}$ in at least one objective $f_j\left(x^{(1)}\right) < f_j\left(x^{(2)}\right)$

- If neither of the two solutions dominates the other, both solutions are non-dominated with respect to each other
- Any non-dominated solution in the entire domain is a Pareto optimal solution
- Function space representation of the Pareto optimal set is Pareto optimal front

Multi-Objective Optimization (MOO)

- Blue solutions are dominated by red solutions but not by green ones
- Red solutions are not dominated by any other solutions
- Collection of red solutions creates Pareto optimal front

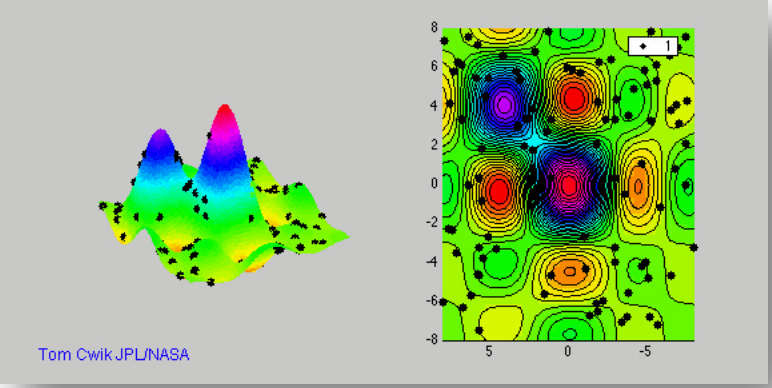


Genetic Algorithms – "Survival of the Fittest"

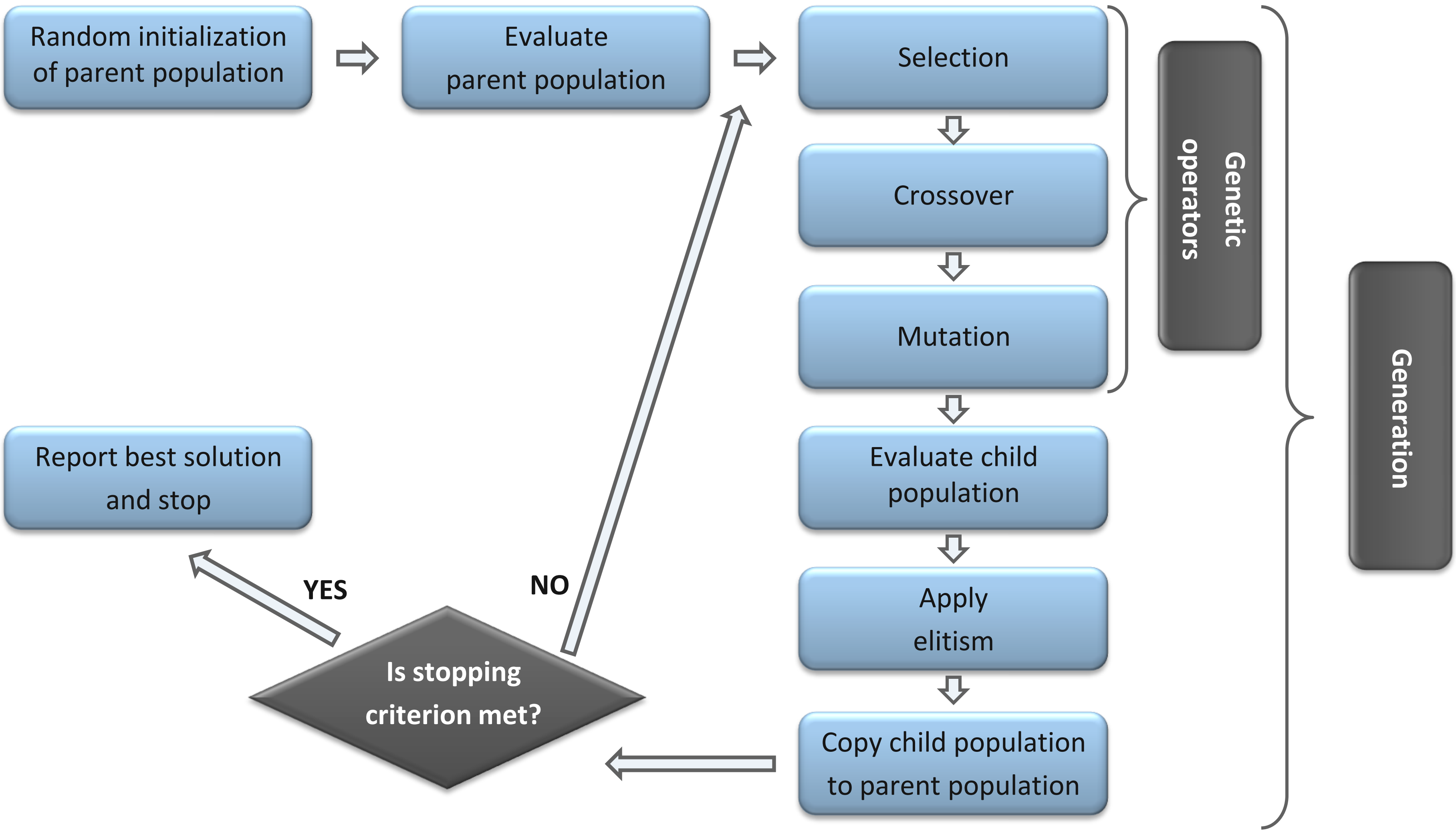
- Based on the evolutionist theory and Darwin's principle "Survival of the Fittest"
- In nature , weak and unfit species within their environment are faced extinction by natural selection
  - GA is a population based approach (uses multiple points at a time)
  - GA is well suited for Pareto Front search
  - GA does not require derivative information to drive the search of optimal points
  - GA is a global optimizer, whereas conventional methods may get stuck in local optima
  - Requires large number of function evaluations

Genetic Algorithms - Terminology

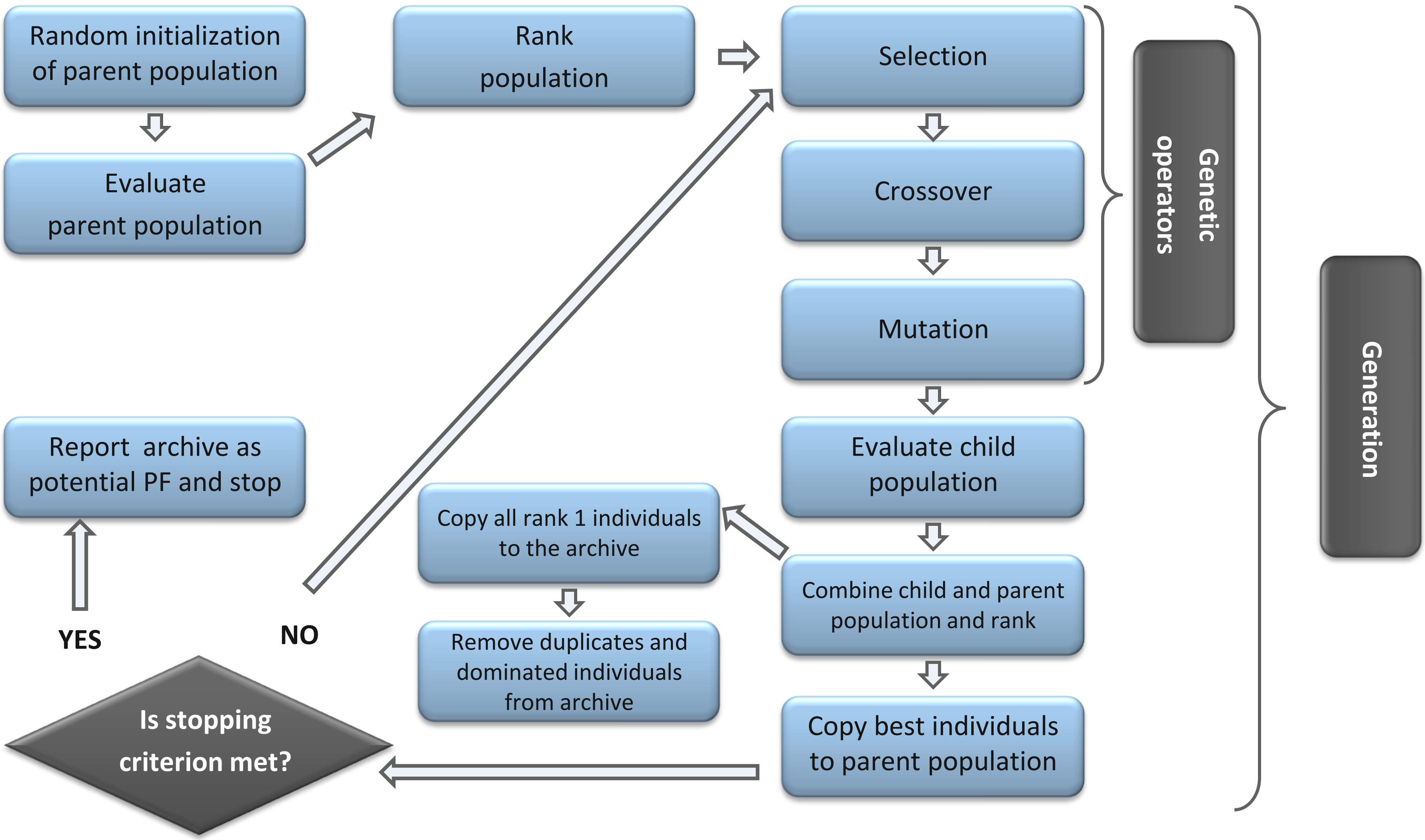
- Gene
  - each design variable x
- Chromosome or individual
  - Group of design variables (vector of design variables)
- Population
  - Group of individuals
- Fitness
  - How good is the individual (analogous to the objective function)
- Genetic operators
  - Selection
  - Crossover
  - Mutation
- Generation
  - A generation comprises of application of genetic operations to create a child population that will become parent population in the next iteration



Genetic Algorithm – Generation Flowchart



Genetic Algorithm for Multiobjective Optimization



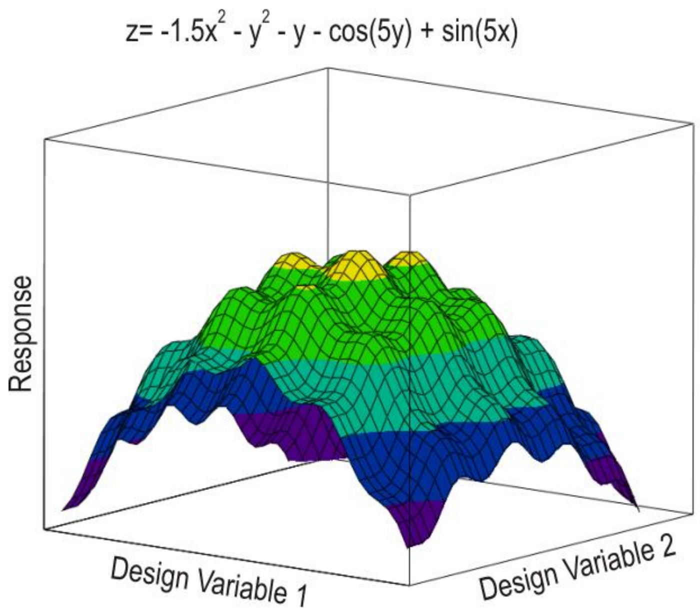
Problem Definition

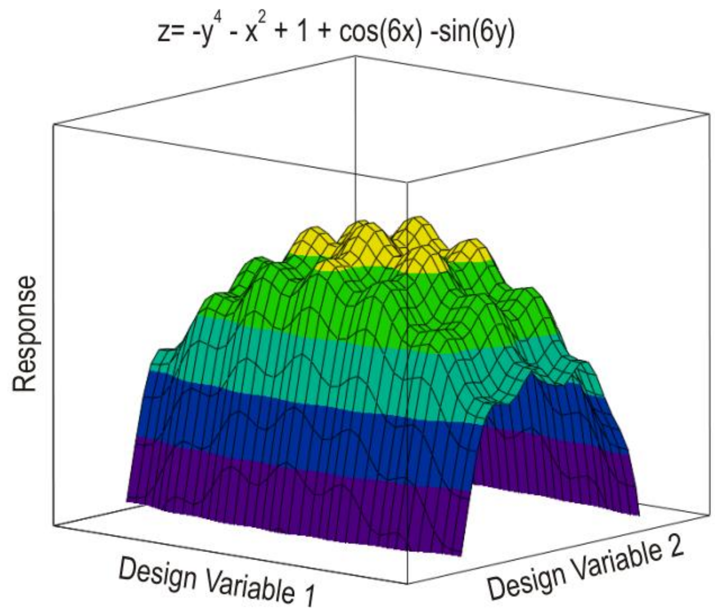
- Objective: find `x` and `y` that would maximize expression `0.33 z1 + 0.67 z2` where:

  $z1=-1.5x^2-y^2-y-\cos(5y)+\sin(5x)$

  $z2=-y^4-x^2+1+\cos(6x)-\sin(6y)$

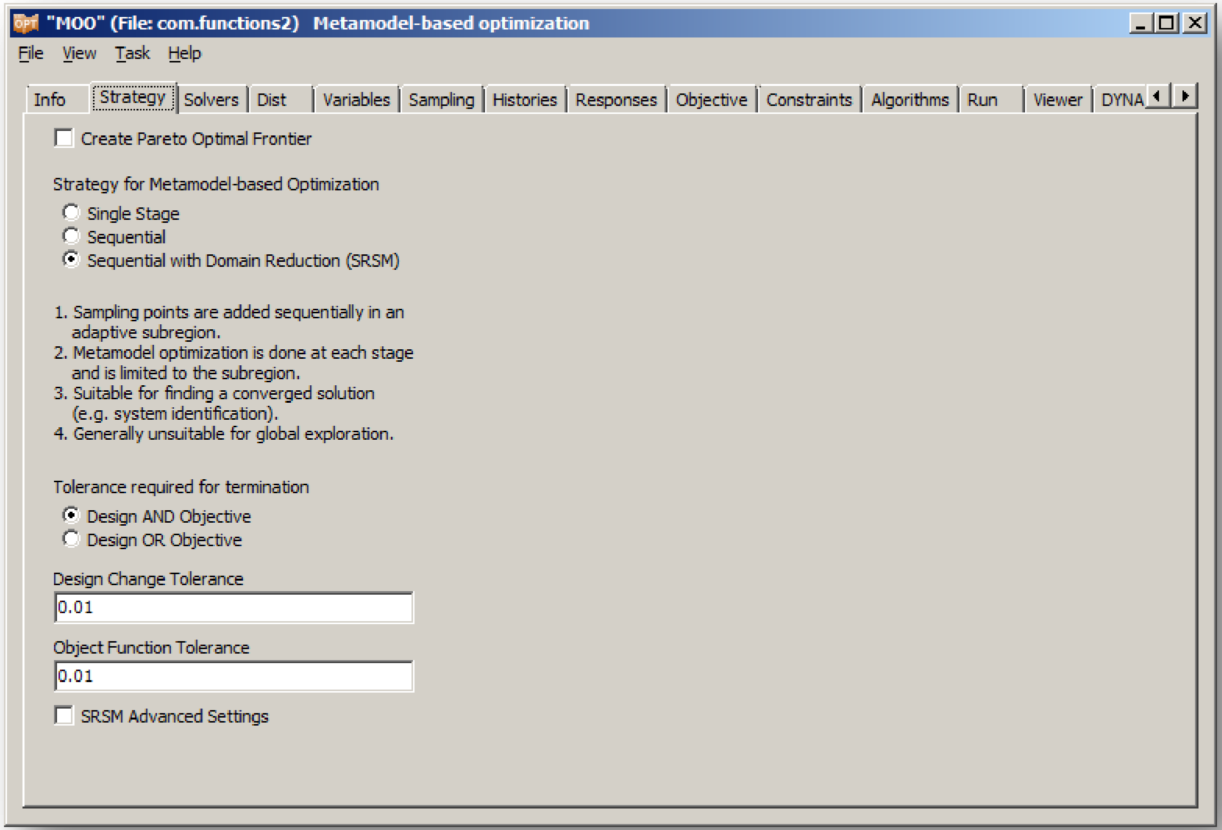
- Two design variables: `x` and `y` in the range `(-3,3)`





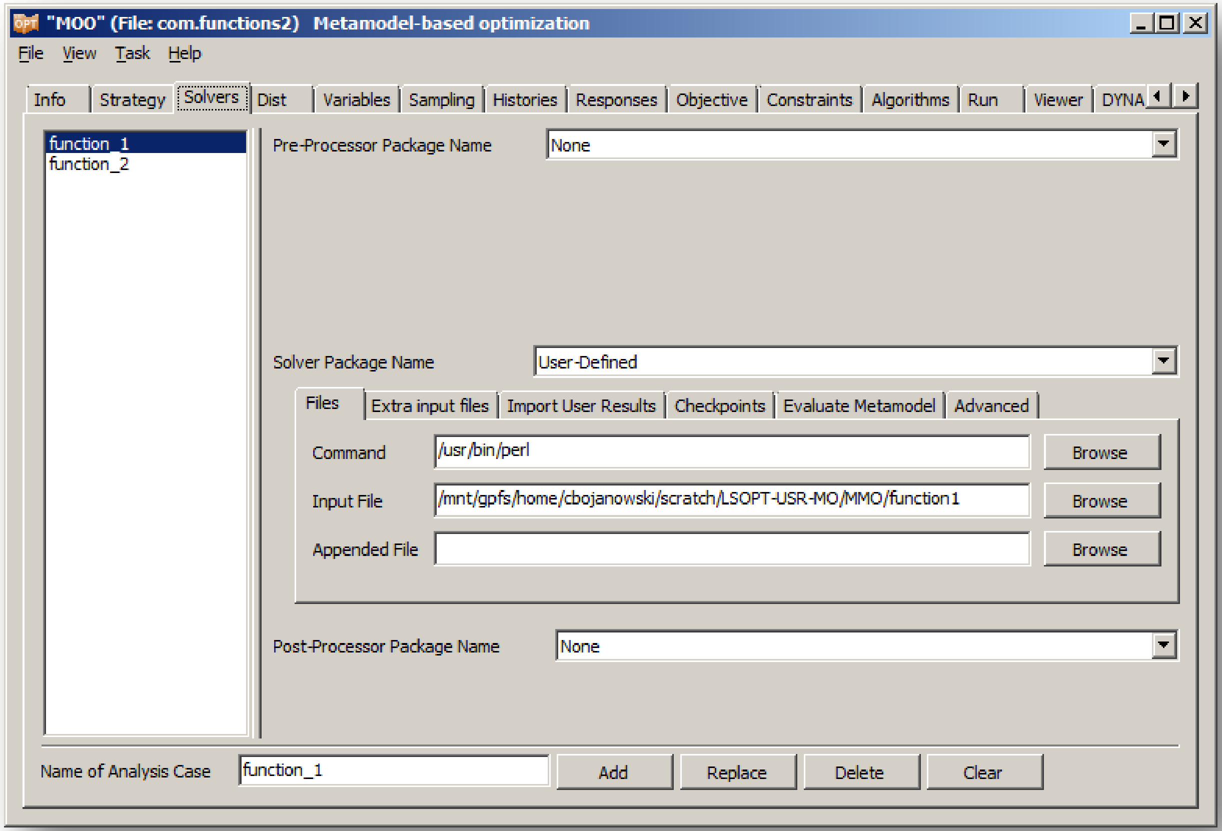
Strategy Panel

- Copy the command file and scripts `function1` and `function2` to new working directory
- Go to Strategy panel and select Sequential with Domain Reduction
- Leave defaults for convergence check criteria



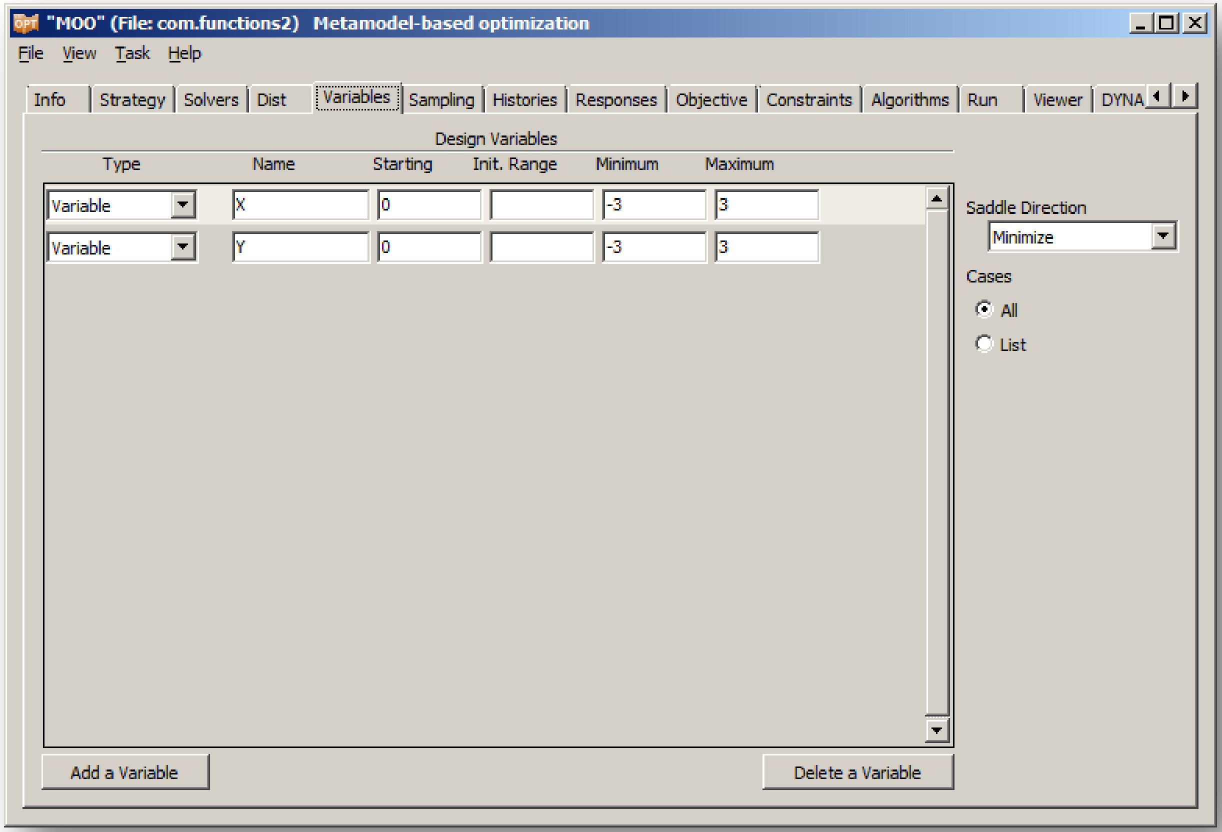
Solvers Panel

- Go to Solvers panel, Analysis Case `function1` should exist already
- Modify the path to the copied input file (`function1`) and press Replace
- Similarly create Analysis Case `function_2` with path to the `function2` input file



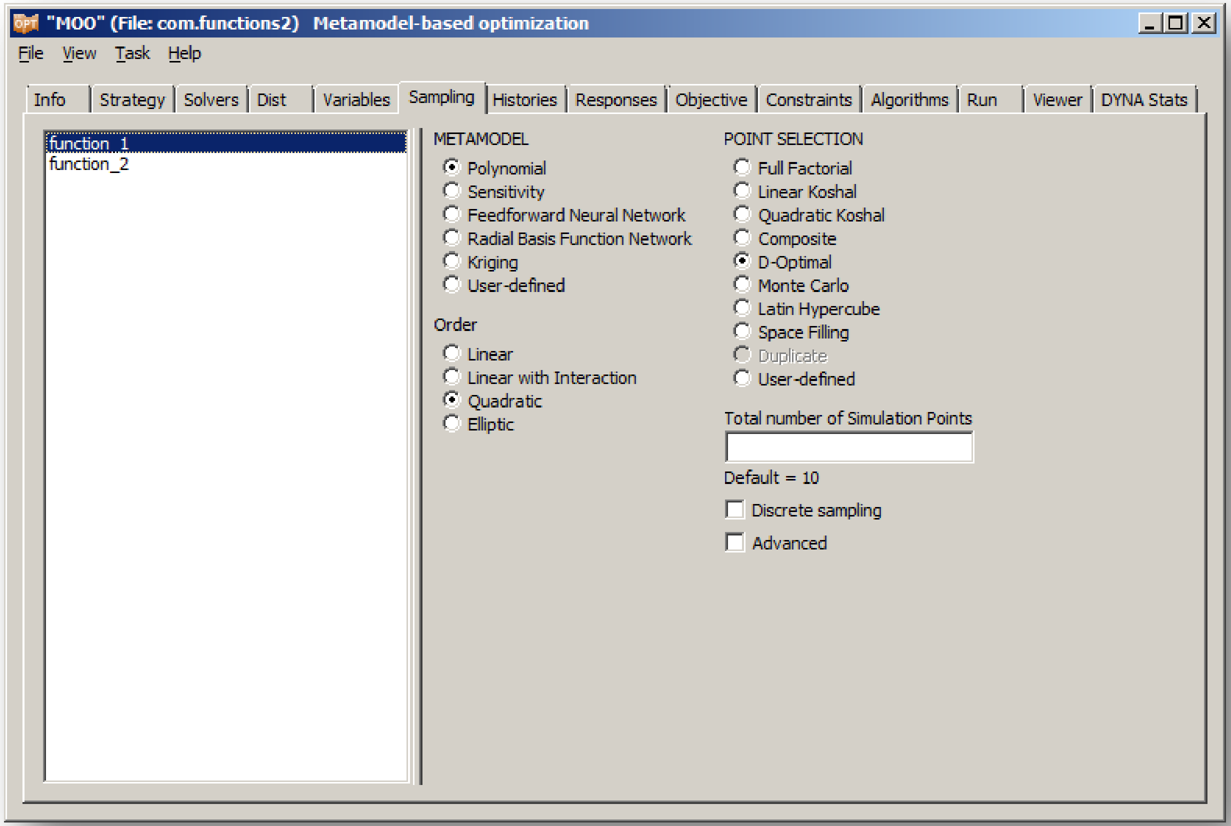
Variables Panel

- Go to Variables panel
- The variables X and Y should exist
- Leave the min and max bounds as they are



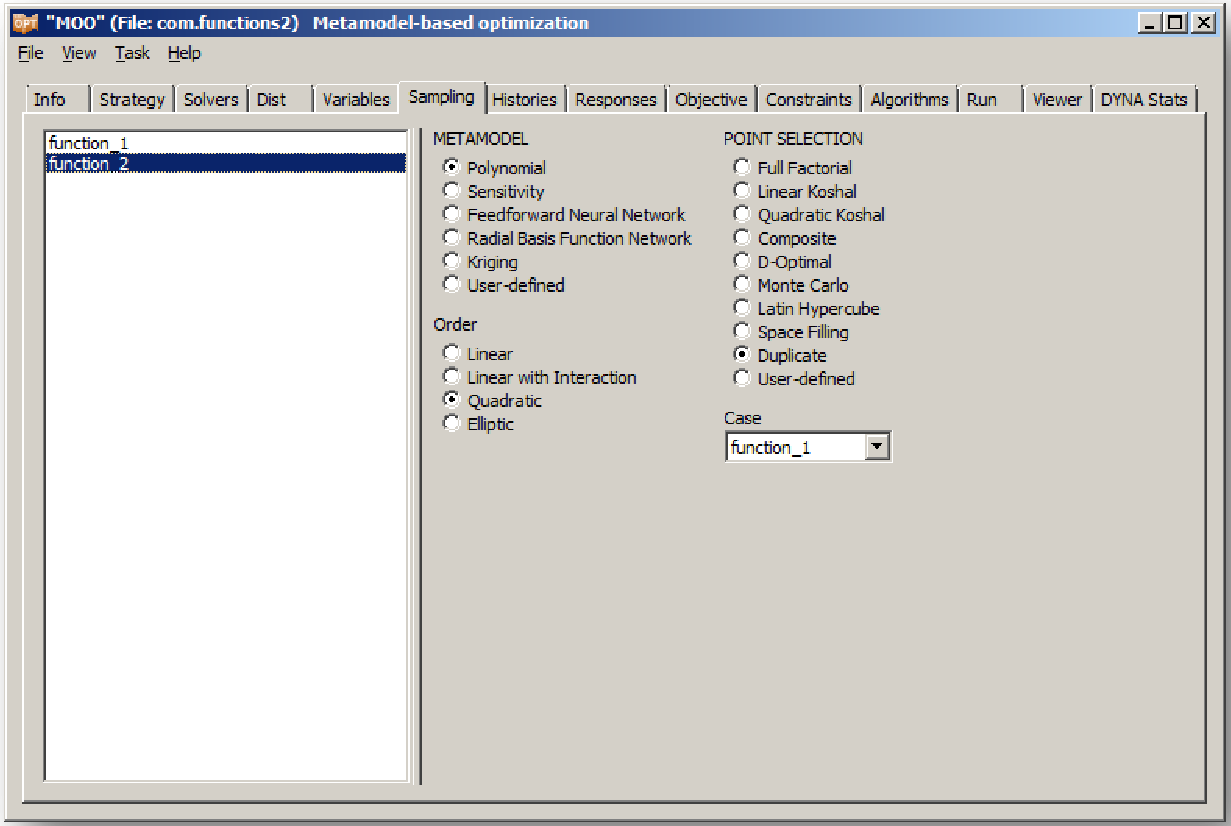
Sampling Panel

- Go to Sampling panel
- For `function_1` choose Quadratic Polynomial Metamodel with D-Optimal Point Selection method



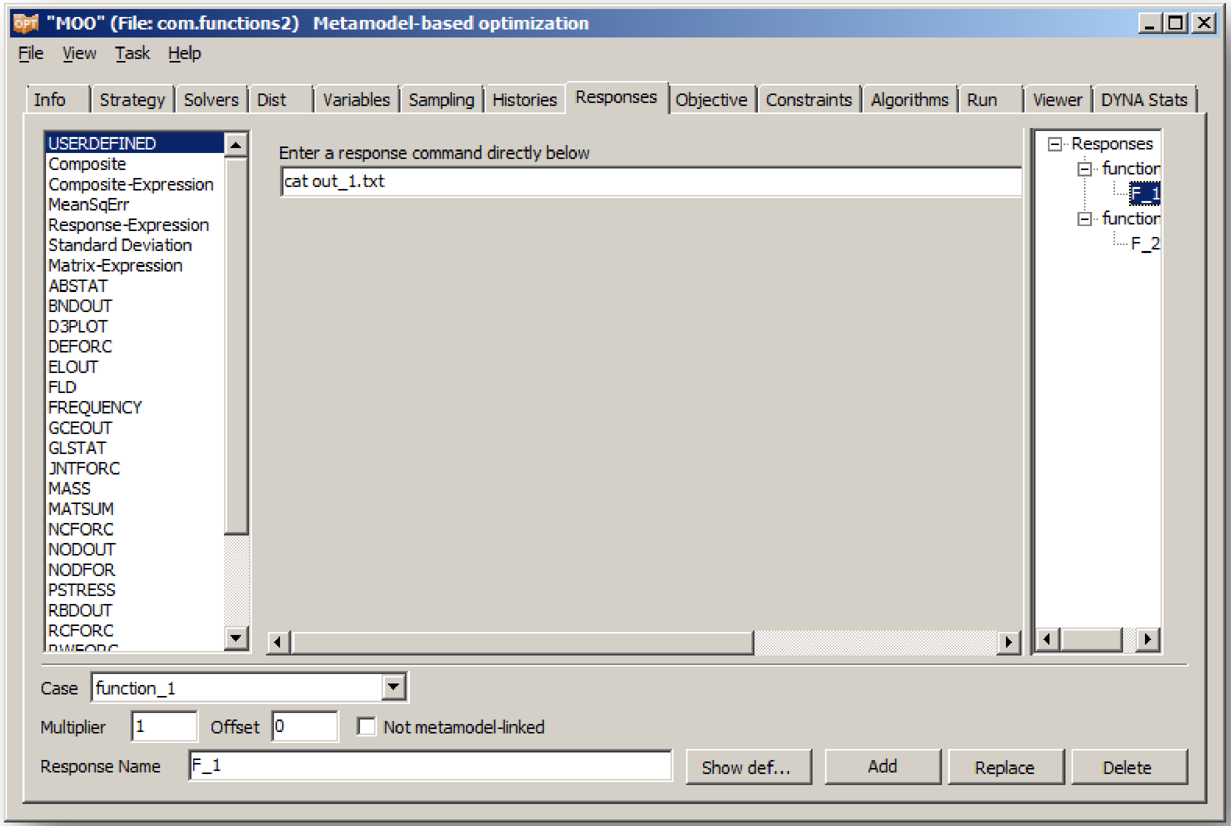
Sampling Panel

- For `function_2` Analysis Case also choose Polynomial Quadratic Metamodel
- For Point selection choose Duplicate and from drop down menu select Case `function_1`



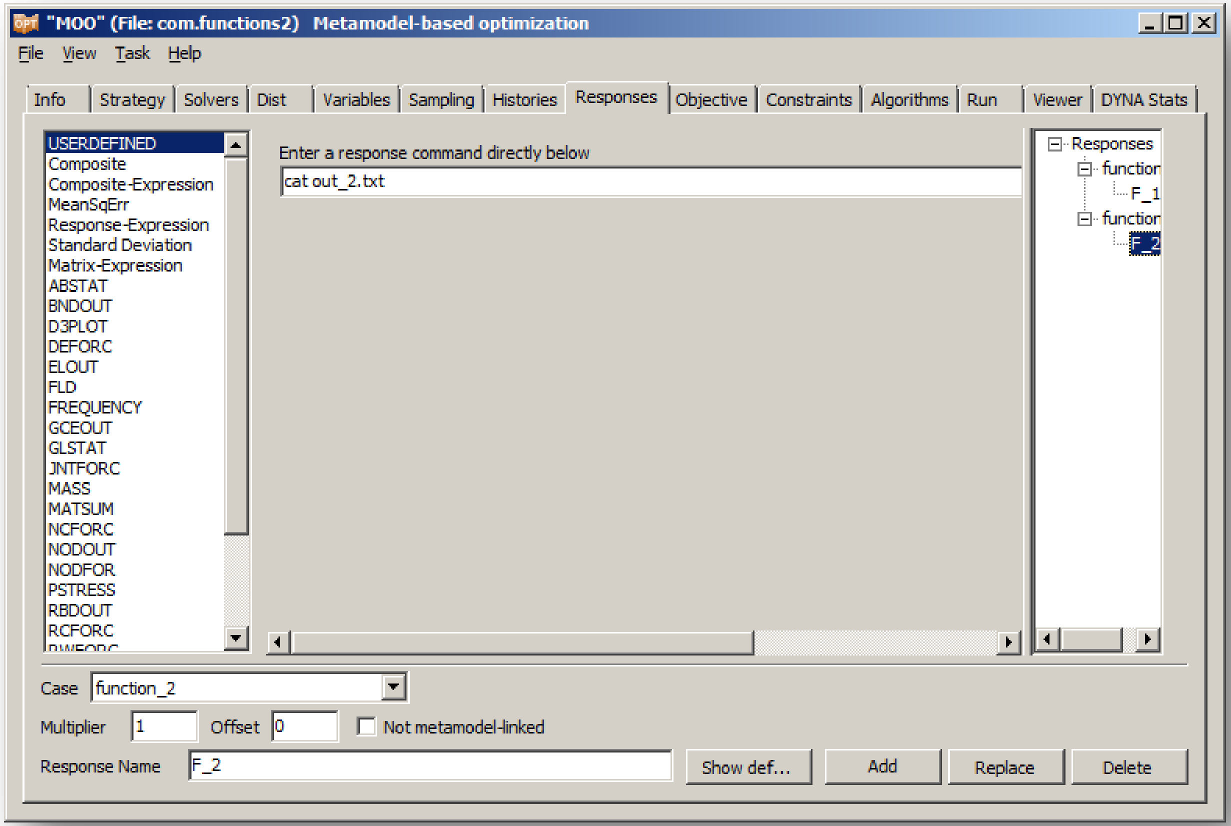
Responses Panel

- Go to Responses panel
- Response `F_1` for case `function_1` should exist



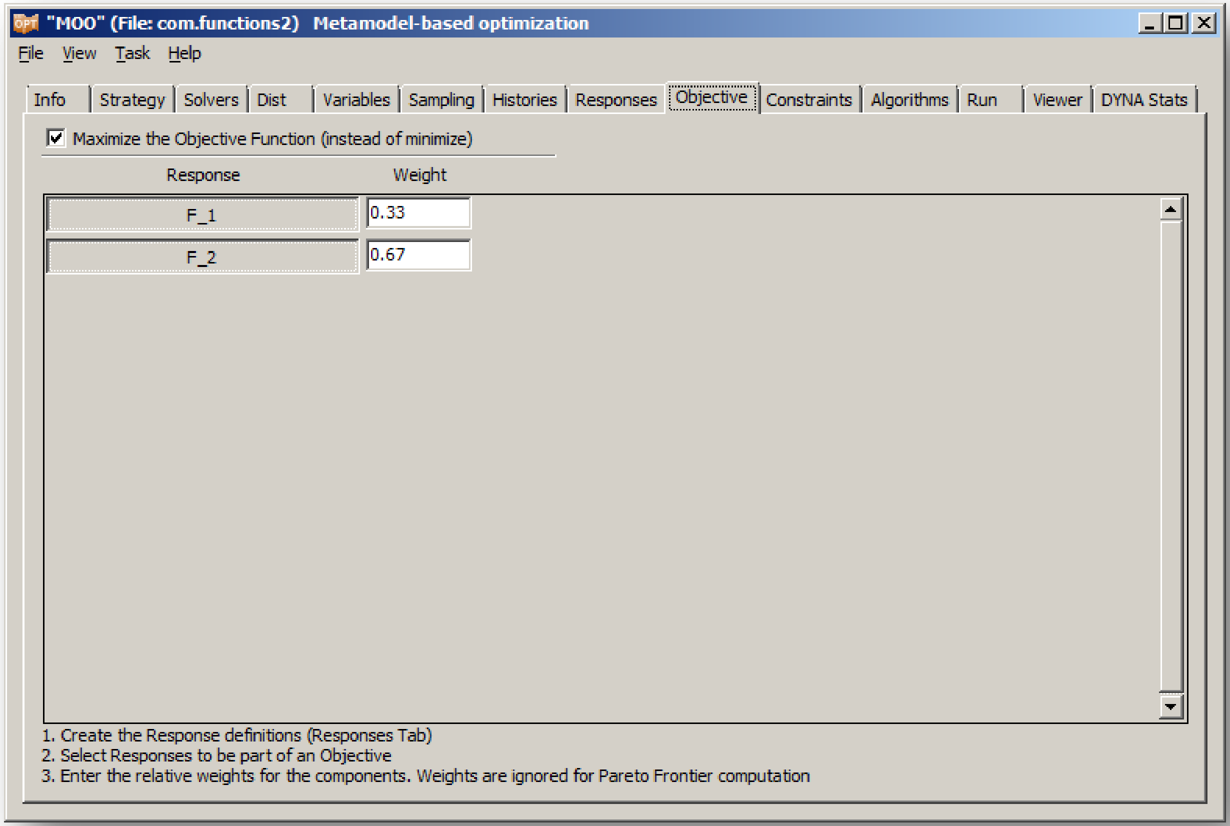
Responses Panel

- Select Case `function_2`
- Enter a response command "`cat out_2.txt`"
- Enter `F_2` in Response Name field and press Add



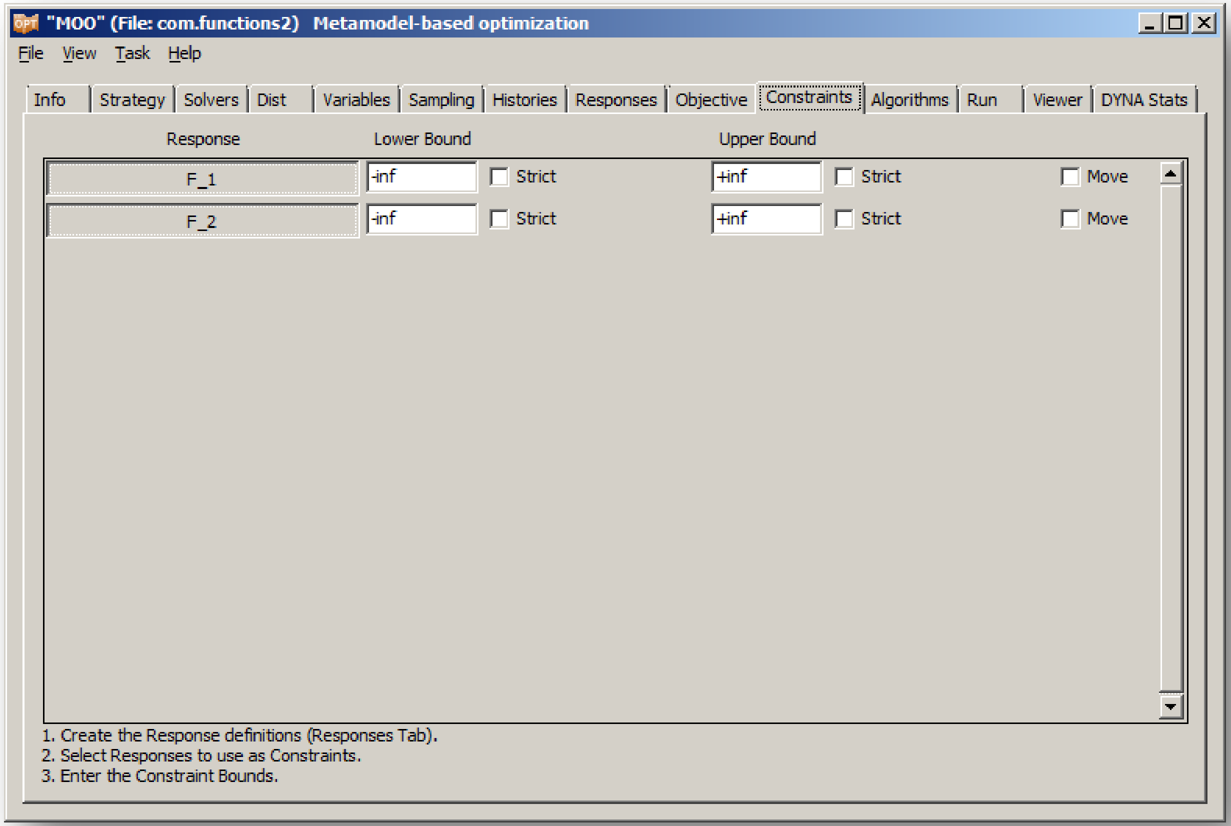
Objective Panel

- Go to Objective Panel
- Select F_1 and F_2 responses for objective
- Assign weight 0.33 to F_1 and 0.67 to F_2 for the multi-objective case
- Note: weights are ignored for Pareto Front computation



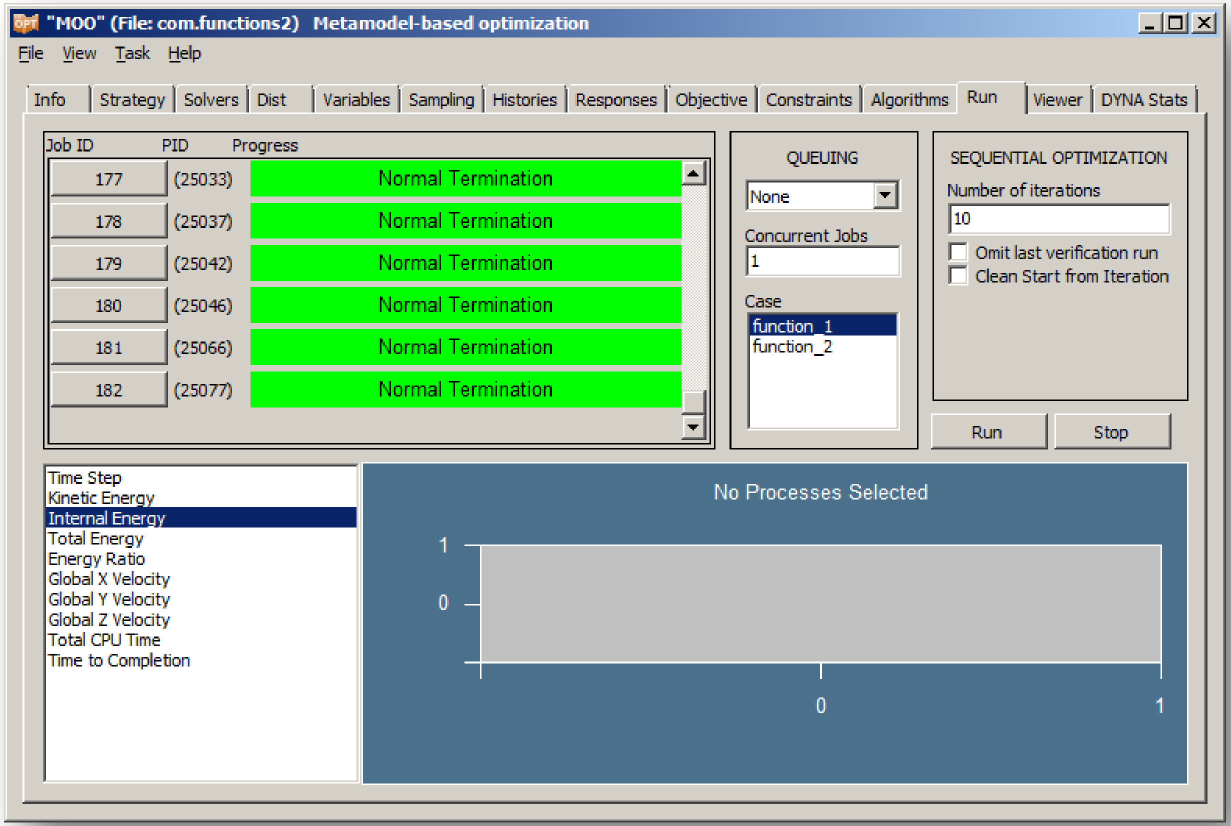
Constraints Panel

- There is no constraints on the functions
- Leave the default infinite bounds for the Responses



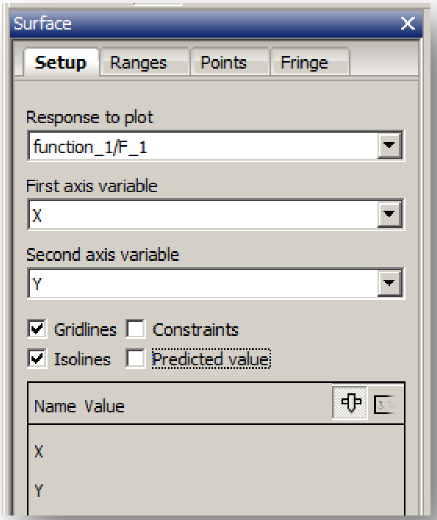
Run Panel

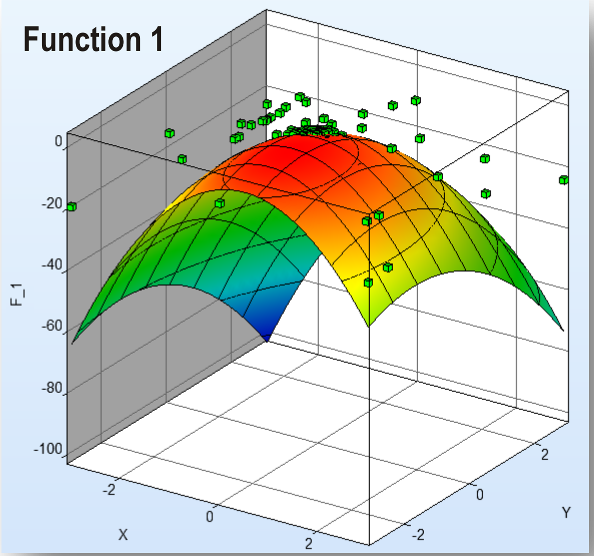
- Go to Run Panel
- Type 10 for Iteration Number
- With None as a Queuing software and one Concurrent Job press Run
- 182 calculations should be performed

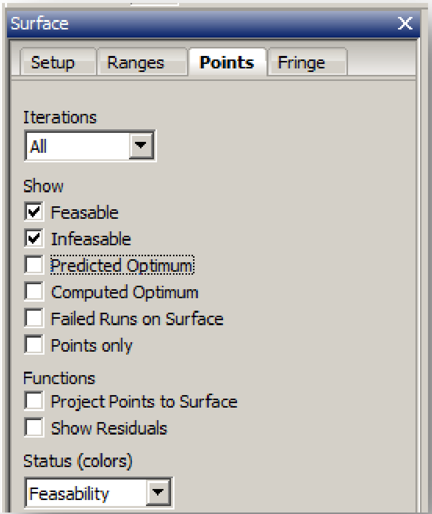


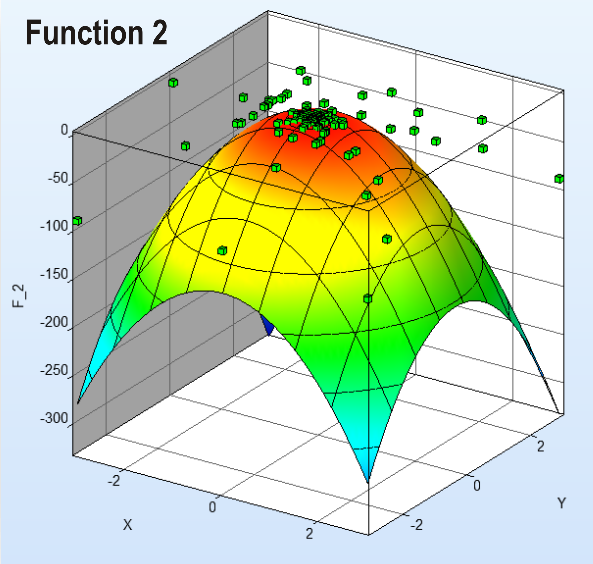
Viewer - Metamodel

- Go to Viewer panel and select Surface from Metamodel menu
- In the Points tab select All Iterations
- Check Feasible and Infeasible Points
- For colors choose Feasibility



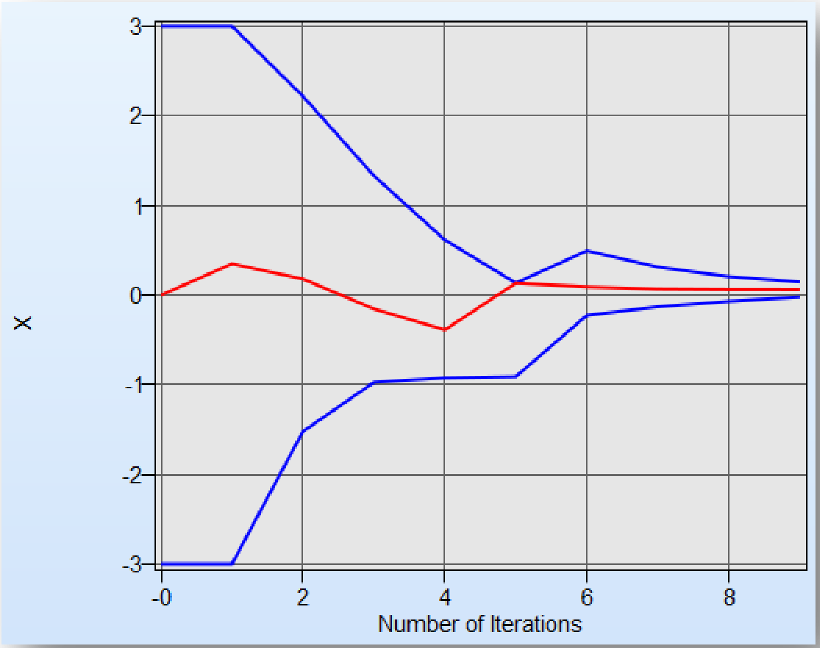


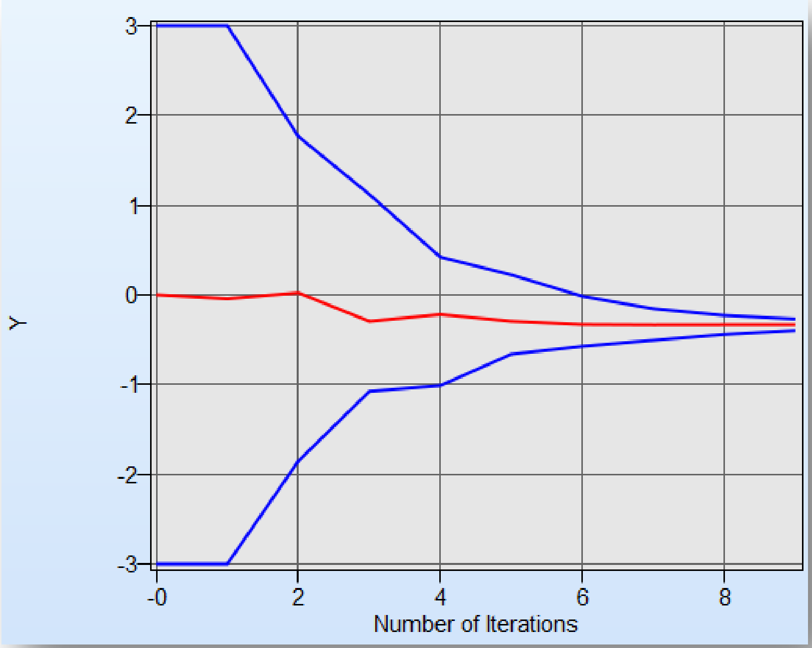




Viewer Panel

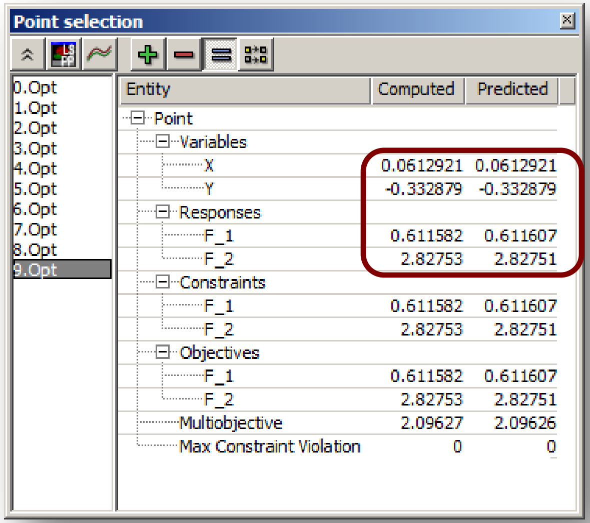
- Go to Viewer panel and press Restart Viewer
- Select Optimization History from the Optimization menu
- See the histories for design variables
- Select the last iteration with clicking close to the plot
- Point selection window should appear

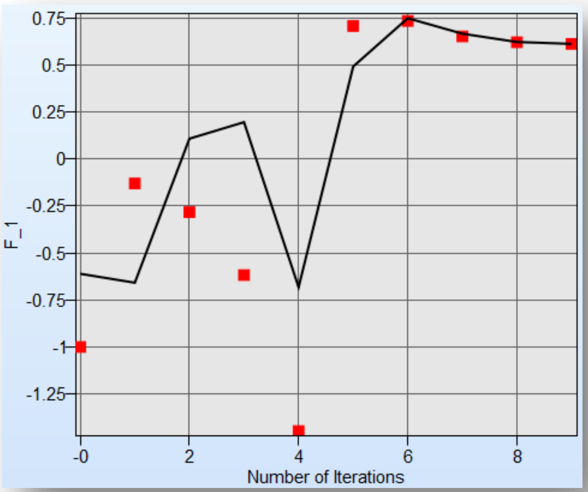


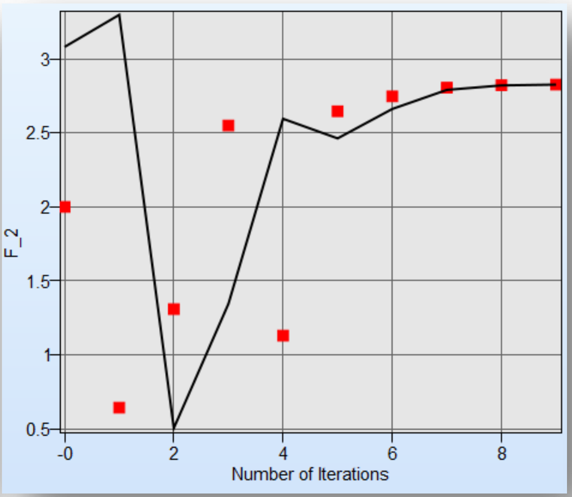


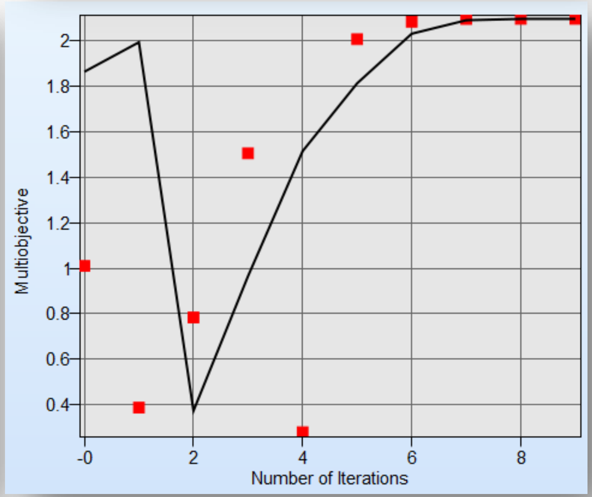
Optimization History

- Check the values of variables for optimal point
- Are they different from the single case problem?
- In the Setup window select Objectives and see the history of F_1, F_2 and Multiobjective function
- The Multiobjective is more influenced by second objective (bigger weight)



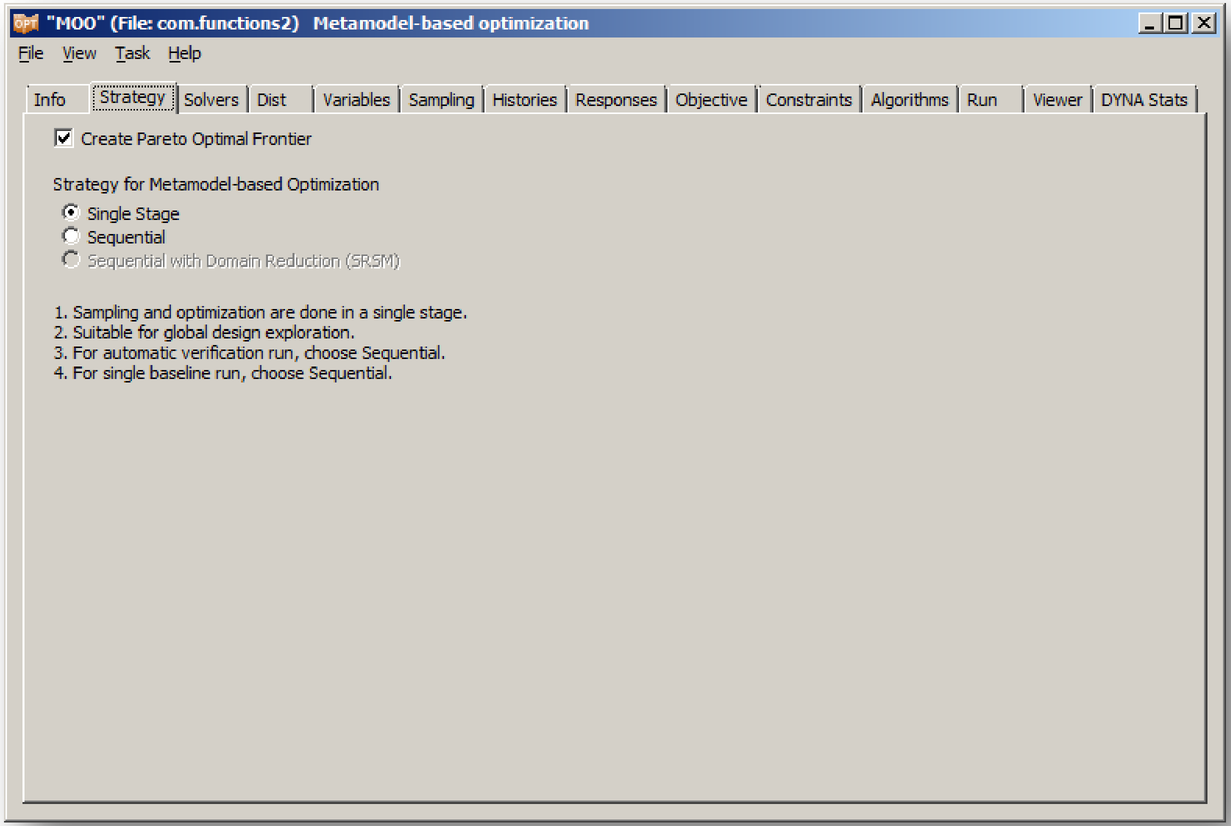






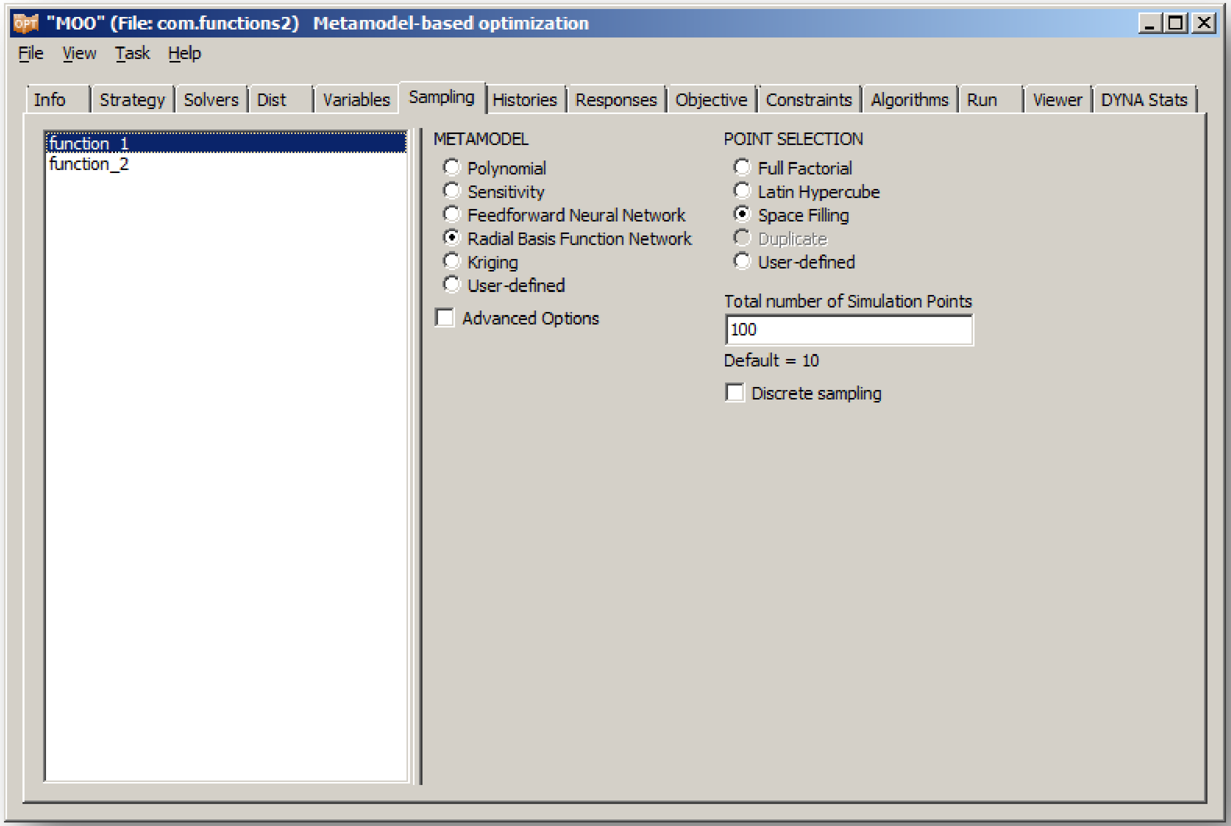
Pareto Front

- Go back to Strategy panel
- Select Single Stage Strategy
- Check the option Create Pareto Optimal Frontier
- Make sure that In Algorithms Tab GA is selected now



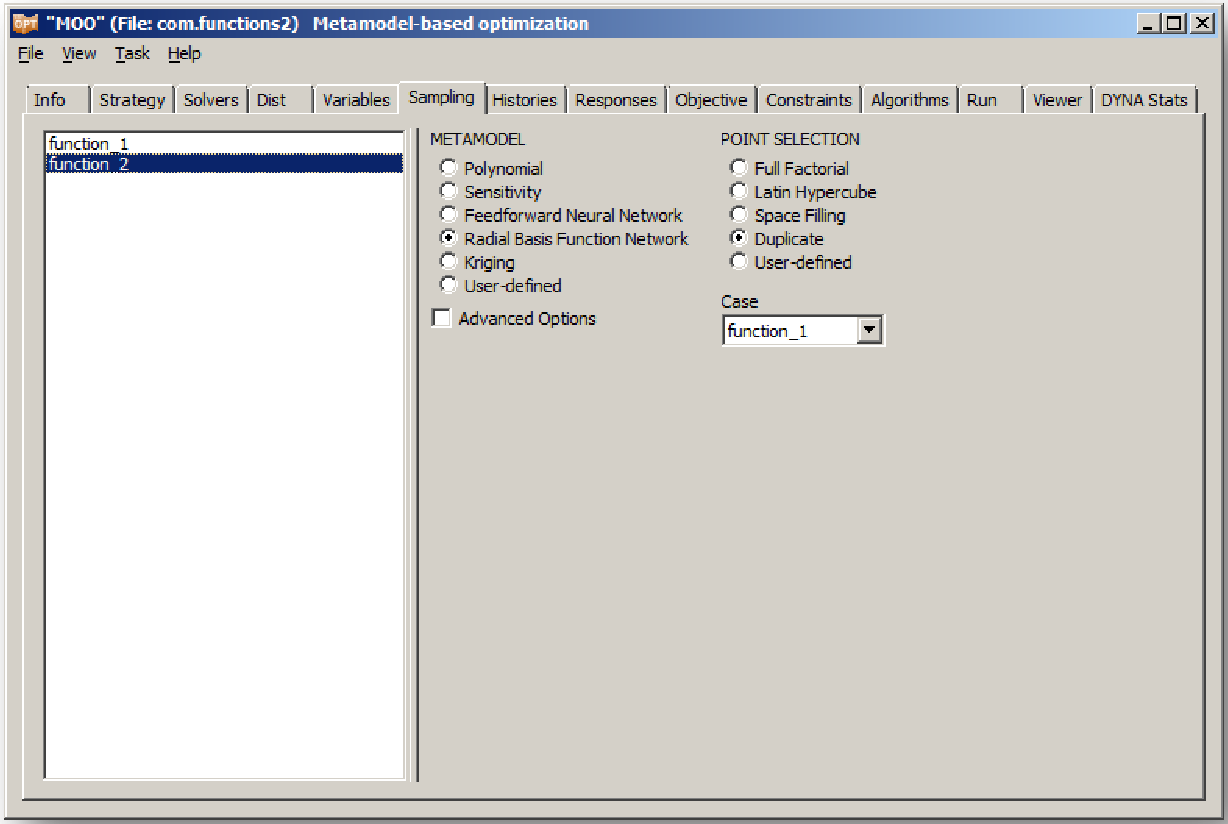
Sampling Panel

- In the Sampling panel make sure that Metamodel has automatically changed to Radial Basis Function Network with Space filling Point Selection for `function_1` case
- Type `100` for the Total number of Simulation Points



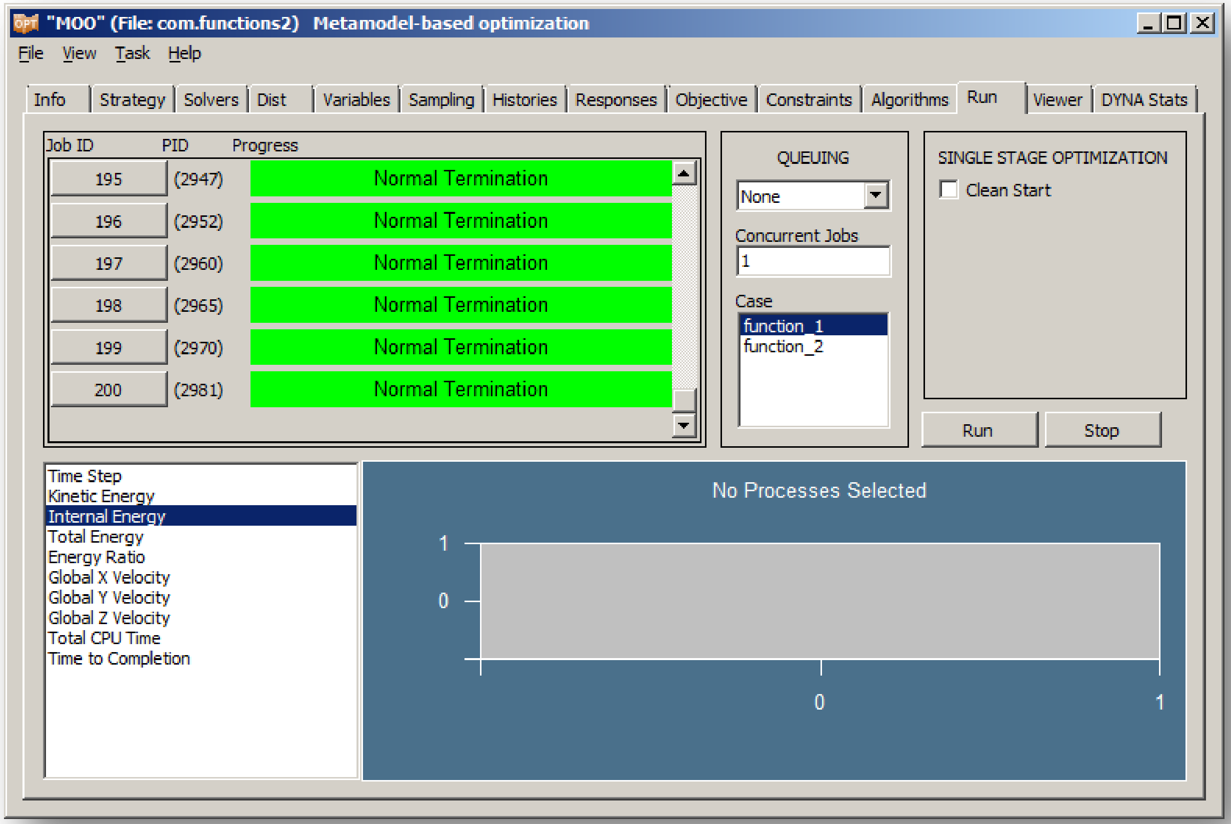
Sampling Panel

- For `function_2` case make sure that the points are duplicated from case 1



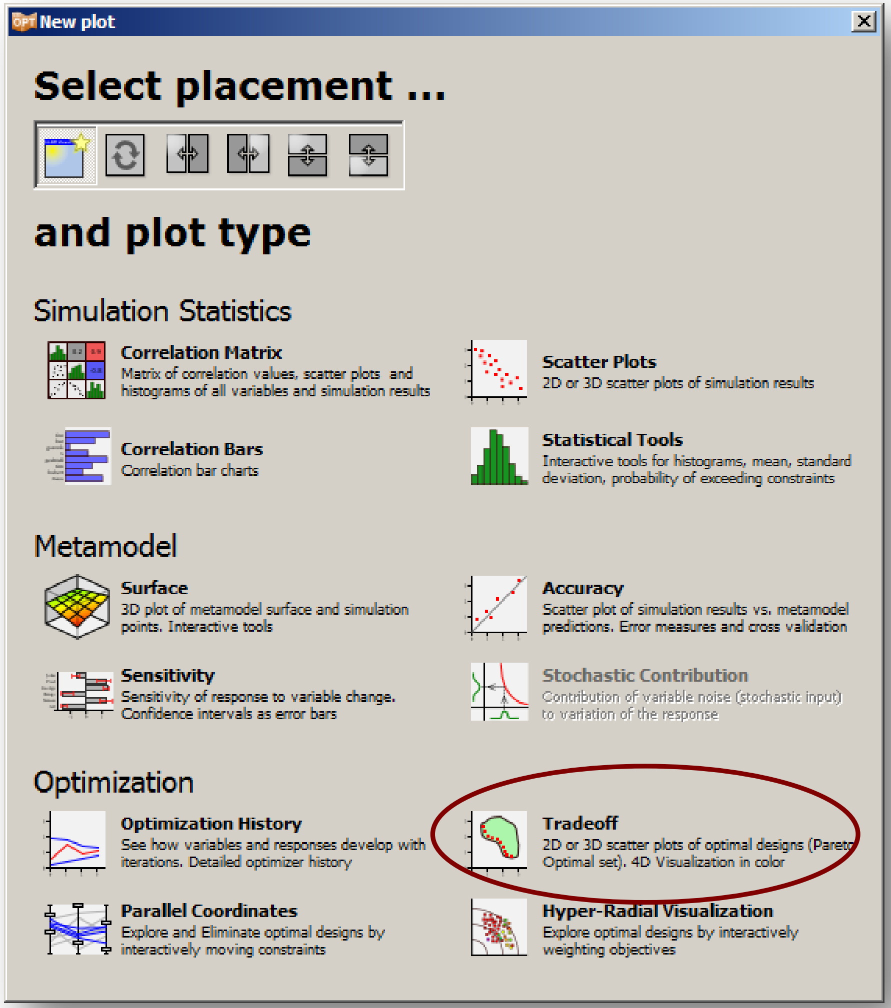
Run Panel

- Go to Run panel
- Hit Run and wait for `200` jobs to be finished



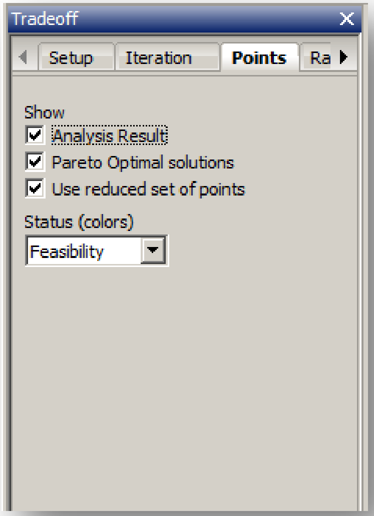
Viewer

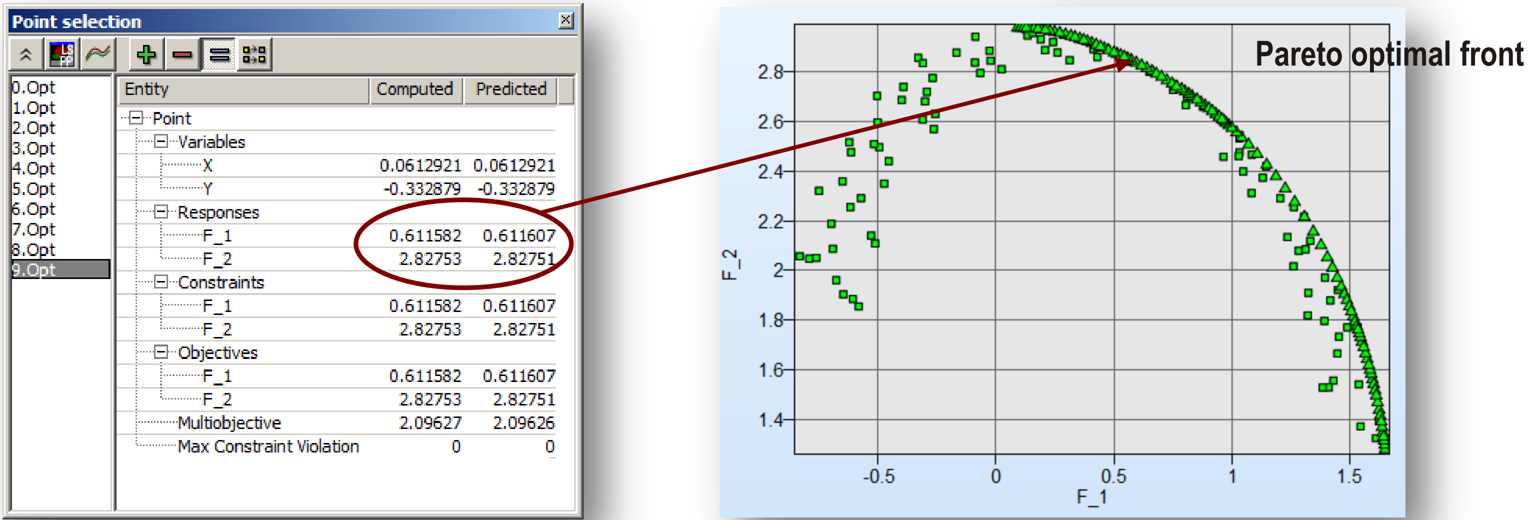
- Go to Viewer panel
- From Optimization select Tradeoff



Tradeoff study

- In the Tradeoff tab go to Points
- Check Analysis Results, Pareto Optimal solution
- For Color leave Feasibility
- Locate the “optimal solution” from Sequential Analysis in the Pareto Front
- Note: weights are ignored here





Tradeoff study - Pareto Front

- In the Tradeoff tab go to Setup
- Select 3D from the menu
- For X-axis select X variable and for Y-axis select Y variable
- For Z-Axis select Responses – F_1